Reliability of Structures – Part 8

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Baysian Methods

- Baysian updating
- Bayes theorem
- Practical application

Bayesian Updating

- Based on the Bayes theorem
- Used when the available information (prior information) is updated as a result of additional information (posterior information).
- Example :

  - Strength of a structural element can be evaluated from the available statistical data.

  - On-site test results can be used to update the strength model.

Bayes Theorem

- Consider k events : A_1, A_2, …, A_k such that

$$A_1 \cup A_2 \cup \ldots \cup A_k = S$$

where S = sample space, and

$$A_i \cap A_j = \Phi \quad \text{for all } i \neq j$$

Then, for any $E \subset S$

$$P(A_j | E) = \frac{P(A_j) P(E | A_j)}{\sum_{i=1}^{k} P(A_i) P(E | A_i)} \quad \text{for } j = 1 \ldots k$$

where $P(A_i)$ = prior probability

Application of Baysian Theorem

- Let A = strength of a structural member (random variable)

- For simplicity, it is assumed that A can take only n values:

  $A = (a_1, a_2, \ldots, a_n)$

- Prior probabilities of occurrence of $(a_1, a_2, \ldots, a_n)$ are $p_1, p_2, \ldots, p_n$ or

$$\text{Prob}\ (a_i) = p_i$$

Additional Measurements

- Additional measurements are performed to improve the estimate of the strength of the member, A′
- The result of these measurements is one of the following values

$$A' = a'_1, a'_2, \ldots, a'_n$$

- However, the results of additional measurements can still involve some uncertainty. It is assumed that the accuracy of these estimates can be verified by more accurate methods.

Application of Baysian Theorem

It is assumed that the following probabilities are available (from the past experience, and/or using more accurate methods)

$$e_{ji} = P\left(a'_j \middle| a_i\right)$$

Application of Baysian Theorem

e_{ij} = Probability of correct estimation of the actual strength a_j, given the estimated strength is a'_i

	a_1	a_2	…	a_i	…	a_n
a'_1	e_{11}	e_{12}	…	e_{1i}	…	e_{1n}
a'_2	e_{21}	e_{22}	…	e_{2i}	…	e_{2n}
…	…	…	…	…	…	…
a'_i	e_{i1}	e_{i2}	…	e_{ii}	…	e_{in}
…	…	…	…	…	…	…
a'_n	e_{n1}	e_{n2}	…	e_{ni}	…	e_{nn}

Application of Baysian Theorem

The posterior probability A = a_j, given the test result A′ = a'_i, is given by the Bayesian formula:

$$P\left(a_i \middle| a'_j\right) = \frac{p_i e_{ji}}{\sum_{k=1}^{n} p_k e_{jk}}$$

where

$$\mathrm{e}_{\mathrm{ji}} = \mathrm{P}\left(\mathrm{a'}_{\mathrm{j}} \middle| \mathrm{a}_{\mathrm{i}}\right)$$

Example

- Consider a steel beam.
- Some corrosion was observed.
- Need to determine the actual shear strength of the web.

- The strength can take on the following 5 values:
  R_v, 0.9 R_v, 0.8 R_v, 0.7 R_v and 0.6 R_v.

	R_v	0.9 R_v	0.8 R_v	0.7 R_v	0.6 R_v
Prior Probability	0	0.15	0.30	0.40	0.15

- Calculations are performed using e_{ij}, the probability of additional tests resulting in $a'_{I,}$ given the actual strength is $a_{j.}$

- Posterior probabilities are calculated using Bayesian Formula

Example

- Matrix of estimates (given):

$$\begin{array}{cc|ccccc}
 & & \multicolumn{5}{c}{a_i} \\
 & & R_v & 0.9\,R_v & 0.8\,R_v & 0.7\,R_v & 0.6\,R_v \\
\hline
 & 0.6\,R_v & 0 & 0 & 0.05 & 0.15 & 0.70 \\
 & 0.7\,R_v & 0 & 0.05 & 0.20 & 0.75 & 0.25 \\
a'_i & \boxed{0.8\,R_v} & 0.10 & 0.25 & 0.70 & 0.10 & 0.05 \\
 & 0.9\,R_v & 0.30 & 0.65 & 0.05 & 0 & 0 \\
 & R_v & 0.60 & 0.05 & 0 & 0 & 0 \\
\hline
\end{array}$$

Measured (or observed)

Example

- The posterior probability of strength = 0.6 R_v, given

  tests showed A′ = 0.8 R_v

$$P\left(0.6R_v \middle| 0.8R_v\right) = \frac{(0.15)(0.05)}{0.295} = 0.025$$

because

$$\sum_{k=1}^{n} p_k e_{jk} = (0)(0.10) + (0.15)(0.25) + (0.3)(0.7) +$$
$$(0.4)(0.10) + (0.15)(0.05) = 0.295$$

Example

- Similarly the calculations are performed for other a_i and a'_i

	Posterior	Prior
$P(0.6\ R_v \| 0.8\ R_v) =$	0.025	0.15
$P(0.7\ R_v \| 0.8\ R_v) =$	0.14	0.40
$P(0.8\ R_v \| 0.8\ R_v) =$	0.71	0.30
$P(0.9\ R_v \| 0.8\ R_v) =$	0.13	0.15
$P(1.0\ R_v \| 0.8\ R_v) =$	0	0