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GENERALIZATION OF UNIQUE GLOBAL AND LOCAL INITIAL IMPERFECTION USED IN EN 1993-1-1 AND EN 1999-1-1

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Abstract. *Derivation of the basic formulae for determination of the flexural buckling resistance of frames with members with non-uniform cross-sections and non-uniform axial compression forces. Generalization of procedure given in EN 1993-1-1 in cl. 5.3.2(11) which is limited to the frames with uniform cross-sections and compression forces. Detailed description of the procedure using iterative calculation. Numerical examples for uniform member. Comparison results with ω-method ones.*

1 FLEXURAL BUCKLING RESISTANCE OF FRAMES WITH NON-UNIFORM MEMBERS AND NON-UNIFORM NORMAL FORCE DISTRIBUTION

Flexural buckling resistance of the frame, which consists of members with variable cross-sections, with any boundary conditions, supports and/or variable foundation and under variable axial forces may be verified by the following condition

$$\left(\frac{N_{Ed}(x)}{N_{Rd}(x)} + \frac{M^{II}_{Ed,ugli}(x)}{M_{Rd}(x)} \right)_{\max} \leq 1 \tag{1}$$

where

$N_{Ed}(x)$ is the axial force distribution, positive if compression. The design values of the axial forces may be calculated by the 1st order theory,

$M^{II}_{Ed,ugli}(x)$ is the bending moment distribution, which is the result of axial forces acting in members of frame having "unique global and local initial" („ugli") imperfection. The design values of this bending moment shall be calculated by the 2nd order theory. The „ugli" imperfection is an equivalent geometrical imperfection, which purpose is to cover in numerical model all imperfections (geometrical and structural) of real structure.

$N_{Rd}(x)$ is the distribution of the axial force resistance depending on the cross-section class,

$M_{Rd}(x)$ is the distribution of the bending moment resistance depending on the cross-section class.

The characteristics relating to critical cross-section, which is the cross-section relevant for assessment of flexural buckling resistance of the frame, are below denoted by index „m“. The most onerous condition (1) occurring in critical cross-section „m“, may be then rewritten in the form

$$\frac{N_{Ed,m}}{N_{Rd,m}} + \frac{M^{II}_{Ed,ugli,m}}{M_{Rd,m}} \leq 1 \tag{2}$$

The “ugli” imperfection is defined as follows

$$\eta_{ugli}(x) = \eta_{0,ugli,m}\eta_{cr}(x) \tag{3}$$

where

$\eta_{cr}(x)$ is the first elastic critical buckling mode, with the amplitude $\left|\eta_{cr}(x)\right|_{\max} = 1$.

$\eta_{0,ugli,m}$ is the amplitude of „ugli“ imperfection depending on characteristics of critical cross-section „m“. Index „0“ will in this paper indicate that a value is amplitude of a function. The amplitude of „ugli“ imperfection may be determined from the condition requiring the following: the critical member of the frame, when under compression axial force, should have the same flexural buckling resistance as its “generalised equivalent member” („gem“). The „gem“ is the member simply supported on its ends, having the same cross-section properties (EI_m, A_m) and axial force ($N_{Ed,m}$) as the critical member of the frame in its critical cross-section „m“, and having such buckling length L_{cr}, that its elastic critical axial force is the same as the elastic critical axial force $N_{cr,m}$ of the critical member of the frame in its critical cross-section „m“. The „ugli“ imperfection amplitude depending on the characteristics of the critical cross-section „m“ is then defined by

$$\eta_{0,ugli,m} = \frac{N_{cr,m}e_{0,d,m}}{EI_m\left|\eta''_{cr,m}\right|} = \alpha_{cr}\frac{N_{Ed,m}e_{0,d,m}}{\left|M_{\eta cr,m}\right|} \tag{4}$$

$$e_{0,d,m} = e_{0,k,m}\frac{1-\dfrac{\chi_m\bar{\lambda}_m^2}{\gamma_{M1}}}{1-\chi_m\bar{\lambda}_m^2} \tag{5}$$

$$e_{0,k,m} = \alpha_m\left(\bar{\lambda}_m - 0.2\right)\frac{M_{Rk,m}}{N_{Rk,m}}, \text{ for } \bar{\lambda}_m > 0.2 \tag{6}$$

where

$\left|M_{\eta cr,m}\right|$ is the absolute value of the fictitious bending moment at the critical cross-section „m“, due to $\eta_{cr}(x)$,

$N_{Ed,m}$ is the design value of compression axial force at the critical cross-section „m“, positive if compression,

$M_{Rk,m}$ is the characteristic value of bending moment resistance of the critical cross-section „m“,

$N_{Rk,m}$ is the characteristic value of axial force resistance of the critical-cross section „m“,

$e_{0,d,m}$, $e_{0,k,m}$ are the design (index „d“) and characteristic (index „k“) values of amplitude of “local initial” („li“) imperfection of the “gem” depending on the characteristics of the critical cross-section „m“. It can be easily shown, that „li“ imperfection is used when analysis of individual member is done,
α_m is the imperfection factor for the critical cross-section „m“ and the relevant buckling curve, see Table 6.1 and Table 6.2 in [1],
γ_{M1} is the partial factor, which should be applied to the various characteristic values of resistance of members to instability,

$$\bar{\lambda}_m = \sqrt{\frac{N_{Rk,m}}{N_{cr,m}}} \tag{7}$$

$\bar{\lambda}_m$ is the relative slenderness of the structure, relating to the critical cross-section „m“,
χ_m is the reduction factor depending on the relevant imperfection factor α_m and the relative slenderness $\bar{\lambda}_m$, see 6.3.1 in [1],
$N_{cr}(x)$ is the distribution of elastic critical force,
α_{cr} is the minimum force amplifier for the values of the axial force configuration $N_{Ed}(x)$ in members to reach the values of elastic critical force configuration $N_{cr}(x)$. For the given frame, α_{cr} is a constant. The ratio $\alpha_{cr} = N_{cr}(x)/N_{Ed}(x)$ gives for all cross-sections „x“ the same numerical value.

The location of the critical cross-section „m“ is generally not known, because it depends on the location of maximum of the sum of two functions in the left side of the condition (1). The value of the second term of the sum in (1) depends on the characteristics of the critical cross-section „m“. The location of the maximum of the first function in (1): $N_{Ed}(x)/N_{Rd}(x)$ usually does not coincide with the location of maximum of the second function in (2): $M^{II}_{Ed,ugli}(x)/M_{Rd}(x)$, which is given by the location of the maximum of the function $|\eta''_{cr}(\mathrm{x})/I(x)|_{\max}$. Generally it is therefore necessary to use iterative calculation. In the special case, when $N_{Ed}(x)/N_{Rd}(x)$ is constant, the location of critical cross-section „m“ is determined by the location of $\left|M^{II}_{Ed,ugli}(x)/M_{Rd}(x)\right|_{\max}$ or $|\eta''_{cr}(\mathrm{x})/I(x)|_{\max}$ and when also $EI(x)$ is constant, by the location of $|\eta''_{cr}(\mathrm{x})|_{\max}$.

Distribution of bending moment $M^{II}_{Ed,ugli}(x)$, which is the effect of axial forces acting in members of frame having the „ugli“ imperfection $\eta_{ugli}(x) = \eta_{0,ugli,m}\eta_{cr}(x)$, may be calculated in the following way:
1) The first eigen-value α_{cr} and the first buckling mode $\eta_{cr}(x)$ and its derivates $\eta'_{cr}(x)$ and $\eta''_{cr}(x)$ are obtained by numerical methods using a computer program.
2) The „ugli“ imperfection amplitude $\eta_{0,ugli,m}$ depending on the characteristics of the critical cross-section „m“ is calculated for the estimated location of the critical cross-section „m“

$$\eta_{0,ugli,m} = \alpha_{cr}\frac{N_{Ed,m}e_{0,d,m}}{\left|M_{\eta cr,m}\right|} \tag{8}$$

3) The distribution of the “ugli” imperfection is then

$$\eta_{ugli}(x) = \eta_{0,ugli,m}\eta_{cr}(x) \tag{9}$$

4) The amplitude $\eta_{0,m}$ of the additive deflection $\eta(x)$, which is the result of axial forces acting in the members of frame with „ugli“ imperfection, may be calculated as

$$\eta_{0,m} = \frac{\eta_{0,ugli,m}}{\alpha_{cr} - 1} \tag{10}$$

5) The distribution of the additive deflection $\eta(x)$ is then

$$\eta(x) = \eta_{0,m}\eta_{cr}(x) = \frac{\eta_{0,ugli,m}}{\alpha_{cr} - 1}\eta_{cr}(x) \tag{11}$$

6) The distribution of the bending moment $M^{II}_{Ed,ugli}(x)$ due to „ugli“ imperfection having shape of $\eta_{cr}(x)$ may be calculated from the formula

$$M^{II}_{Ed,ugli}(x) = -EI(x)\eta''(x) = -EI(x)\frac{\eta_{0,ugli,m}}{\alpha_{cr} - 1}\eta''_{cr}(x) = kN_{Ed,m}e_{0,d,m}\frac{-EI(x)\eta''_{cr}(x)}{\left|M_{\eta cr,m}\right|} \tag{12}$$

where
k is the well known ratio of the bending moment calculated according to the 2nd order theory to the bending moment calculated according to the 1st order theory, which is in the case of using elastic critical buckling mode $\eta_{cr}(x)$ constant value for the whole frame

$$k = \frac{\alpha_{cr}}{\alpha_{cr} - 1} = \frac{1}{1 - \dfrac{1}{\alpha_{cr}}} \tag{13}$$

It may be also written

$$\begin{aligned} M^{II}_{Ed,ugli}(x) &= kN_{Ed,m}e_{0,d,m}\frac{-EI(x)\eta''_{cr}(x)}{EI_m\left|\eta''_{cr,m}\right|} = kN_{Ed,m}e_{0,d,m}\frac{-EI(x)\eta''_{C,cr}(x)}{EI_m\left|\eta''_{C,cr,m}\right|} = \\ &= kN_{Ed,m}e_{0,d,m}\frac{M_{\eta cr}(x)}{\left|M_{\eta cr,m}\right|} = M^{II}_{0,Ed,ugli,m}\frac{M_{C\eta cr}(x)}{\left|M_{C\eta cr,m}\right|} \end{aligned} \tag{14}$$

where

$$\eta_{C,cr}(x) = C_0\eta_{cr}(\mathrm{x}) \tag{15}$$

is the first elastic critical buckling mode with amplitude C_0, which may have any numerical value, and

$$M^{II}_{0,Ed,ugli,m} = kN_{Ed,m}e_{0,d,m} \tag{16}$$

From (14) it may be seen that, the first elastic critical buckling mode $\eta_{C,cr}(x)$ with any value of the amplitude C_0 may be used, and not only $\eta_{cr}(\mathrm{x})$ having $C_0 = 1$, when computing ratio of bending moments

$$M_{\eta cr}(x)/\left|M_{\eta cr,m}\right| . \tag{17}$$

7) After the distribution of the function on the left side of the condition (1) is known, the condition (2) may be evaluated and checked, if the location of maximum of this function will coincide with estimated location of the critical cross-section „m“ from the first iteration. If the answer is no, the procedure shall be repeated in an iterative way.
8) If the answer is yes, the condition (2) may be evaluated and the frame verified.

The proposed procedure was first time published in [2] and was verified by calculating of several numerical examples [3-8]. Prof. Chladný derived $e_{0,d}$ and was the first who generalized the method given in 5.3.2(11) of EN 1993-1-1 [1] also for non-uniform cross-sections and non-uniform compression forces. He applied it in design of bridges in practice, e.g. in design of basket handle arch type Apollo bridge in Bratislava and in investigations of continuous truss bridges [9-11]. The derivation of basic formulae used in this paper differs from the ones published by Prof. Chladný. His method was accepted in 5.3.2(11) of EN 1999-1-1 [12].

2 NUMERICAL EXAMPLE

Given input values: uniform member with cross-section HE 260 B (ARBED), steel grade S 355, partial safety factor $\gamma_{M1} = 1.1$. The member length $L = 4.6m$. The action: the axial normal force N_{Ed} which equals to the member resistance. For $N_{Ed} = N_{b,Rd}$ the utility factor $U = 1.0$. The flexural buckling about minor axis z-z is investigated (buckling curve „c“, $\alpha = 0.49$).
Material properties

$$f_y = 355\,MPa, \quad \gamma_{M1} = 1.1, \quad f_{y,d} = f_y / \gamma_{M1} = 322.727\,MPa, \quad E = 210\,GPa \tag{18}$$

Properties of cross-section

$$h = 0.26m, \quad b = 0.26m, \quad buckling\ curve\ "c", \quad \alpha_m = 0.49, \quad \bar{\lambda}_0 = 0.2 \tag{19}$$

$$A = 11.84x10^3\,m, \quad I_z = 51.35x10^6\,mm^4, \quad W_{el,z} = 395x10^3\,mm^3 \tag{20}$$

Index “m” denotes that properties relate to the critical section. In this example the member has uniform cross-section, therefore

$$A_m = A, \quad I_{z,m} = I_z, \quad W_{el,z,m} = W_{el,z} \tag{21}$$

The resistances of the critical cross-section “m”

$$N_{Rk} = Af_y = 4203\,kN, \quad N_{Rd,m} = \frac{N_{Rk,m}}{\gamma_{M1}} = 3821\,kN, \quad M_{z,Rk} = W_{el,z} f_y = 140.225\,kNm \tag{22}$$

$$N_{Rk,m} = N_{Rk}, \quad M_{z,Rk,m} = M_{z,Rk} \tag{23}$$

The critical force in the critical cross-section “m”

$$\beta = 0.699, \quad L_{z,cr} = \beta L = 3.216\,m, \quad N_{cr,m} = \frac{\pi^2 EI_{z,m}}{L_{z,cr}^2} = 10290\,kN \tag{24}$$

The buckling resistance in the critical cross-section "*m*"

$$\bar{\lambda}_m = \sqrt{\frac{N_{Rk,m}}{N_{cr,m}}} = 0.639, \quad \Phi_m = 0.5\left[1 + \alpha_m\left(\bar{\lambda}_m - \bar{\lambda}_0\right) + \bar{\lambda}_m^2\right] = 0.812 \tag{25}$$

$$\chi_m = \frac{1}{\Phi_m + \sqrt{\Phi_m^2 - \bar{\lambda}_m^2}} = 0.762, \quad N_{b,Rd,m} = \chi_m N_{Rd,m} = 2911.5kN \tag{26}$$

In the example it is supposed that the value of action equals to the buckling resistance

$$\alpha_{ult,d} = \frac{N_{Ed}}{N_{b,Rd,m}}, \quad \alpha_{ult,d} = 1.0, \quad N_{Ed} = \alpha_{ult,d} N_{b,Rd,m} = 2911.5kN \tag{27}$$

The characteristic and design values of the part of the amplitude of the unique global and local initial (ugli) imperfection with the shape of elastic critical buckling mode depending on the properties of the critical cross-section "*m*". The formula (28) is based on the requirement that the imperfection $\eta_{ugli}(\xi)$ (38) having the shape of the elastic buckling mode $\eta_{cr}(\xi)$ (30) or (33) should have the same maximum curvature as the equivalent uniform member.

$$e_{0,k,m} = \alpha_m\left(\bar{\lambda}_m - \bar{\lambda}_o\right)\frac{M_{z,Rk,m}}{N_{Rk,m}} = 7.179\,mm, \quad e_{0,d,m} = e_{0,k,m}\frac{1 - \dfrac{\chi_m \bar{\lambda}_m^2}{\gamma_{M1}}}{1 - \chi_m \bar{\lambda}_m^2} = 7.473\,mm \tag{28}$$

$$\frac{L}{e_{o,d,m}} = 615.5, \quad \alpha_{cr} = \frac{N_{cr,m}}{N_{Ed}} = 3.534, \quad k = \frac{\alpha_{cr}}{\alpha_{cr} - 1} = 1.395, \quad \varepsilon = \frac{\pi}{\beta} = 4.494, \quad \xi = \frac{x}{L} \tag{29}$$

The shape of the first elastic critical buckling mode

$$\bar{\eta}(\xi) = \varepsilon(1 - \xi) - \varepsilon\cos(\varepsilon\xi) + \sin(\varepsilon\xi) \tag{30}$$

$$\left[\bar{\eta}(\xi)\right]_{\max} \text{ is in the section } \xi_{\max} = \frac{x_{\max}}{L} = \frac{2.768}{4.6} = 0.602 \tag{31}$$

$$\left[\bar{\eta}(\xi)\right]_{\max} = C = \left[\varepsilon(1 - \xi) - \varepsilon\cos(\varepsilon\xi) + \sin(\varepsilon\xi)\right]_{\max} = 6.283 \tag{32}$$

The normalized first elastic critical buckling mode. Note that the normalizing is not necessary to perform. The results will be the same.

$$\eta_{cr}(\xi) = \frac{\varepsilon(1 - \xi) - \varepsilon\cos(\varepsilon\xi) + \sin(\varepsilon\xi)}{C}, \quad \left[\eta_{cr}(\xi)\right]_{\max} = 1.0 \tag{33}$$

The second derivation of the normalized first elastic critical buckling mode

$$\eta_{cr}''(\xi) = \left(\frac{\varepsilon}{L}\right)^2 \frac{\varepsilon\cos(\varepsilon\xi) - \sin(\varepsilon\xi)}{C} \tag{34}$$

$[\eta''_{cr}(\xi)]_{\max}$ is in the critical section "*m*" $\xi_m = \frac{x_m}{L} = \frac{L-0.5L_{cr}}{L} = \frac{2.992}{4.6} = 0.650$ (35)

$$[\eta''_{cr}(\xi)]_{\max} = \eta''_{cr}(\xi_m) = -0.6991\,m^{-1} \tag{36}$$

The amplitude of the unique global and local initial (ugli) imperfection

$$\eta_{0,ugli,m} = \frac{\alpha_{cr} e_{0,d,m} N_{Ed}}{EI_{z,m}\left|-\eta''_{cr}(\xi_m)\right|} = \frac{e_{0,d,m} N_{cr,m}}{EI_{z,m}\left|-\eta''_{cr}(\xi_m)\right|} = 10.201\,mm \tag{37}$$

The unique global and local initial (ugli) imperfection. In the Eurocodes the symbol $\eta_{init}(\xi)$ is used for it.

$$\eta_{ugli}(\xi) = \eta_{0,ugli,m}\eta_{cr}(\xi) = \frac{\alpha_{cr} e_{0,d,m} N_{Ed}}{EI_{z,m}\left|-\eta''_{cr}(\xi_m)\right|}\eta_{cr}(\xi) = \frac{e_{0,d,m} N_{cr,m}}{EI_{z,m}\left|-\eta''_{cr}(\xi_m)\right|}\eta_{cr}(\xi) \tag{38}$$

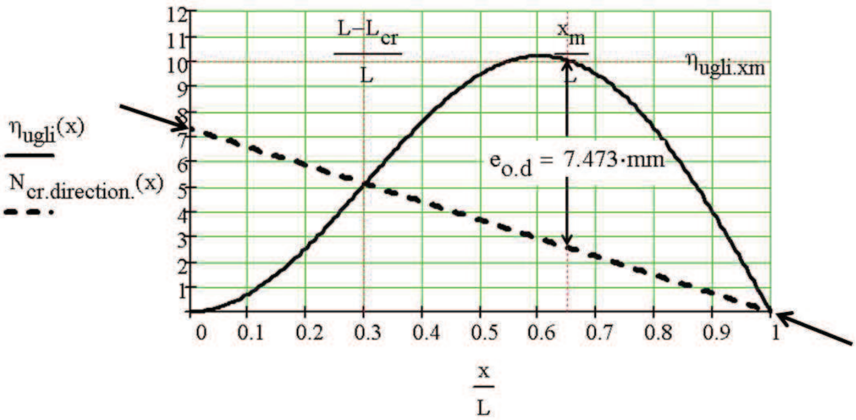

Figure 1: Uniform global and local initial ("ugli") imperfection valid for buckling about z-z

The additive deflection due to N_{Ed} acting on the member with the unique global and local initial (ugli) imperfection.

$$\eta_{II}(\xi) = \frac{\eta_{0,ugli,m}}{\alpha_{cr}-1}\eta_{cr}(\xi) \tag{39}$$

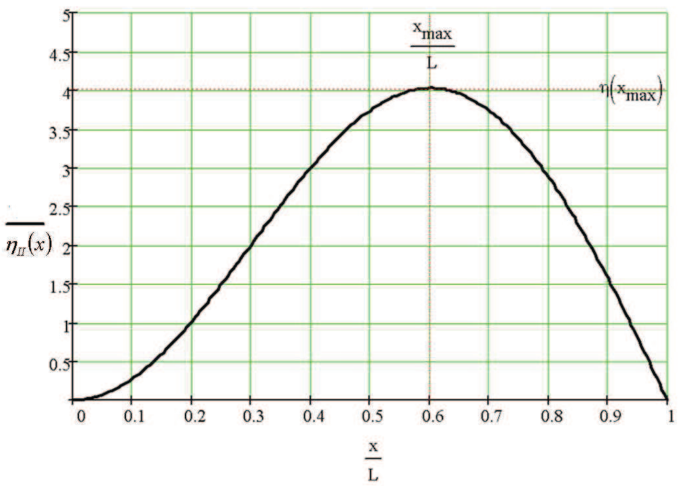

Figure 2: Additive deflection $\eta_{II}(x)$ due to N_{Ed} for flexural buckling about minor axis z-z

The bending moment distribution in the member due to $\eta_{II}(\xi)$ with allowing for second order effects

$$M^{II}_{\eta_{ugli}}(\xi) = -EI_{z,m}\eta''_{II}(\xi) \tag{40}$$

The bending moment due to $\eta_{II}(\xi)$ in the critical section "*m*".

$$M^{II}_{\eta_{ugli}}(\xi_m) = -EI_{z,m}\eta''_{II}(\xi_m) = 30.346\,kNm \tag{41}$$

This value may be calculated without any analysis from this formula

$$M^{II}_{\eta_{ugli}}(\xi_m) = kM^{I}(\xi_m) = ke_{0,d,m}N_{Ed} = 30.346\,kNm \tag{42}$$

The bending moment due to $\eta_{II}(\xi)$ in the fixed end of the member.

$$M^{II}_{\eta_{ugli}}(0) = -EI_{z,m}\eta''_{II}(0) = -29.621\,kNm \tag{43}$$

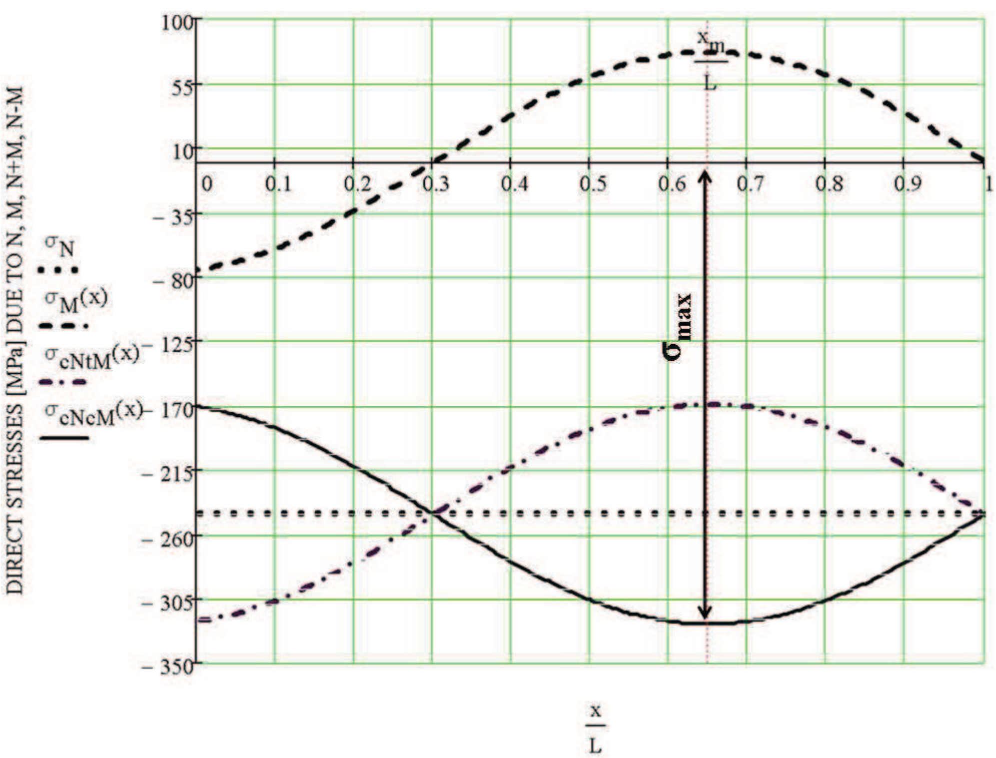

Figure 3: Direct stresses distributions for flexural buckling about minor axis z-z

The utility measure in the fixed end of the member.

$$U^{II}(0) = \frac{N_{Ed}}{N_{Rd}} + \left|\frac{M^{II}_{\eta_{ugli}}(0)}{M_{Rd}}\right| = 0.762 + 0.232 = 0.994 \tag{44}$$

The utility measure in the critical section "*m*".

$$U^{II}(\xi_m) = \frac{N_{Ed}}{N_{Rd}} + \left| \frac{M^{II}_{\eta_{ugli}}(\xi_m)}{M_{Rd}} \right| = 0.762 + 0.238 = 1.0 \tag{45}$$

The utility measure $U^{II}(\xi)$ in the critical section "*m*" must be the same as the utility measure in equivalent member method when $N_{Ed} = N_{b,Rd}$.

$$U^{I} = \frac{N_{Ed}}{N_{b,Rd}} = 1.0 \tag{46}$$

The comparison with ω-method [13]

$$\omega(x) = \frac{1}{\chi + (1-\chi)\sin\left[\frac{\pi(L-x)}{L_{cr}}\right]} \tag{47}$$

$$\Delta M(x) = -N_{Ed}\frac{M_{Rd}}{N_{Rd}}\left(\frac{1}{\chi\omega(x)} - 1\right) \tag{48}$$

$$\Delta M(x_m) = -30.347\, kNm \tag{49}$$

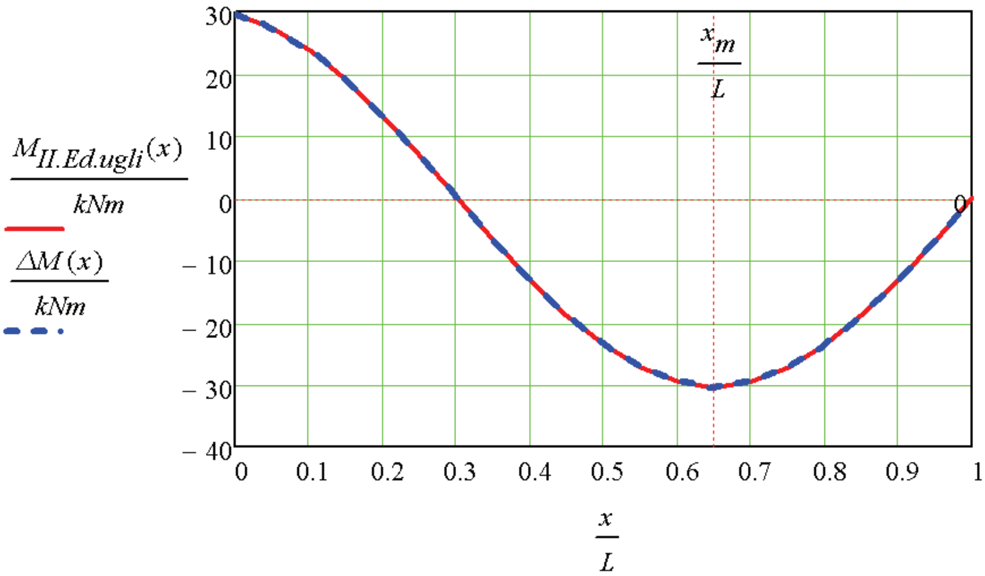

Figure 4: The comparison of the bending moments distributions for $N_{Ed} = N_{b,Rd}$

The utility measure

$$U = \frac{N_{Ed}}{N_{Rd}} + \left| \frac{\Delta M(\xi_m)}{M_{Rd}} \right| = 0.762 + 0.238 = 1.0 \tag{50}$$

The utility measures will differ when $N_{Ed} < N_{b,Rd}$. For example for $N_{Ed} = 0.5 N_{b,Rd}$ we obtain

$$U^{II}(\xi_m) = \frac{N_{Ed}}{N_{Rd}} + \left| \frac{M^{II}_{\eta_{ugli}}(\xi_m)}{M_{Rd}} \right| = 0.381 + 0.099 = 0.480 \tag{51}$$

$$U^I = \frac{N_{Ed}}{N_{b,Rd}} = 0.5\,, \quad U = \frac{N_{Ed}}{N_{Rd}} + \left|\frac{\Delta M(\xi_m)}{M_{Rd}}\right| = 0.381 + 0.119 = 0.5 \tag{52}$$

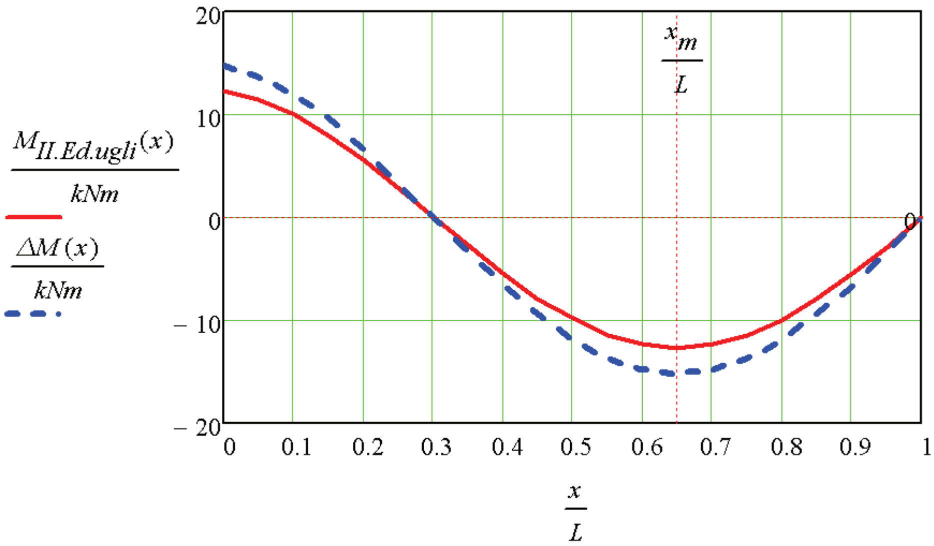

Figure 5: The comparison of the bending moments distributions for $N_{Ed} = 0.5N_{b,Rd}$

Generally it is necessary to use iterative calculation. The iterative procedure was not necessary to do in given numerical example. In the special case, when $N_{Ed}(x)/N_{Rd}(x)$ is constant, and the cross-section is uniform (our case), the location of critical cross-section "*m*" is determined by the location of $|\eta''_{cr}(\mathrm{x})|_{\max}$.

The graphical interpretation of the $e_{0,d}$ in the case of uniform member with uniform normal force distribution, which found by the first author, is very important for designers in practice. After drawing the shape of the elastic buckling mode it is possible for simple frames (Fig. 6, 7, 8) to find the exact or approximate location of the critical section "*m*" (see explanation in Fig. 1). The locations of critical cross-section "*m*" for 14 special cases are shown in Fig. 6, 7 and 8.

For such cases the value of the bending moment due to $\eta_{II}(\xi)$ in the critical section "*m*" may be calculated without any analysis only from the formula (42).

3 CONCLUSION

As an alternative to global initial sway imperfections and initial local bow imperfections the shape of the elastic critical buckling mode $\eta_{cr}(\xi)$ of the structure or of the verified member may be applied as a unique global and local imperfection (ugli) imperfection.

It was shown how to apply the Chladný`s method [9, 10, 11] which was derived by another way in [2] for the uniform member with uniform normal force distribution. The first author found that the $e_{0,d}$ has the graphical interpretation. This fact enables for simple frames to find the exact or approximate location of the critical section "*m*" and to calculate the bending moment due to $\eta_{II}(\xi)$ in the critical section "*m*" without any analysis only from the formula

$$M^{II}_{\eta_{ugli}}(\xi_m) = kM^I(\xi_m) = ke_{0,d,m}N_{Ed} \tag{53}$$

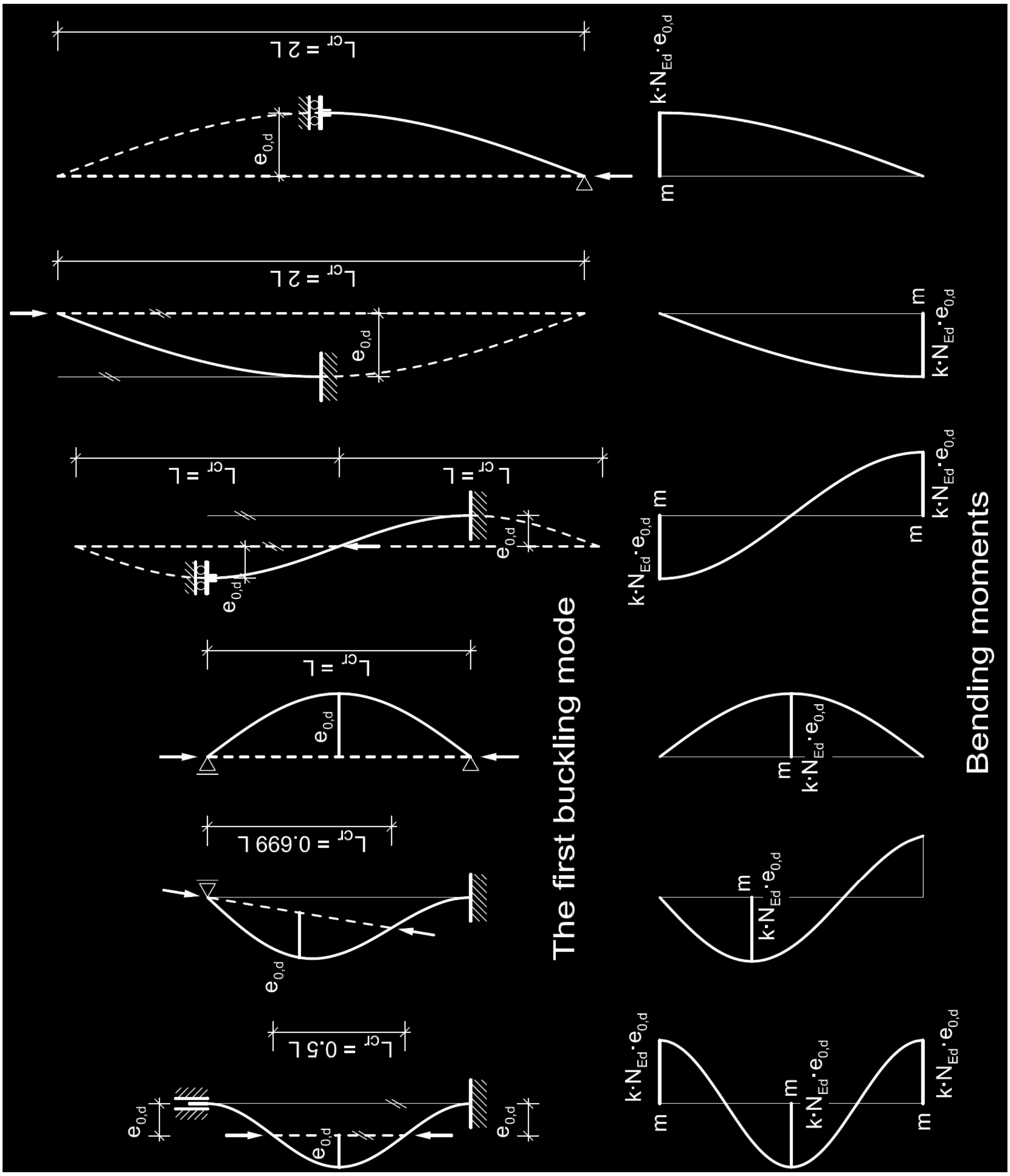

Figure 6: The shapes of elastic buckling mode applied as "ugli" imperfections. Bending moments due to "ugli" imperfection and location of "*m*".

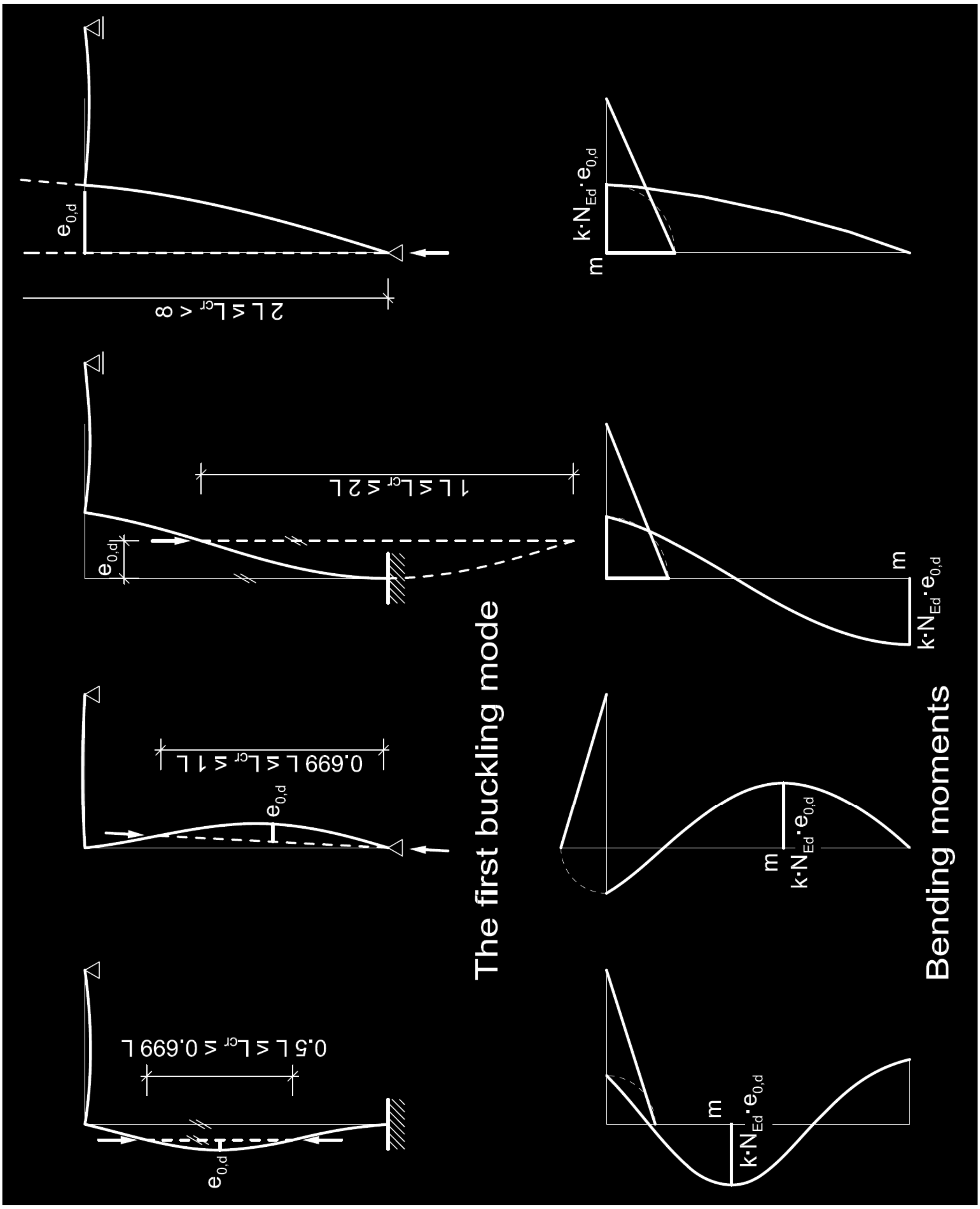

Figure 7: The shapes of elastic buckling mode applied as "ugli" imperfections. Bending moments due to "ugli" imperfection and location of "*m*".

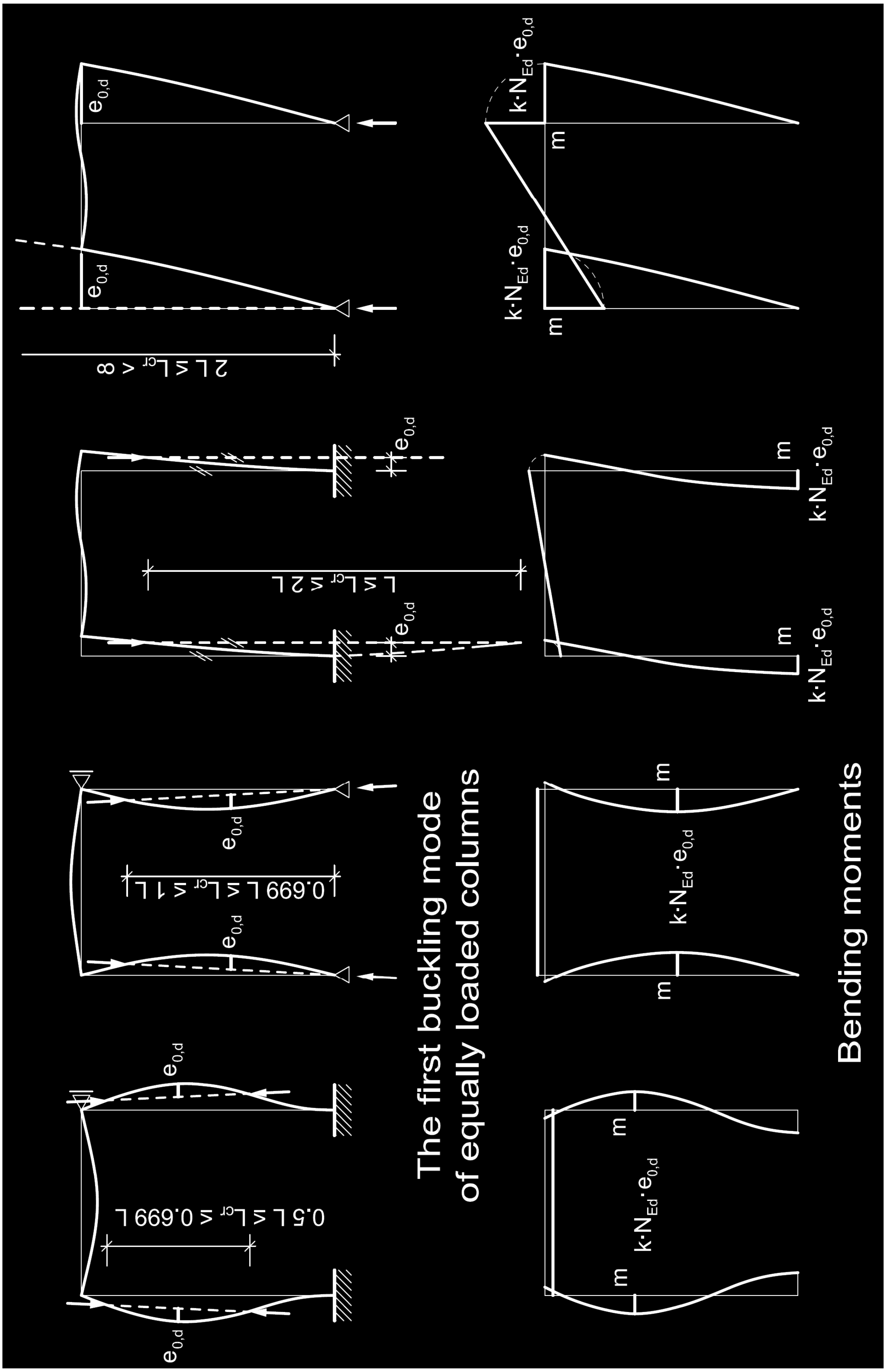

Figure 8: The shapes of elastic buckling mode applied as “ugli” imperfections. Bending moments due to “ugli” imperfection and location of “*m*”.

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REFERENCES

[1] STN EN 1993-1-1: *Design of steel structures. Part 1-1: General rules and rules for buildings.* SÚTN Bratislava, November 2006. (EN 1993-1-1, CEN Brussels May 2005).

[2] Baláž I., Determination of the flexural buckling resistance of frames with members with non-uniform cross-section and non-uniform axial compression forces. Zborník XXXIV. aktívu pracovníkov odboru OK so zahraničnou účasťou „Teoretické a konštrukčné problémy oceľových a drevených konštrukcií a mostov". 16.-17. 10. 2008, Pezinok, KKDK SvF STU v Bratislave, str.17-22.

[3] Baláž I., Resistance of metal frames with UGLI imperfections. XII. Mezinárodní vědecká konference u příležitosti 110. výročí založení FAST VUT v Brně, 20-22 duben 2009. Sekce: Inženýrske konstrukce, s.11-14.

[4] Baláž I. and Koleková Y., Metal frames with non-uniform members and/or non-uniform normal forces with imperfections in the form of elastic buckling mode. Engineering Research. Aniversary volume honoring Amália and Miklós Iványi. Pollack Mihály Faculty of Engineering. University of Pécs. October 25-26, 2010, pp. B:3-B:15.

[5] Baláž I. and Koleková Y., In plane stability of two-hinged arches. Proceedings of European Conference on Steel and Composite Structures, Eurosteel 2011. 31. August – 2. September 2011. Budapest, Vol. C, pp.1869-1874.

[6] Baláž I. and Koleková Y., Structures with UGLI imperfections. Proceedings of abstracts of 18th International conference Engineering mechanics 2012. Svratka, Czech republic, May 14-17, 2012, paper No. 233, p.18-19.

[7] Baláž I. and Koleková Y., Bending Moments due to Elastic Buckling Mode Applied as Uniform Global and Local Initial Imperfection. Steel structures and bridges 2012. Elsevier. SciVerse ScienceDirect, *Procedia Engineering* 40 (2012), 32-37.

[8] Baláž I. and Koleková Y., Bending Moments due to Elastic Buckling Mode Applied as Uniform Global and Local Initial Imperfection. Girders. Steel structures and bridges 2012. Proceedings of extended abstracts of 23rd Czech and Slovak International Conference. September 26-28, 2012, Hotel Permon, Podbanské. Slovakia, p.20.

[9] Chladný E. and Štujberová M., Frames with unique global and local imperfection in the shape of the elastic buckling mode (Part 1). *Stahlbau* 82, 2012, Heft 8.

[10] Chladný E. and Štujberová M., Frames with unique global and local imperfection in the shape of the elastic buckling mode (Part 2). *Stahlbau* 82, 2012, Heft 9.

[11] Chladný E. and Štujberová M., Berichtigung zu: Chladný E. and Štujberová,M., Frames with unique global and local imperfection in the shape of the elastic buckling mode. *Stahlbau* 83 (2013), Heft 1.

[12] EN 1999-1-1: *Design of aluminium structures. Part 1-1: General structural rules.* CEN Brussels March 2007.

[13] Höglund T., A unified method for the design of steel beam-columns. *Steel Construction* 7 (2014), No. 4, pp. 230-245.