Reliability of Structures – Part 5

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Resistance Models

- Non-composite Steel
- Composite steel
- Reinforced concrete
- Prestressed concrete
- Wood

Parameters of resistance

- Material : uncertainty in the strength of material, modulus of elasticity, cracking stresses, and chemical composition.
- Fabrication : uncertainty in the overall dimensions of the component which can affect the cross-section area, moment of inertia, and section modulus.
- Analysis : uncertainty resulting from approximate methods of analysis and idealized stress/strain distribution models.

Parameters of Resistance

$$R = R_n \, M \, F \, P$$

where :

R_n = nominal value of resistance

M = material factor

F = fabrication factor

P = professional factor

Parameters of Resistance

- The mean value of R is

$$\mu_R = R_n \mu_M \mu_F \mu_P$$

- Coefficient of variation

$$V_R = \sqrt{(V_M)^2 + (V_F)^2 + (V_P)^2}$$

- Bias factor

$$\lambda_R = \lambda_M \lambda_F \lambda_P$$

STEEL COMPONENTS

- Hot-Rolled Steel Beams (Non-composite Behavior)

$$M_p = F_y Z$$

- $\lambda_F = 1.0$ and $V_F = 0.05$ for all cases. However, if a component contains fillet welds, it is recommended that $V_F = 0.15$ be used.

Hot-Rolled Steel Beams (Ellingwood et al, 1980)

Element Type	μ_P	V_P	μ_M	V_M	μ_F	V_F	λ_R (1)	V_R
Tension Member (Yielding)	1.00	0	1.05	0.10	1.00	0.05	1.05	0.11
Tension Member (Ultimate)	1.00	0	1.10	0.10	1.00	0.05	1.10	0.11
Compact beam (Uniform moment)	1.02	0.06	1.05	0.10	1.00	0.05	1.07	0.13
Compact beam (Continuous)	1.06	0.07	1.05	0.10	1.00	0.05	1.11	0.13
Elastic beam (Lat.-Torsional Buckling)	1.03	0.09	1.00	0.06	1.00	0.05	1.03	0.12
Inelastic beam (Lat.-Torsional Buckling)	1.06	0.09	1.05	0.10	1.00	0.05	1.11	0.14
Plate Girders (Flexure)	1.03	0.05	1.05	0.10	1.00	0.05	1.08	0.12
Beam-Columns	1.02	0.10	1.05	0.10	1.00	0.05	1.07	0.15

Non-composite steel girder bridges

Moment Capacity of non-composite steel Girders

- Evaluation of the response to bending moment for representative sizes using a computer procedure developed by Tabsh (1990).
- From simulations :

$$\lambda=1.075 \qquad V=0.10$$

- From American Iron and Steel Institute

$$\lambda=1.095 \qquad V=0.075$$

- Professional factor : $\lambda_P=1.02$ $V_P=0.06$

Resistance : $\lambda_R=1.12 \quad V_R=0.10$

Non-composite steel girder bridges

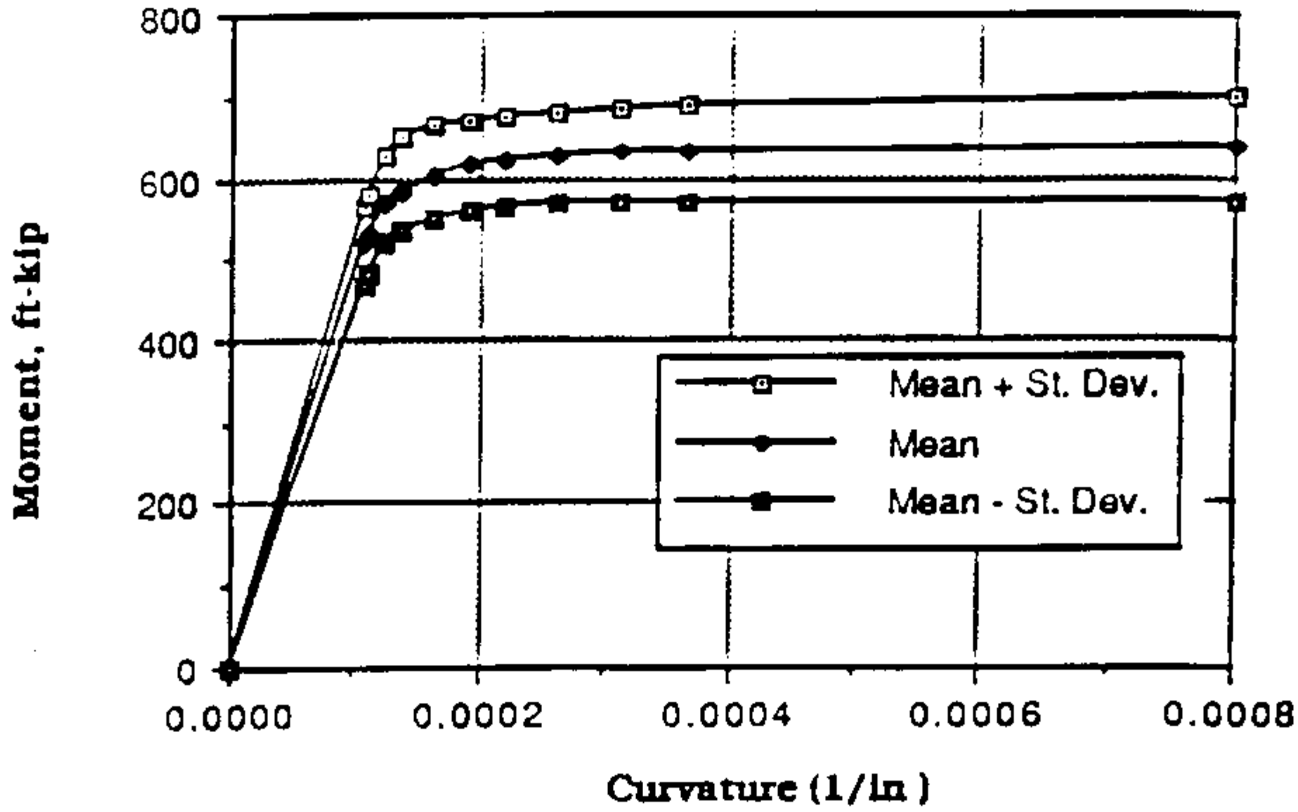

Fig. 8-2. Moment-Curvature Curves for a Non-Composite W24x76 Steel Section.

Non-composite steel girder bridges

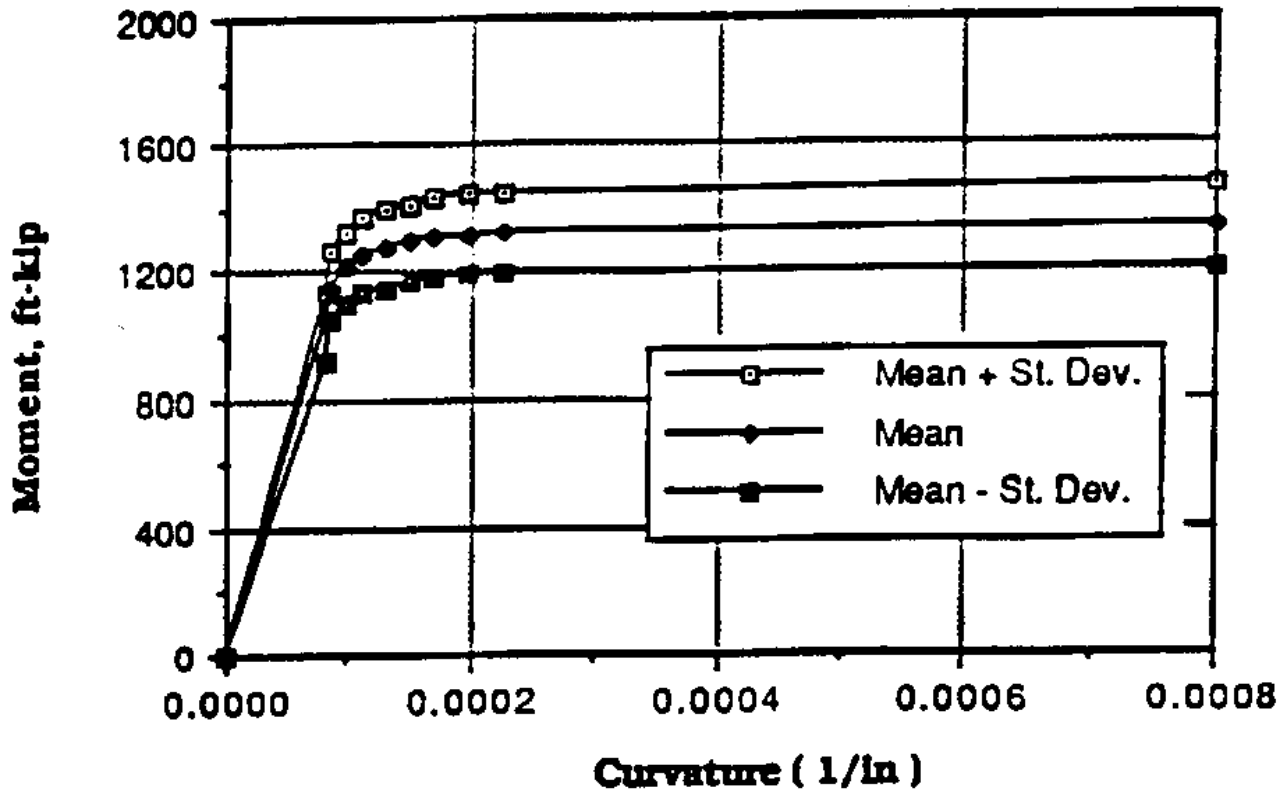

Fig. 8-3 Moment-Curvature Curves for a Non-Composite W33x118 Steel Section.

Non-composite steel girder bridges

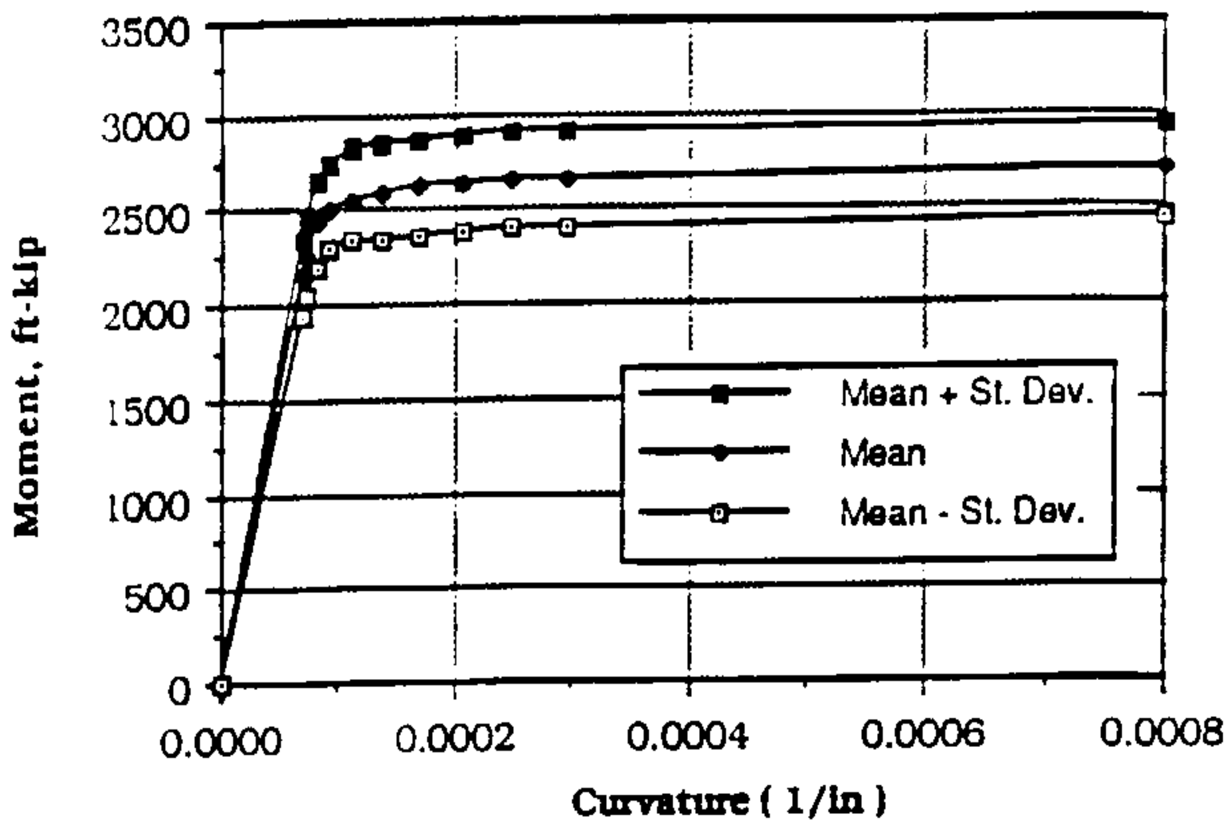

Fig. 8-4. Moment-Curvature Curves for a Non-Composite W36x210 Steel Section.

Non-composite steel girder bridges

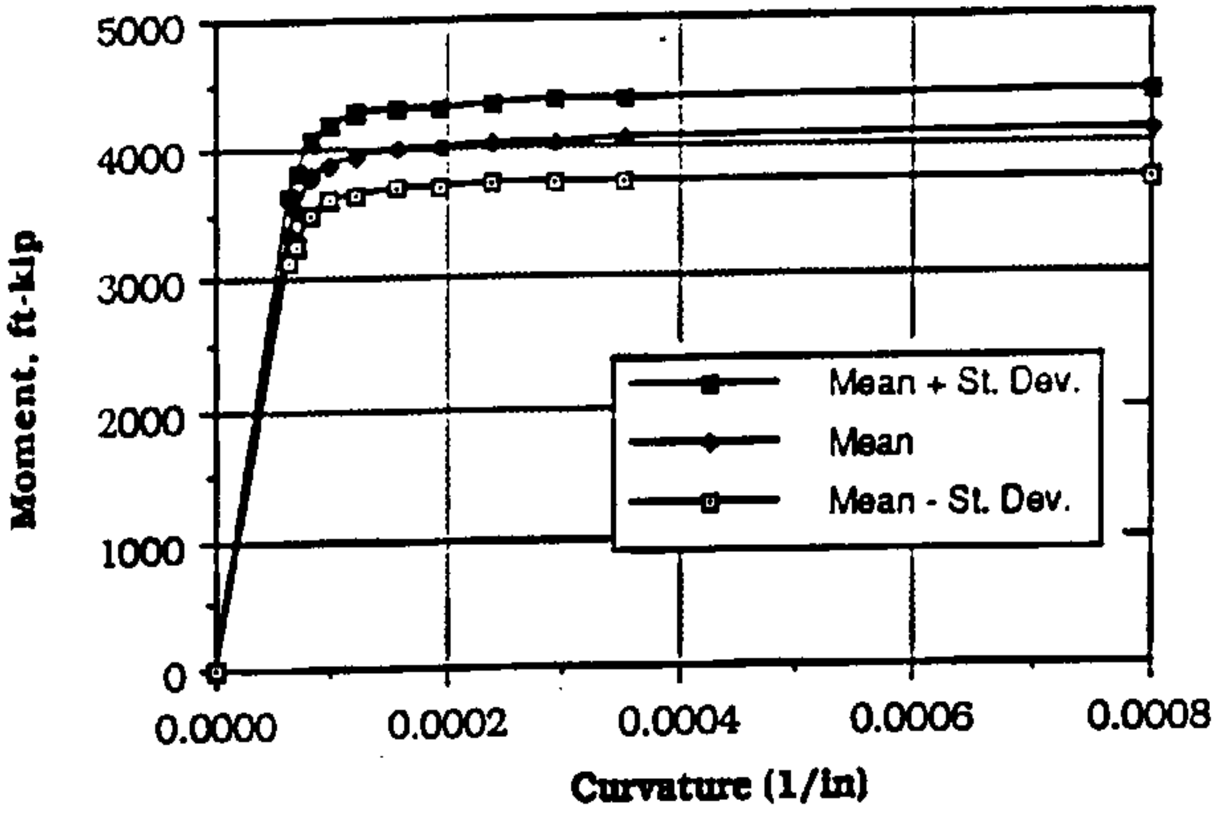

Fig. 8-5. Moment-Curvature Curves for a Non-Composite W36x300 Steel Section.

Composite Steel Girders

- Moment-curvature relationship

$$\phi = \frac{M}{EI_e} + C_1\left(\frac{M}{M_y}\right)^{C_2}$$

- Simulation technique: Monte Carlo
- Moment-curvature relationships by Tabsh and Nowak (1991)
- Concrete slab width considered is 6 ft (1.8 m),
- Slab thickness is assumed to be 7 in (175 mm).
- Based on data from the American Iron and Steel Institute (AISI), the statistical parameters for MF are $\boldsymbol{\lambda_{MF} = 1.07}$ and $\boldsymbol{V_{MF} = 0.08}$. For the analysis factor, P, $\boldsymbol{\lambda_P = 1.05}$ and $V_P = 0.06$.
- The ultimate moment, $\boldsymbol{\lambda_R = 1.12}$ and $\boldsymbol{V_R = 0.10}$.

Composite Steel Girders

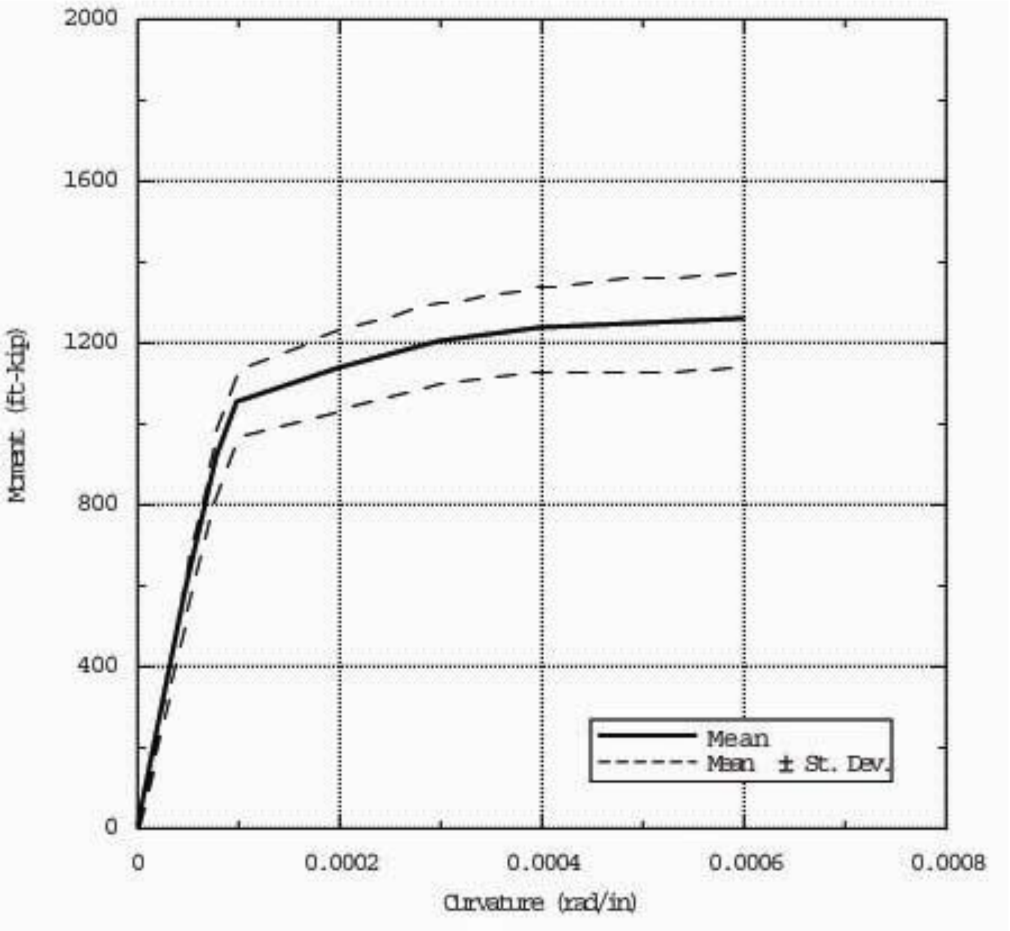

Figure 7-5 Moment-Curvature Curves for a Composite W24x76 Steel Section.

Composite Steel Girders

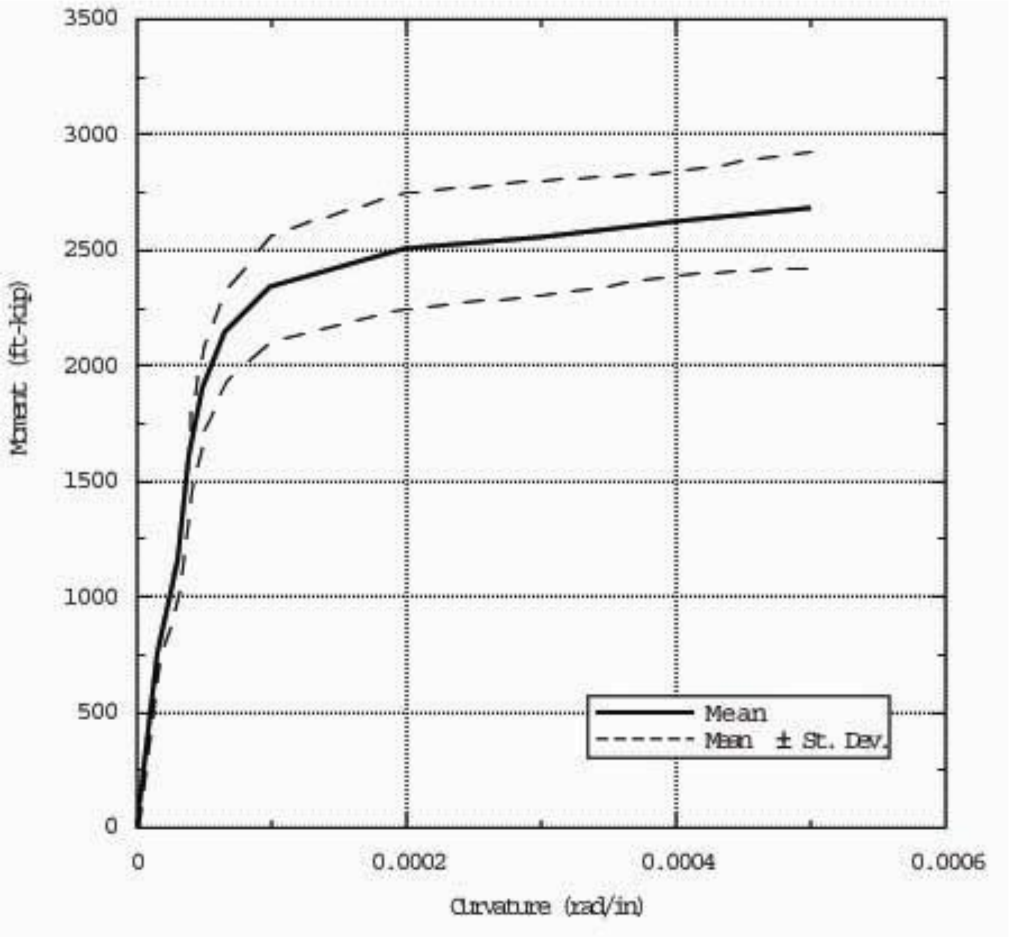

Figure 7-6 Moment-Curvature curves for a Composite W33x130 Steel Section.

Composite Steel Girders

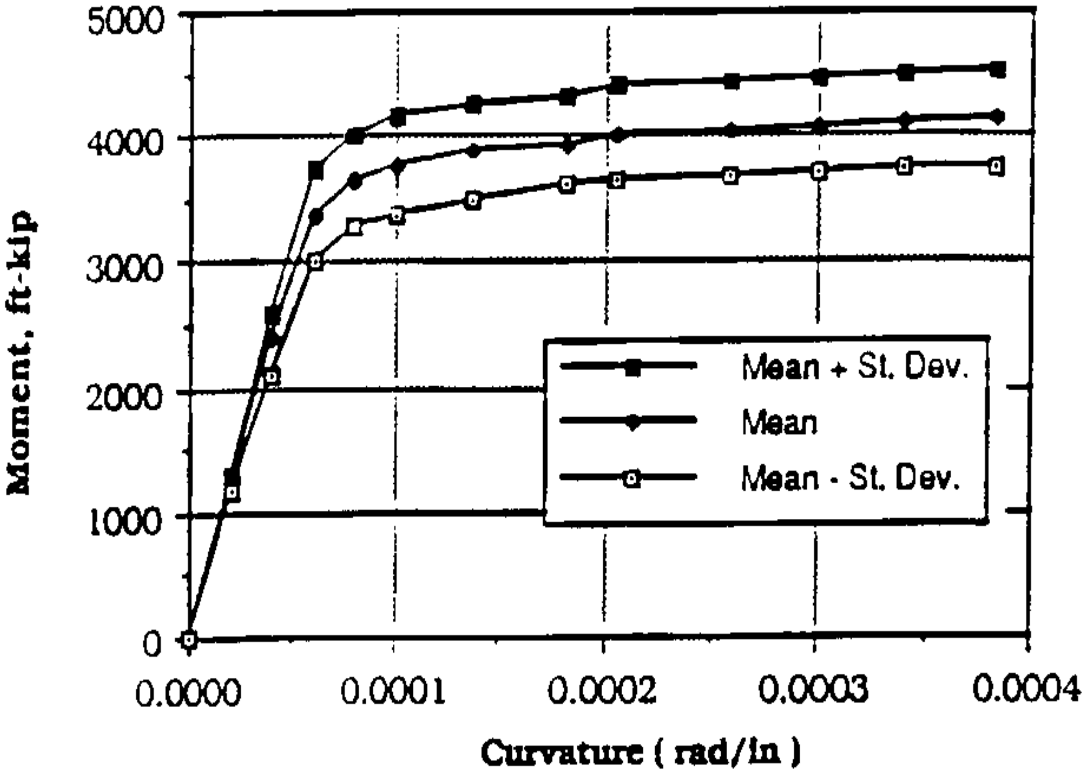

Fig. 8-8. Moment-Curvature Curves for a Composite W36x210 Steel Section.

Composite Steel Girders

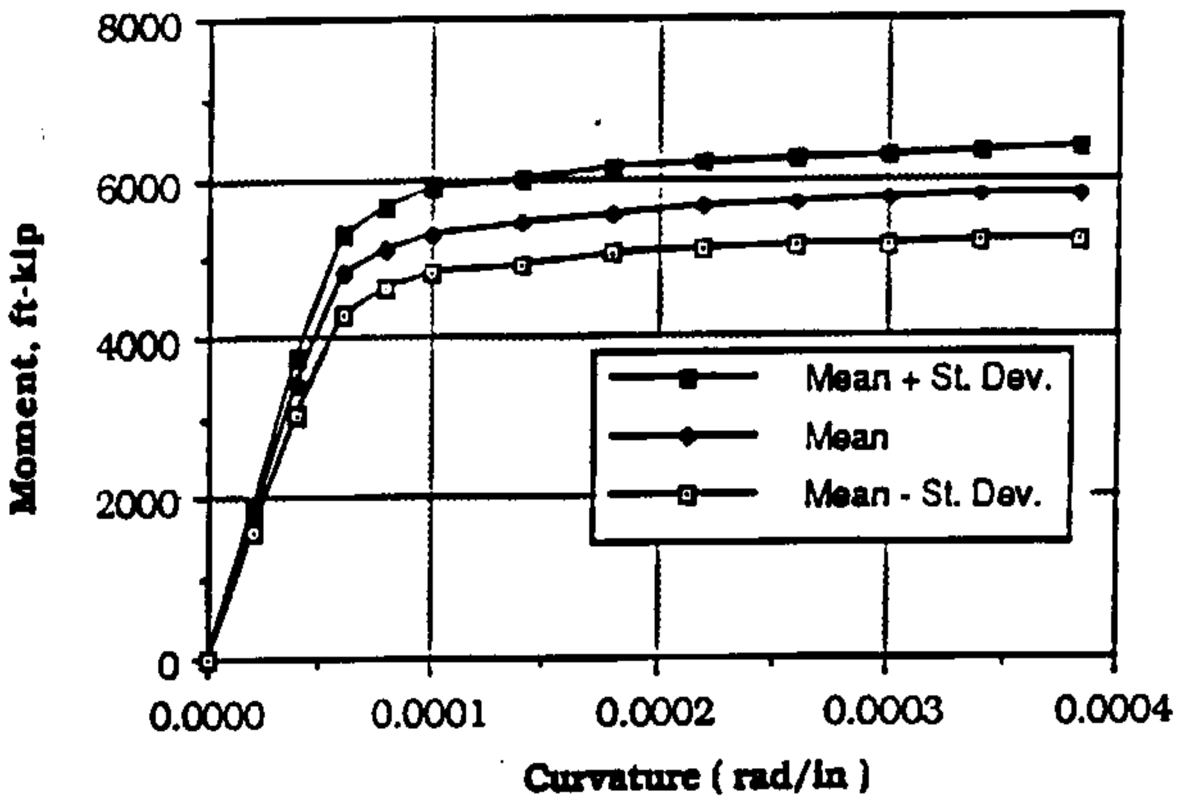

Fig. 8-9. Moment-Curvature Curves for a Composite W36x300 Steel Section.

Shear Capacity of Steel Beams

- Ultimate shear capacity of steel sections

  $V_u = (1/3)\ A_w\ F_y$

- Statistical parameters derived by Yamani (1992).

- Based on recent test data provided by the American Iron and Steel Institute, the statistical parameters for MF combined are $\lambda_{MF} = 1.12$, and $V_{MF} = 0.08$. The parameters for the analysis factor are taken as $\lambda_P = 1.02$ and $V_P = 0.07$.

- Resulting parameters of the shear resistance are λ_R **= 1.14** and $\mathbf{V_P}$ **= 0.106**.

Steel Columns

- The parameters depend on the slenderness ratio. Statistical data is provided by Ellingwood *et al*. (1980)

$$\chi = \frac{KL}{r}\frac{1}{\pi}\sqrt{\frac{F_{ys}}{E}}$$

- Critical stress defined as

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

- mean value

$$\frac{\sigma_{cr}}{F_{ys}} = \begin{cases} 1 & \chi \le 0.15 \\ 1.035 - 0.202\,\chi - 0.222\,\chi^2 & 0.15 \le \chi \le 1.0 \\ -0.111 + \dfrac{0.636}{\chi} + \dfrac{0.087}{\chi^2} & 1.0 \le \chi \le 2.0 \end{cases}$$

Steel Columns

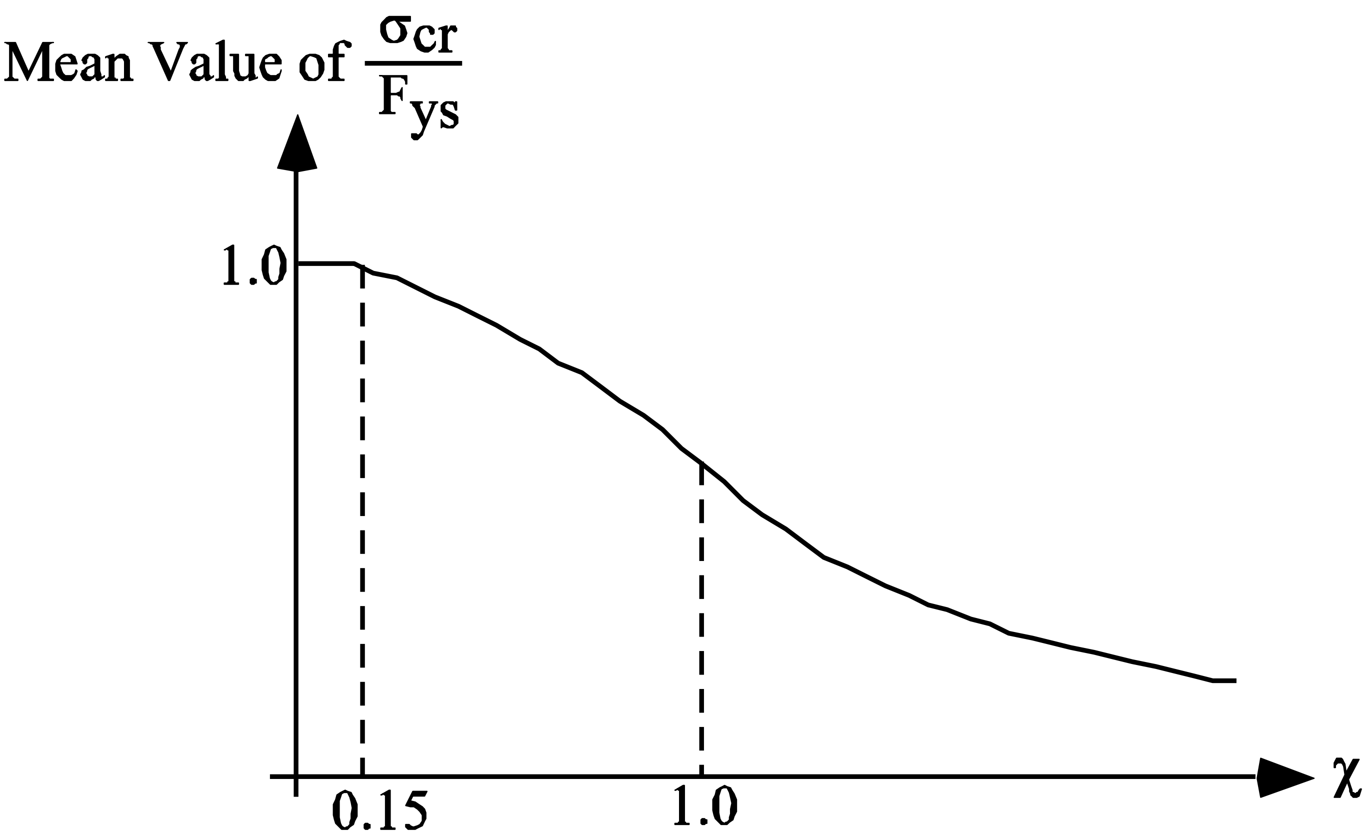

Figure 7-9 Critical Stress for Steel Columns.

Steel Columns

- Mean resistance

$$\mu_R = \left(\frac{\sigma_{cr}}{F_{ys}}\right)_{mean} \mu_P \mu_M \mu_F$$

- According to Ellingwood *et al*. (1980), the product $\mu_M \mu_F \mu_P$ is **1.08**
- Values of χ ranging from **0.3 to 1.9**
- For the same range of χ values, the coefficient of variation, $\mathbf{V}_R$, varies between **0.12 and 0.15**.

Cold-Formed Members

Member Type	λ_R	V_R
Tension member	1.10	0.11
Braced beams in flexure[(1)]	1.17-1.60	0.17-0.28
Laterally unbraced beams	1.15	0.17
Columns[(2)]	0.97-1.68	0.09-0.26

ALUMINUM STRUCTURES

Table 8-6 Resistance Statistics for Aluminum Structures, from Ellingwood et al (1980).

Type Member	F.S.	$\bar{R}/R_n$	V_R
1. Tension members, limit state yield	1.65	1.10	0.08
2. Tension members, limit state ultimate	1.95	1.10	0.08
3. Beams, limit state yield	1.65	1.10	0.08
4. Beams, limit state lateral buckling	1.65	1.03	0.13
5. Beams, limit state inelastic local buckling	1.65	1.00	0.09
6. Columns, limit state yield	1.82	1.10	0.08
7. Columns, limit state local buckling	1.95	1.0	0.09
8. Columns, limit state overall buckling, $\lambda = 1$	1.95	0.92	0.14
9. Columns, limit state overall buckling, $\lambda = 1.6$	1.95	0.87	0.13
10. Columns, limit state overall buckling, $\lambda = 2.0$	1.95	0.91	0.14

REINFORCED CONCRETE COMPONENTS - BUILDINGS

Assumptions

- Variability of the material properties and dimensions correspond to average quality of construction

- Material strengths are assumed to be representative of relatively slow loading rates for dead load, live load and snow.

- Long time strength changes of the concrete and steel due to increasing maturity of the concrete and possible future corrosion of the reinforcement were ignored.

Concrete

How the long-term aging of concrete can affect its strength, consider this real-world example ?

from Gardiner and Hatcher (1970)
Concrete strength was tested at 99 points (locations) in a 22 year old building.

- Average Strength = 8,050 psi (56 N/mm^2)
- Standard deviation = 500 psi (3.5 N/mm^2).
- Average 28-day cylinder strength = 3,780 psi (26 N/mm^2),
- Specified design strength (nominal strength) = 3,000 psi (21 N/mm^2)

Ratio of 22-year strength and 28-day strength = 2.13.

Stress-Strain Relationship for Concrete

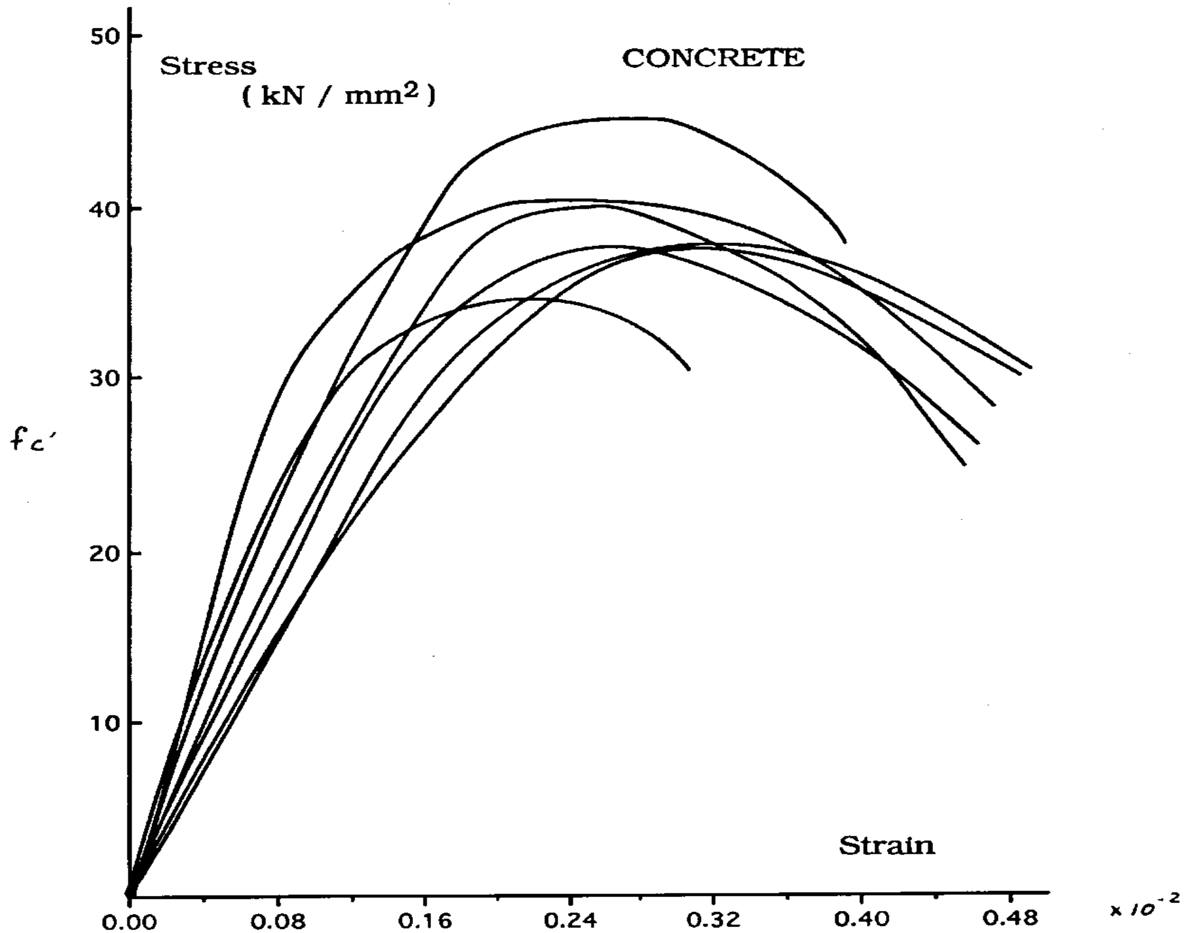

Fig. 8-10. Examples of Stress- Strain Relationships for Concrete.

Stress-Strain Relationship for Prestressing Steel

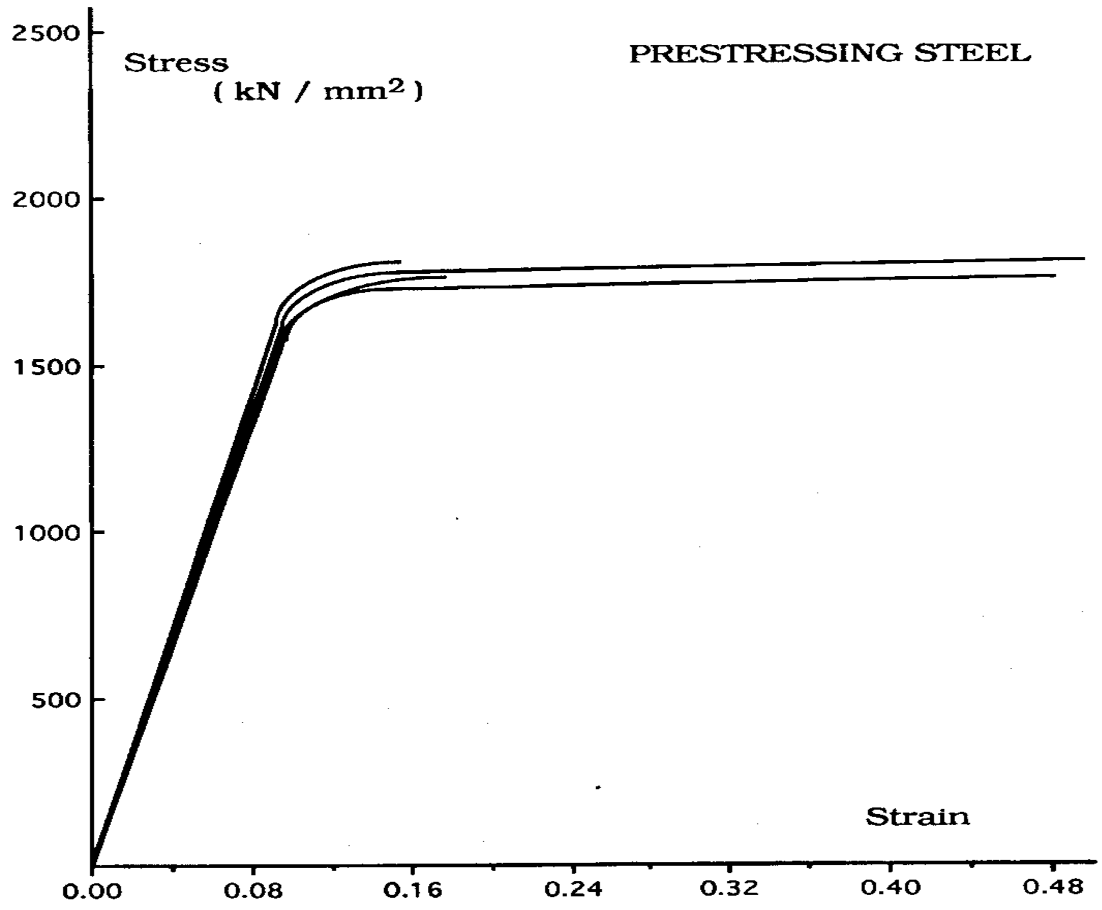

Fig. 8-11. Stress- Strain Relationship for Prestressing Steel.

Resistance parameters, Ellingwood et al, 1980

Property	Mean	V*	σ**	Ref.
Concrete Normal Control				
Compressive strength in structure loaded to failure in one hour.				
f'_c = 3000 psi	2760 psi	0.18	-	B.1
= 4000 psi	3390 psi	0.18	-	B.1
= 5000 psi	4028 psi	0.15	-	B.1
Tensile strength in structure, loaded to failure in one hour.				
f'_c = 3000 psi	306 psi	0.18	-	B.1
= 4000 psi	339 psi	0.18	-	B.1
= 5000 psi	366 psi	0.18	-	B.1
Reinforcement				
Grade 40, Static Yield	45.3 ksi	0.116	5.3 ksi	B.2
Grade 60, Static Yield	67.5 ksi	0.098	6.6 ksi	B.2
Grade 270 Prestressing Strand, Tensile Strength in Static Test	281 ksi	0.025	7.0 ksi	B.9
Dimensions				
Overall depth - Nominal				
Slab (1696 Swedish Slabs)	+0.03 in	-	0.47 in	B.3
(99 Slabs)	+0.21 in	-	0.26 in	B.5
Beam (108 beams)	-0.12 in	-	0.25 in	B.3
(24 beams)	+0.81 in	-	0.55 in	B.5
Effective depth - Nominal				
One-way Slab; Top Bars				
(1696 Swedish Slabs)	-0.75 in	-	0.63 in	B.3
(99 Slabs)	-0.04 in	-	0.37 in	B.5
Values Used	-0.40 in	-	0.50 in	
One-way Slab; Bottom Bars				
(2805 Swedish Slabs)	-0.13 in	-	0.34 in	B.3
(96 Slabs)	-0.16 in	-	0.35 in	B.5
Values Used	-0.13 in	-	0.35 in	
Beam, Top Bars	-0.22 in	-	0.53 in	B.3
Beam Stem Width - Nominal Width	+0.10 in	-	0.15 in	B.3
Column width, breadth - Nominal	+0.06 in	-	0.25 in	B.3
Cover, bottom steel in beams	+0.06 in	-	0.45 in	B.3
	-0.35 in	-	0.28 in	B.5

Statistical Parameters of beams and columns

Action	Type of Member	Details	$\bar{R}/R_n$	V_R
Flexure Reinforced Concrete	Continuous one-way slabs	5 in. thick, Grade 40	1.22	0.16
		5 in. thick, Grade 60	1.21	0.15
	Two-way slabs	5 in. thick, Grade 60	1.16	0.15
		7 in. thick, Grade 60	1.12	0.14
	One-way pan joists	13 in. overall depth, Grade 60	1.13	0.135
	Beams, Grade 40, f'_c = 5 ksi	$\rho = 0.005 = 0.09\ \rho_b$	1.18	0.14
		$\rho = 0.019 = 0.35\ \rho_b$	1.14	0.14
	Beams, Grade 60, f'_c = 5 ksi	$\rho = 0.006 = 0.14\ \rho_b$	1.04	0.08
		$\rho = 0.015 = 0.31\ \rho_b$	1.09	0.11
		$\rho = 0.027 = 0.57\ \rho_b$	1.05	0.11
		$\rho = 0.034 = 0.73\ \rho_b$	1.01	0.12
Flexure, Reinforced Concrete - Overall Values			1.05	0.11
Flexure Prestressed Concrete	Plant Precast Pretensioned	$\omega_p = 0.054$	1.06	0.057
		$\omega_p = 0.122$	1.05	0.061
		$\omega_p = 0.228$	1.06	0.083
		$\omega_p = 0.295$	1.04	0.097
	Cast-in-Place Post-tensioned	$\omega_p = 0.054$	1.02	0.061
		$\omega_p = 0.122$	1.05	0.083
		$\omega_p = 0.228$	1.03	0.111
		$\omega_p = 0.295$	1.05	0.144
Flexure, Plant Precast Pretensioned, Overall Value			1.06	0.08
Cast-in-Place Post-tensioned, Overall Value			1.04	0.095
Axial Load and Flexure	Short Columns, Compression Failures	f'_c = 3 ksi	1.05	0.16
		f'_c = 5 ksi	0.95	0.14
	Short Columns, Tension Failures	f'_c = 3 and 5 ksi	1.05	0.12
	Slender Columns, $k\ell/h = 20$, Compression Failures	f'_c = 5 ksi	1.10	0.17
	Slender Columns, $k\ell/h = 20$, Tension Failures	f'_c = 5 ksi	0.95	0.12
Shear	Beams with $a/d \geq 2.5$, $\rho_w = 0.008$	No stirrups	0.93	0.21
		Min stirrups	1.00	0.19
		$\rho_v f_y$ = 150 psi	1.09	0.17

ACI 318-99
ACI 318R-99

Building Code Requirements for Structural Concrete (318-99) and Commentary (318R-99)

Reported by ACI Committee 318

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FARMINGTON HILLS, MICHIGAN 48333-9094

ACI 318-02
ACI 318R-02

Building Code Requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318R-02)

An ACI Standard

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Concrete Strength

- Compressive strength, cylinders with diameter = 6” (150mm) and height 12” (300mm)
- Mostly 28 day strength, for precast concrete also 1 day or 56 day strength


Holcim
3/15
1/7 +10
2/13 +10
2/8


FORNEY
F96
FORNEY


NOVAK
1
28/02/03
PLATEN


34
MAX PLATEN

Results of Material Tests

- Cumulative distribution functions (CDF)
- For an easier interpretation of the results, CDF’s are plotted on the normal probability paper
- CDF of a normal random variable is represented by a straight line
- Any straight line on the normal probability paper represents a normal CDF

Strength of Ordinary Concrete

Ready mix concrete	Plant-cast concrete
3,000 psi (21 MPa)	5,000 psi (35 MPa)
3,500 psi (24 MPa)	5,500 psi (38 MPa)
4,000 psi (28 MPa)	6,000 psi (42 MPa)
4,500 psi (31 MPa)	6,500 psi (45 MPa)
5,000 psi (35 MPa)	
6,000 psi (42 MPa)	

Strength of Concrete

Light-weight concrete

3,000 psi (21 MPa)
3,500 psi (24 MPa)
4,000 psi (28 MPa)
5,000 psi (35 MPa)

High strength concrete

7,000 psi (49 MPa)
8,000 psi (56 MPa)
9,000 psi (62 MPa)
10,000 psi (70 MPa)
12,000 psi (84 MPa)

Objectives

- Update the materials strength models using new statistical data
- Update the resistance models for reliability analysis
- Calculate reliability indices for components designed using ACI 318-05
- Provide a basis for selection of resistance factors

New Materials Data

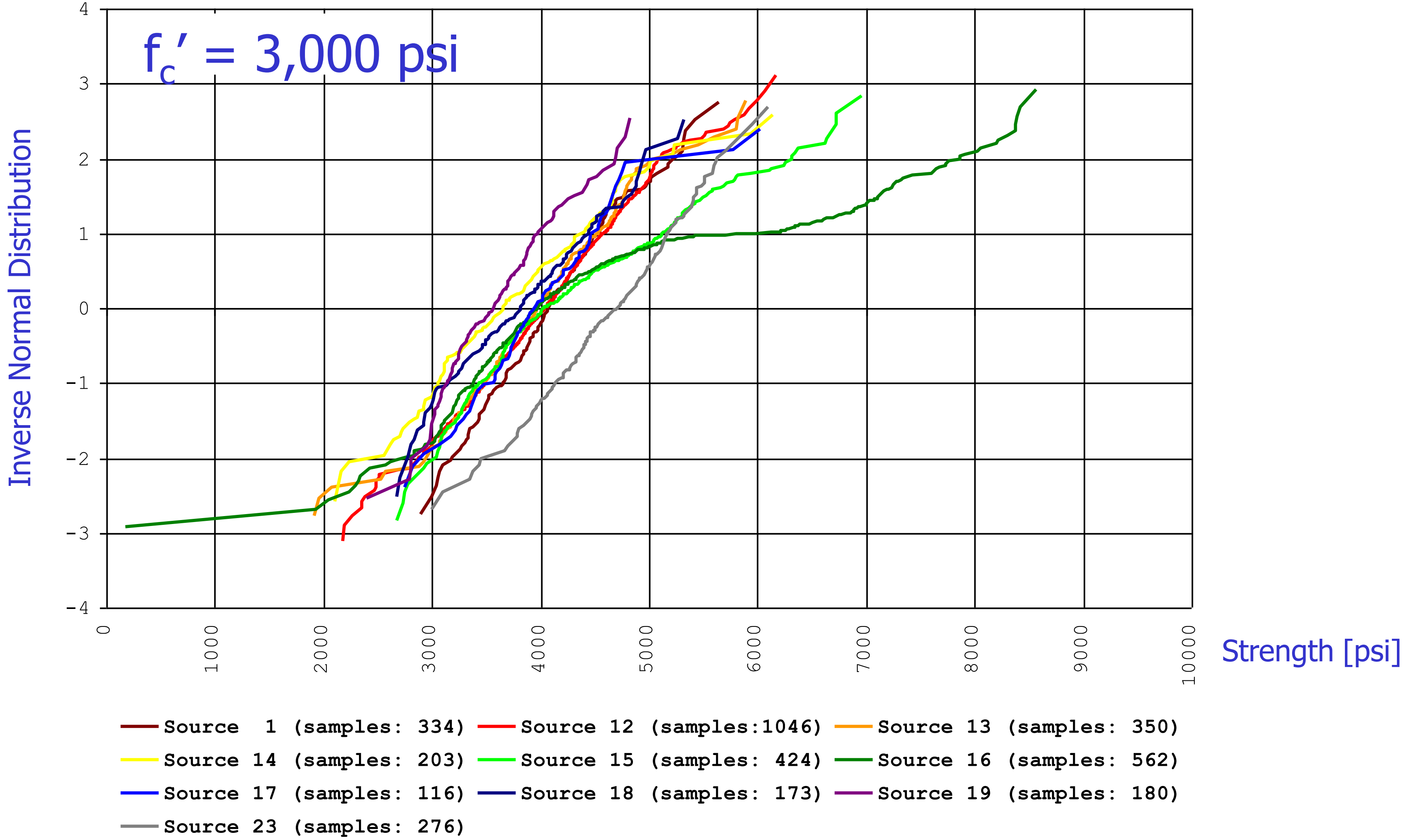
- Compressive Strength of Ordinary Concrete, Ready mixed, fc’: 3,000 3,500 4,000 4,500 5,000 and 6,000 psi
- Yield Stress of Reinforcing Steel Bars, Grade 60 Bar Sizes: #3, #4, #5, #6, #7, #8, #9, #10, #11 and #14
- Breaking Stress of Prestressing Steel (7-wire strands), Grade 270, Nominal Diameters: 0.5 in and 0.6 in

Ordinary Concrete – Number of Samples

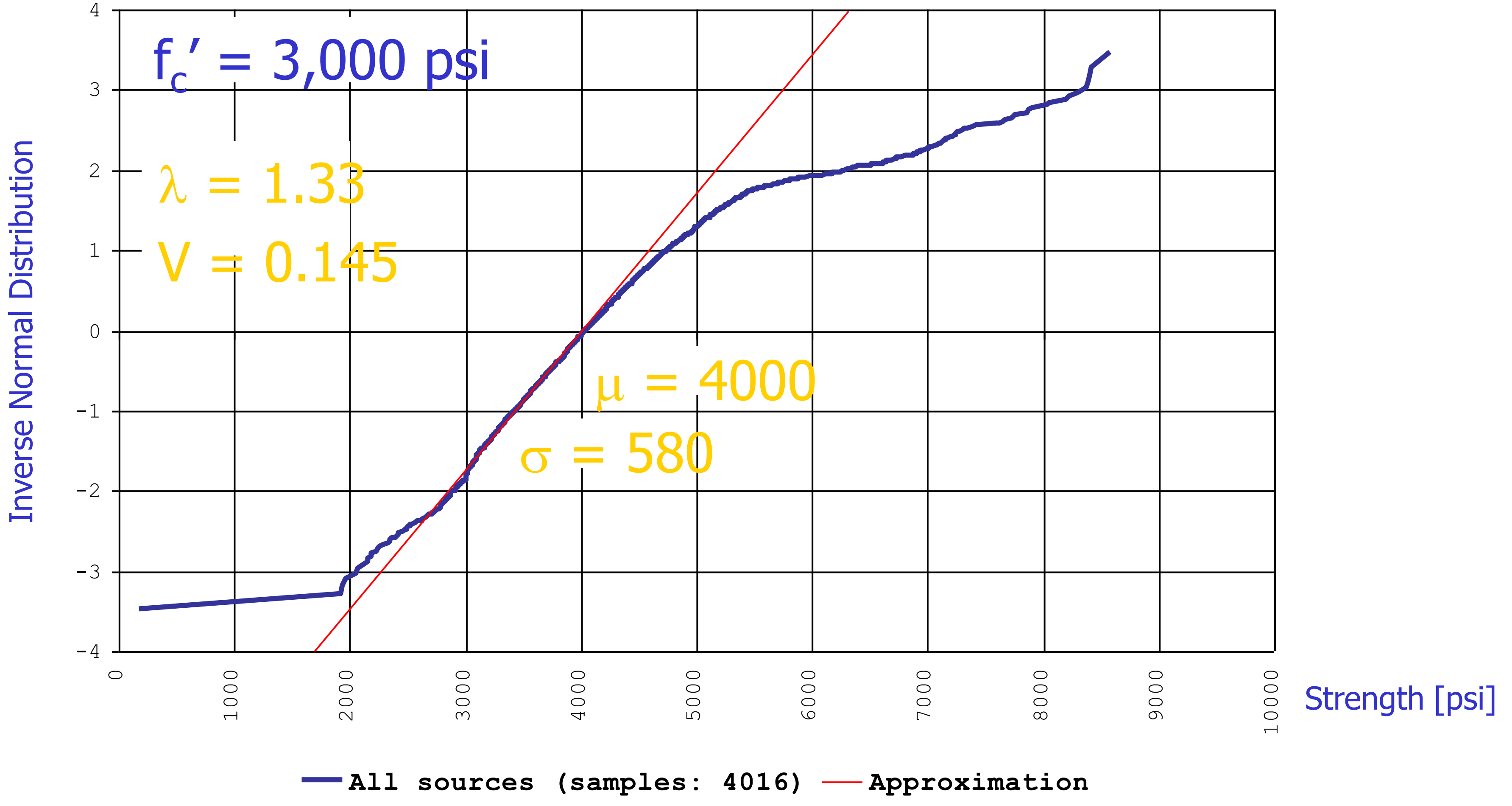
Specified Compressive Strength of Concrete f_c' [psi]:

Source:	3,000	3,500	4,000	4,500	5,000	6,000	
1	334		316		138	130	
2			156				
3	96		71		84		
4	82				66		
5	52						
6			54				
7	36						
8			60				
9			50				
10					206		
11					294		
12	1046	21	27	839			
13	350		274	298	2		
14	203	54	269		263		
15	424	8	220	164	8		
16	562	339	584	52	100		
17	116			90			
18	173	6	99	36	133		
19	180	99	533	80	422		
20			18		6		
21	8			2			
22	78		26	12			
23	276			346			
24			27				Total:
Total:	**4016**	**527**	**2784**	**1919**	**1722**	**130**	**11098**

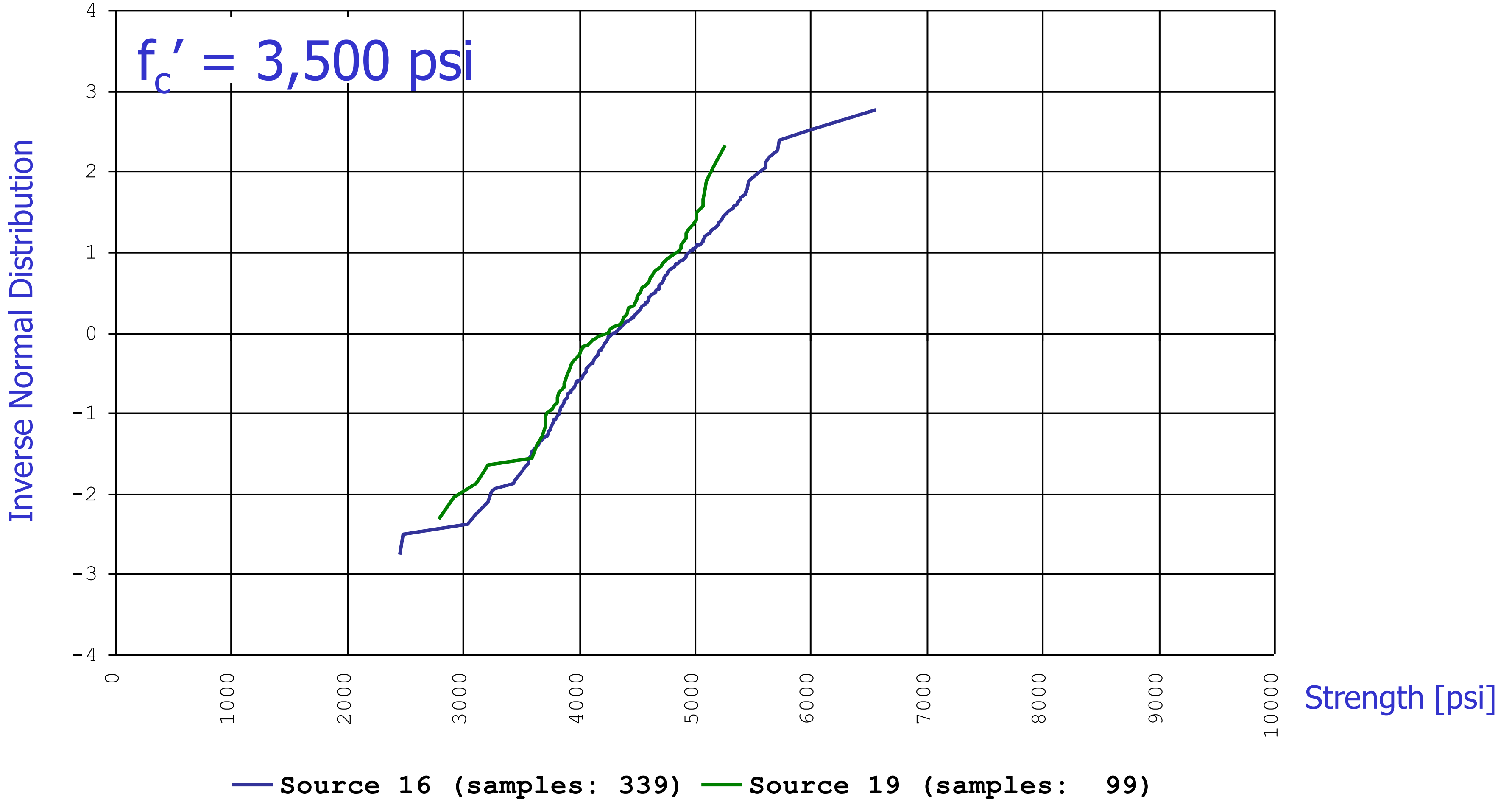
Ordinary Concrete – CDF of Strength



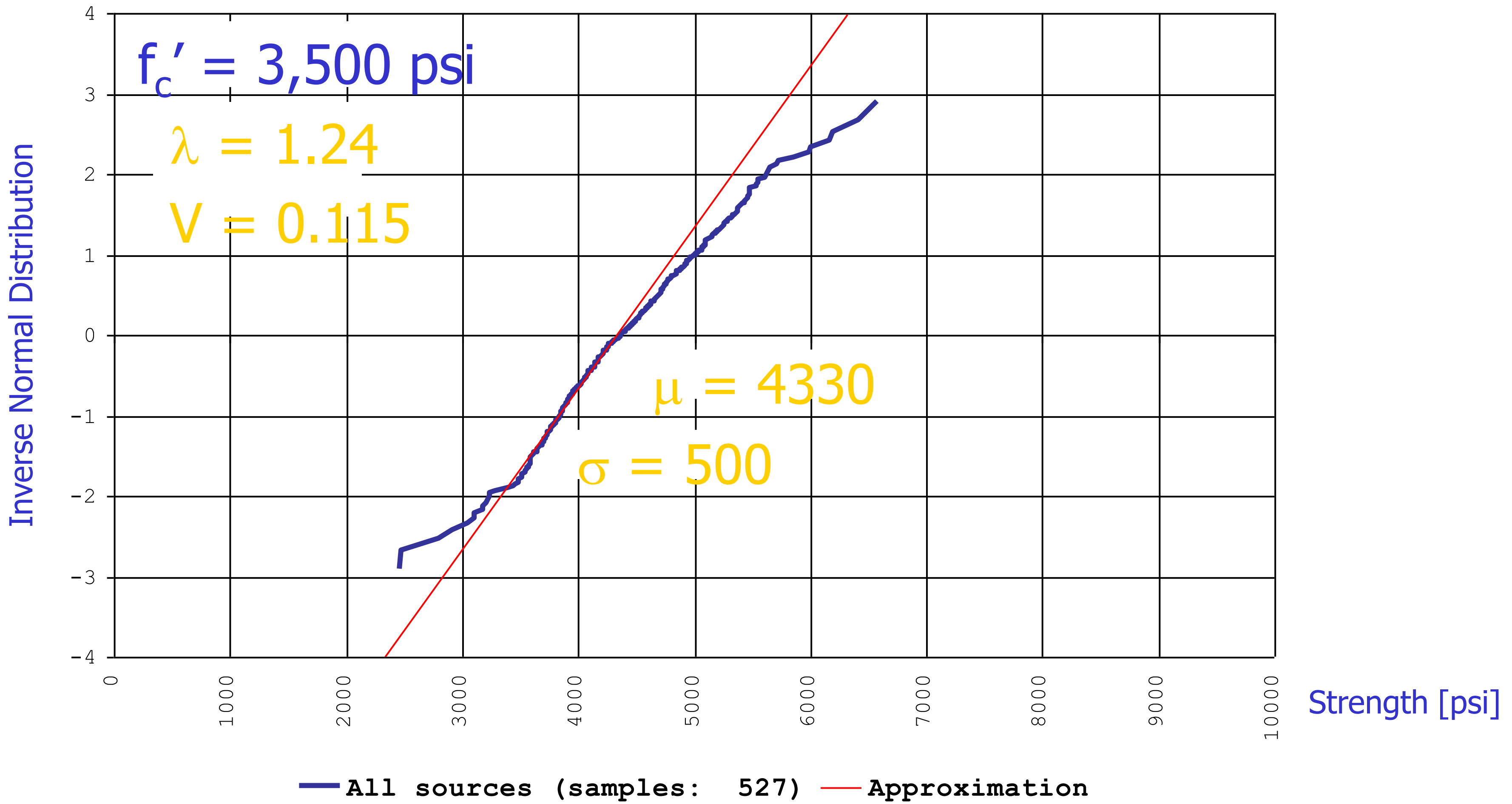
Ordinary Concrete – CDF of Strength



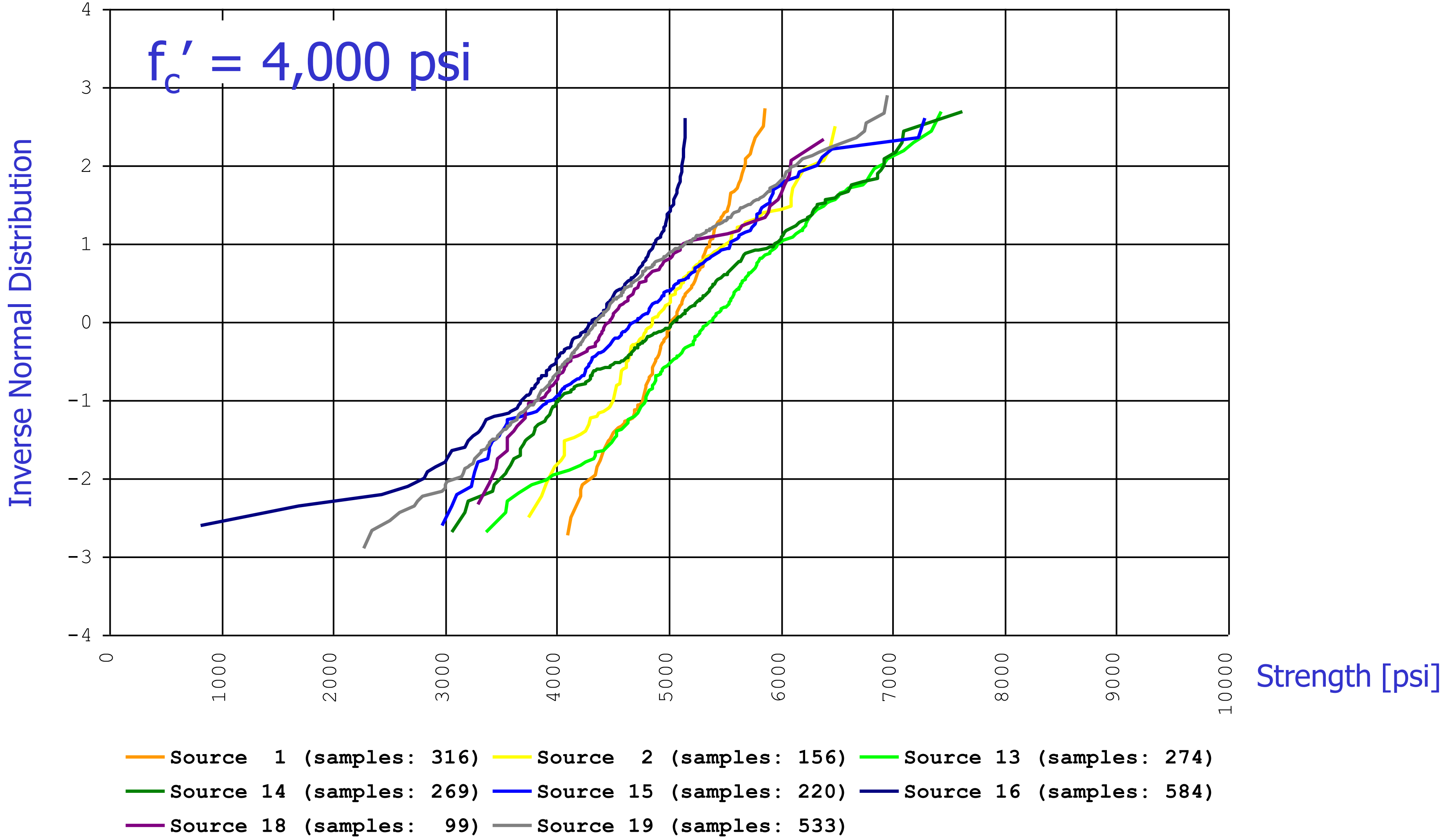
Ordinary Concrete – CDF of Strength



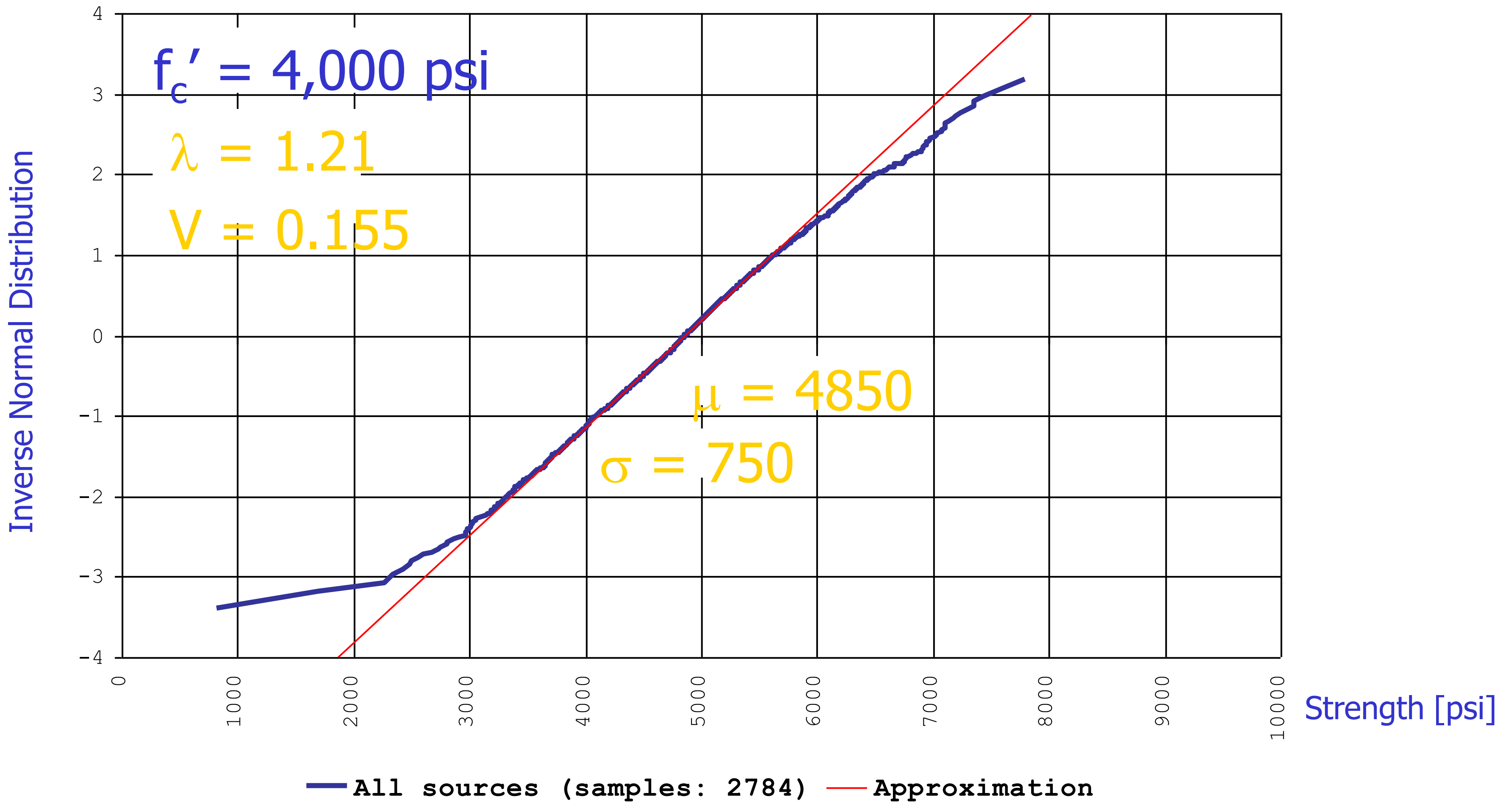
Ordinary Concrete – CDF of Strength



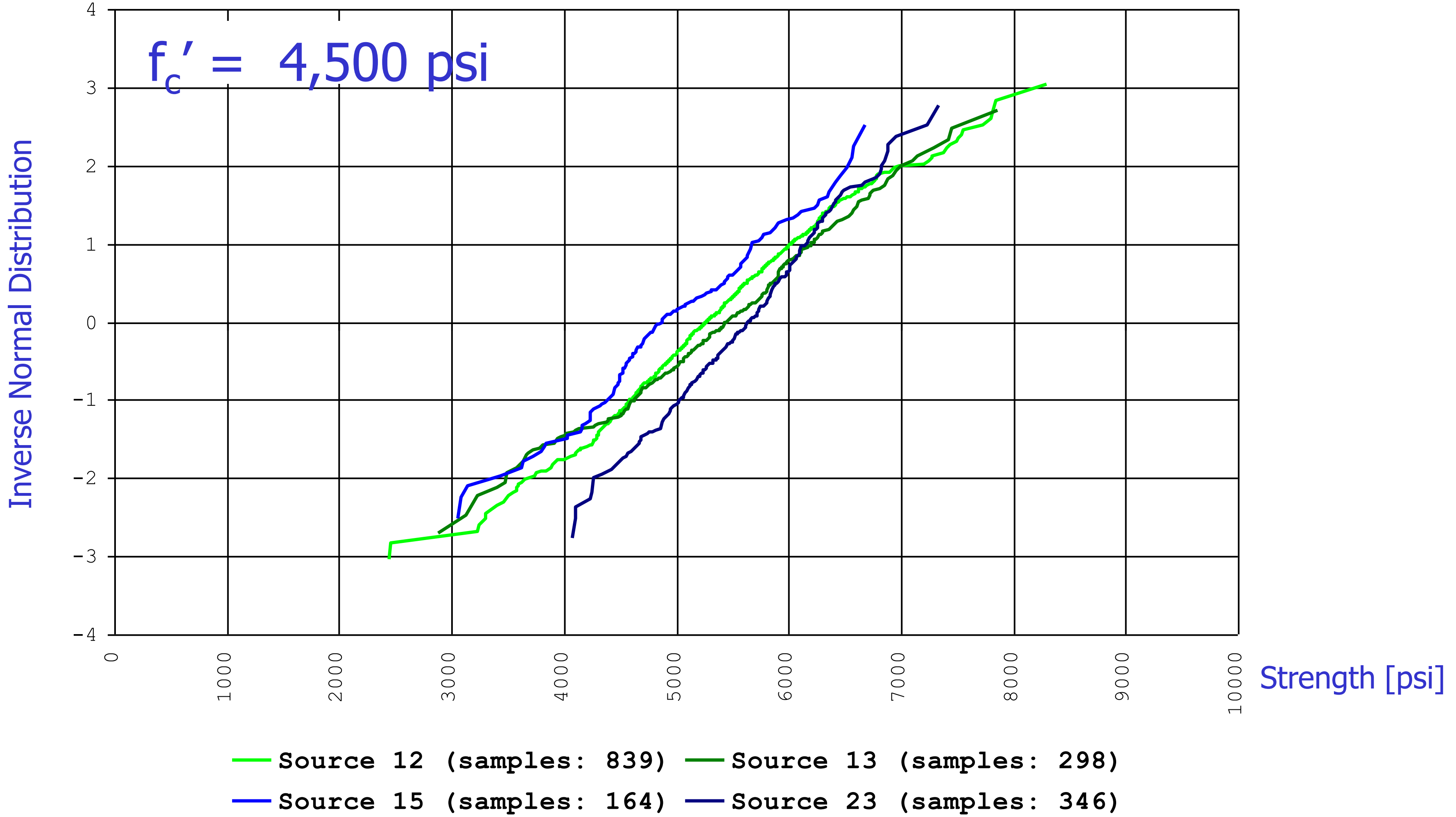
Ordinary Concrete – CDF of Strength



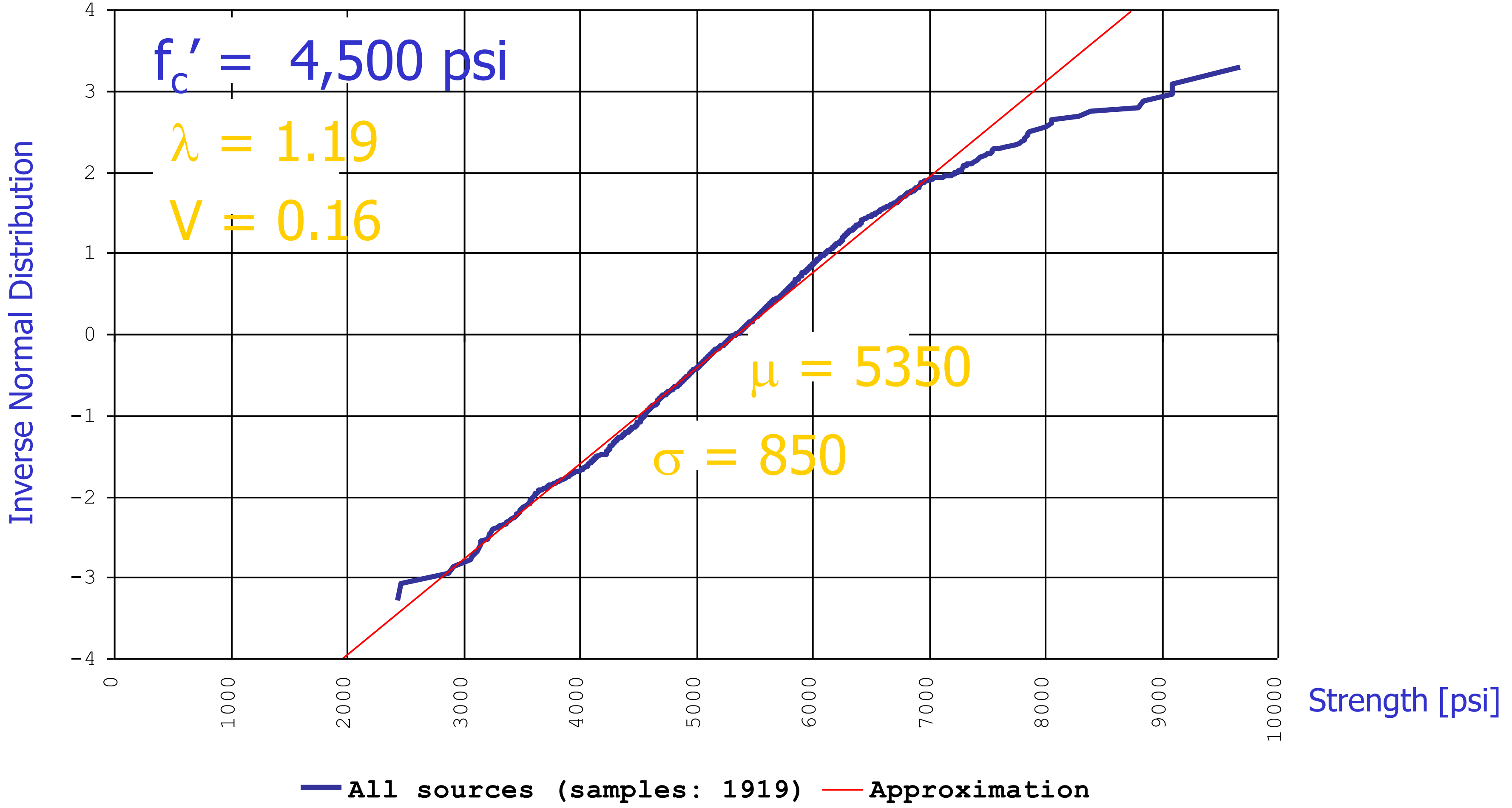
Ordinary Concrete – CDF of Strength



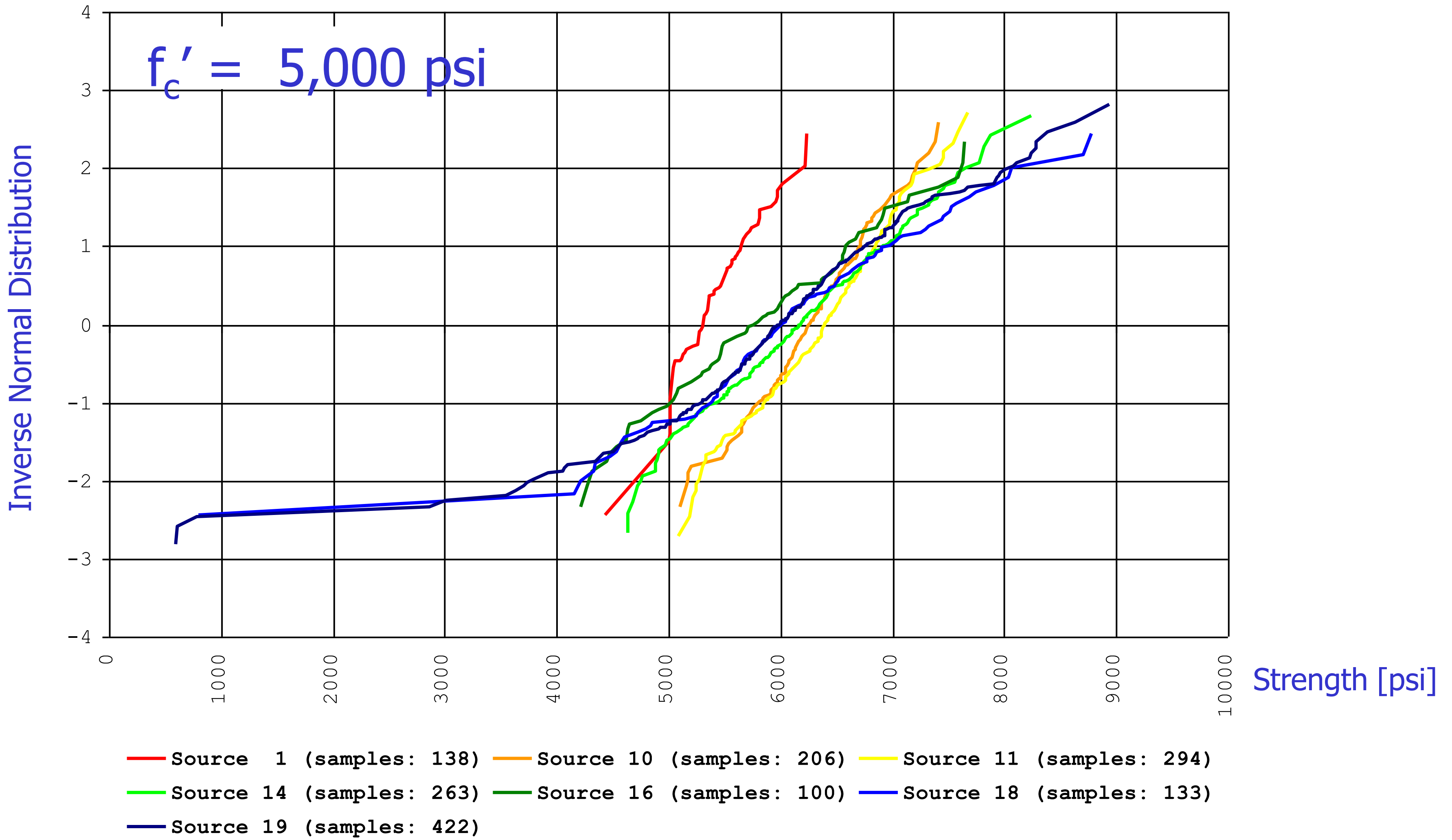
Ordinary Concrete – CDF of Strength



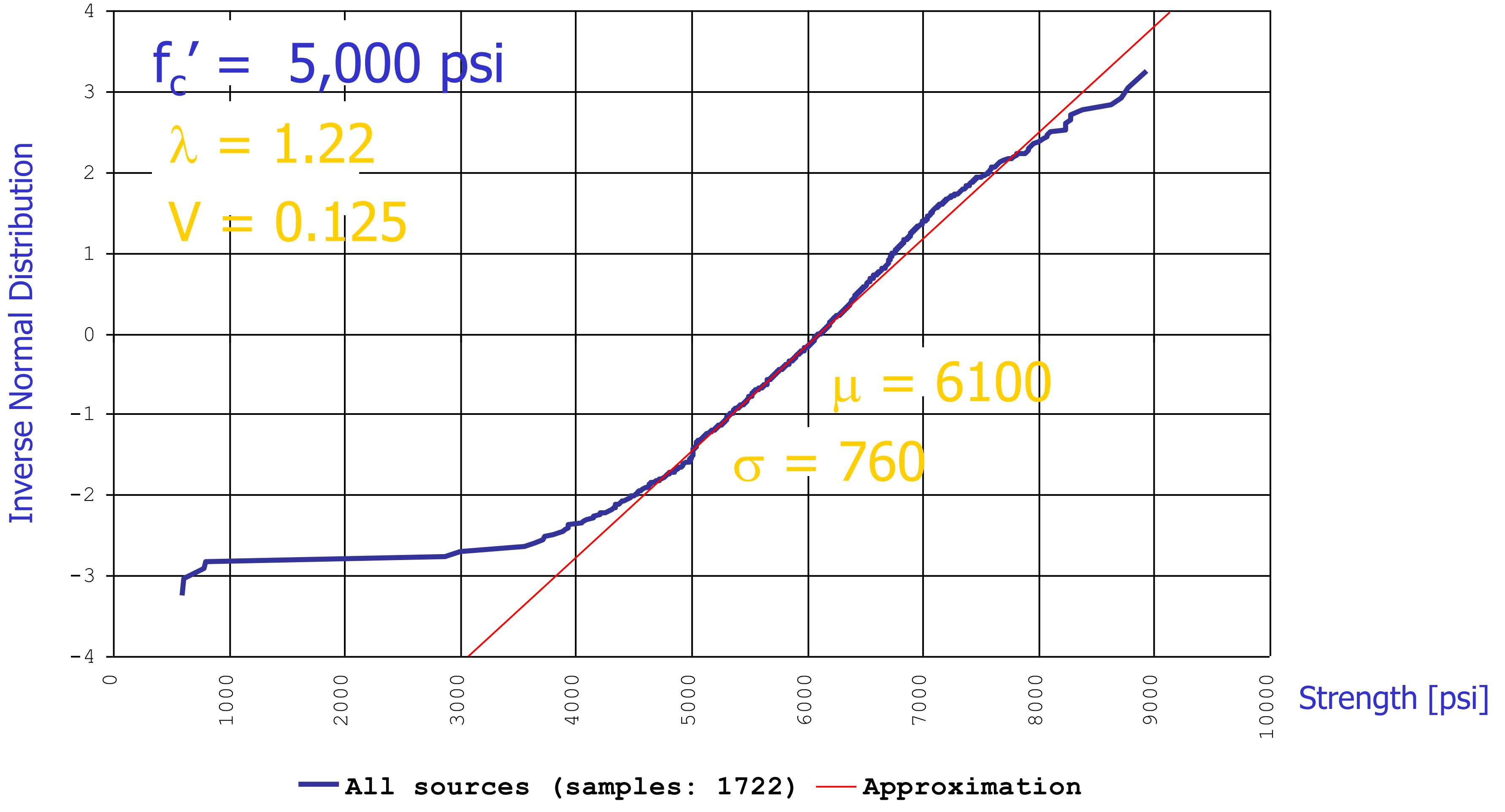
Ordinary Concrete – CDF of Strength



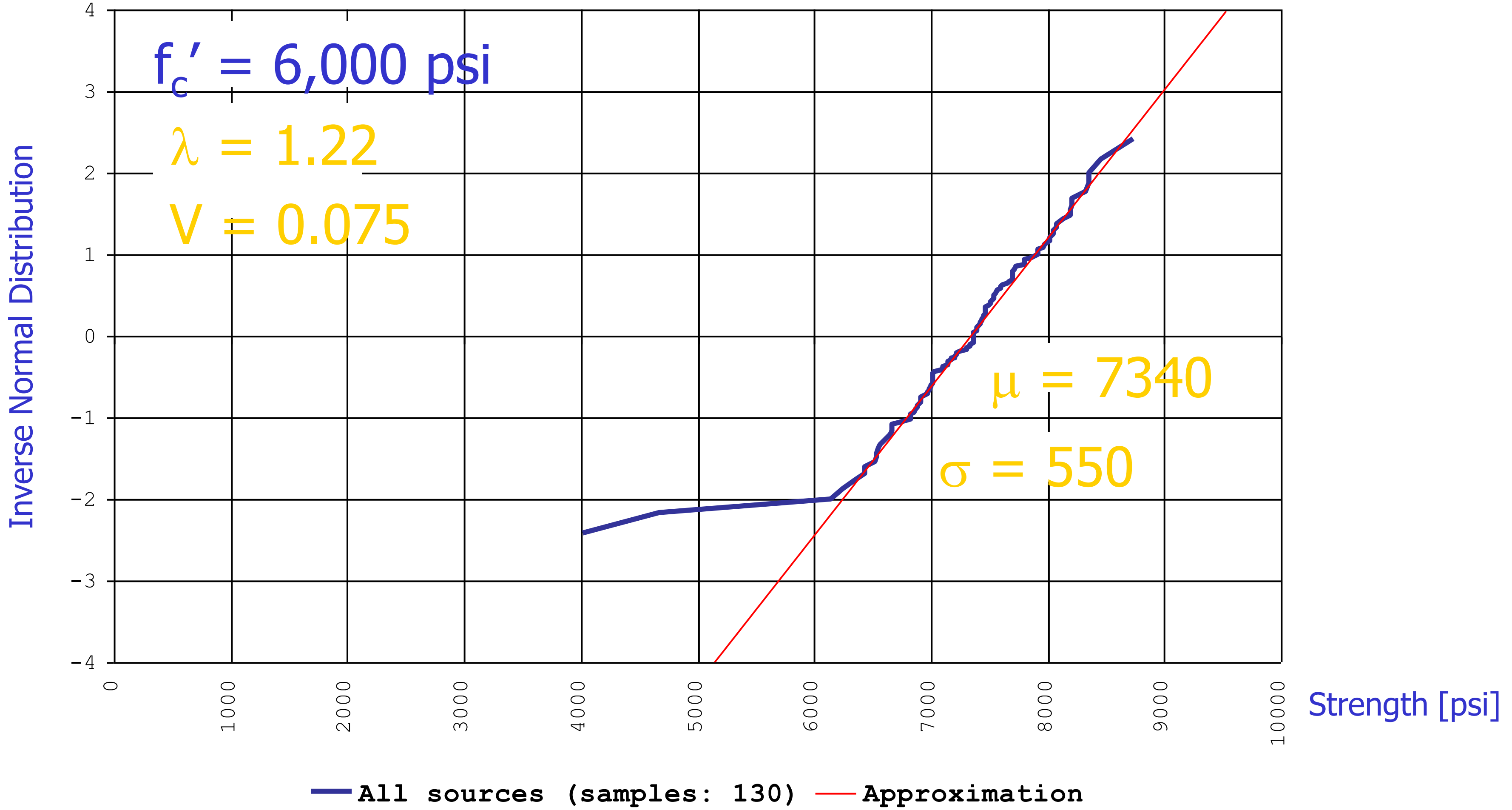
Ordinary Concrete – CDF of Strength



Ordinary Concrete – CDF of Strength

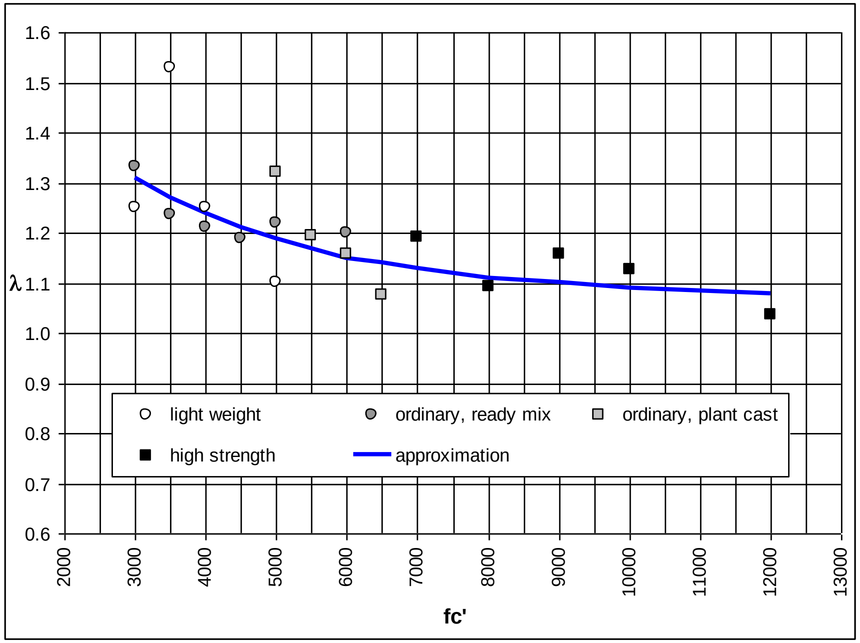


Ordinary Concrete – CDF of Strength



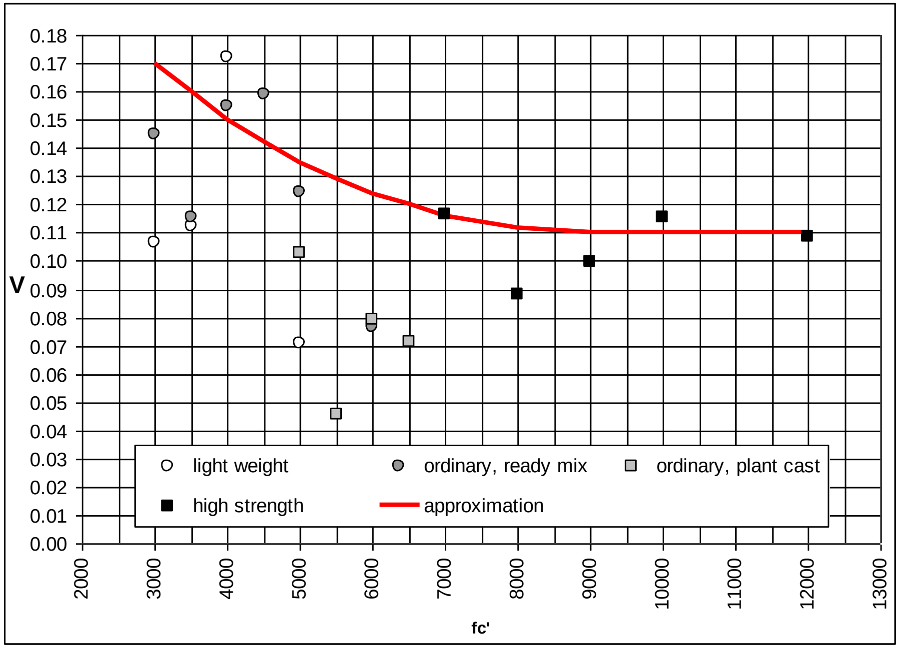
Summary of the Statistical Parameters for Concrete

λ



f_c' [psi]

V



f_c' [psi]

Recommended Bias Factor and Coefficient of Variation for Compressive Strength, and Shear Strength of Concrete

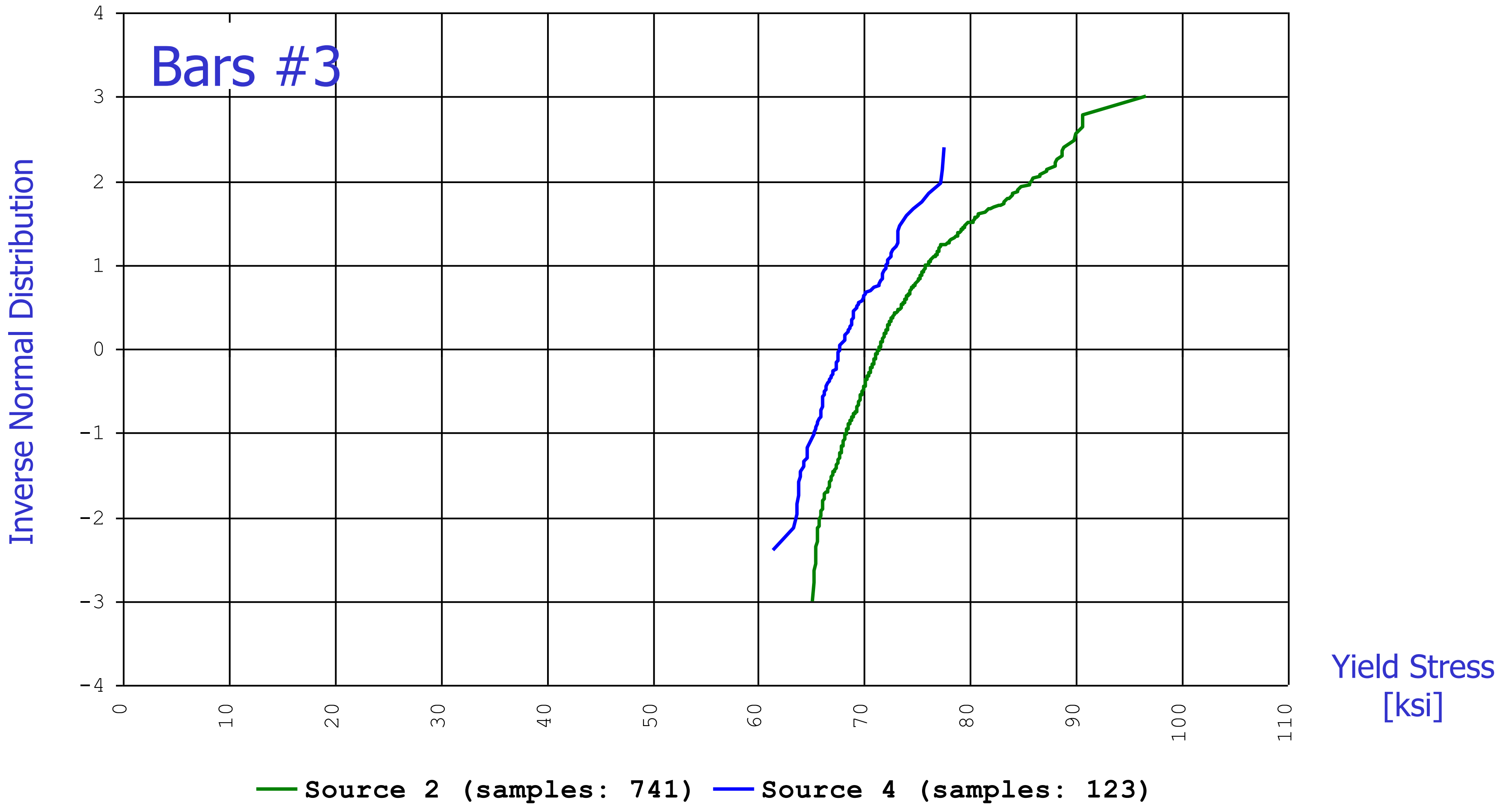
Concrete Grade f_c' (psi)			Shear Strength	
	λ	V	λ	V
3000	1.31	0.17	1.31	0.205
3500	1.27	0.16	1.27	0.19
4000	1.24	0.15	1.24	0.18
4500	1.21	0.14	1.21	0.17
5000	1.19	0.135	1.19	0.16
5500	1.17	0.13	1.17	0.155
6000	1.15	0.125	1.15	0.15
6500	1.14	0.12	1.14	0.145
7000	1.13	0.115	1.13	0.14
8000	1.11	0.11	1.11	0.135
9000	1.10	0.11	1.10	0.135
10,000	1.09	0.11	1.09	0.135
12,000	1.08	0.11	1.08	0.135

Reinforcing Steel Bars, Grade 60 – Number of Samples

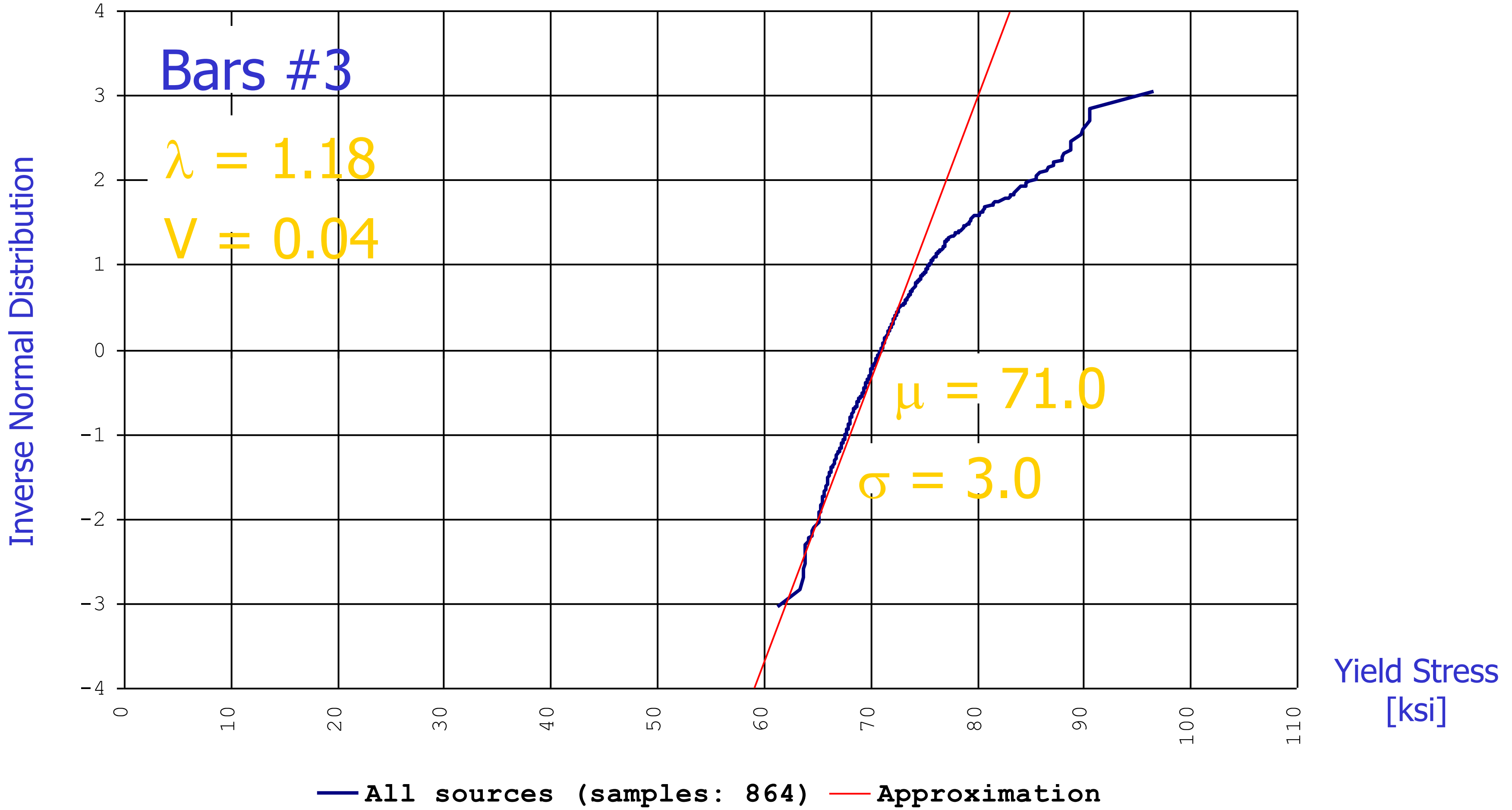
Bar Size:

Source:	#3	#4	#5	#6	#7	#8	#9	#10	#11	#14	Total:
1		60	60	60	60	60	60	60	60		
2	741	2369	3333	1141	1318	1146	1290	825	1019		
3		60	60	60	60	60	60	60	60	12	
4	123	106	179	104	79	90	73	70	87		
5		90	90	90	90	90	90	74	90		
Total:	864	2685	3722	1455	1607	1446	1573	1089	1316	12	15769

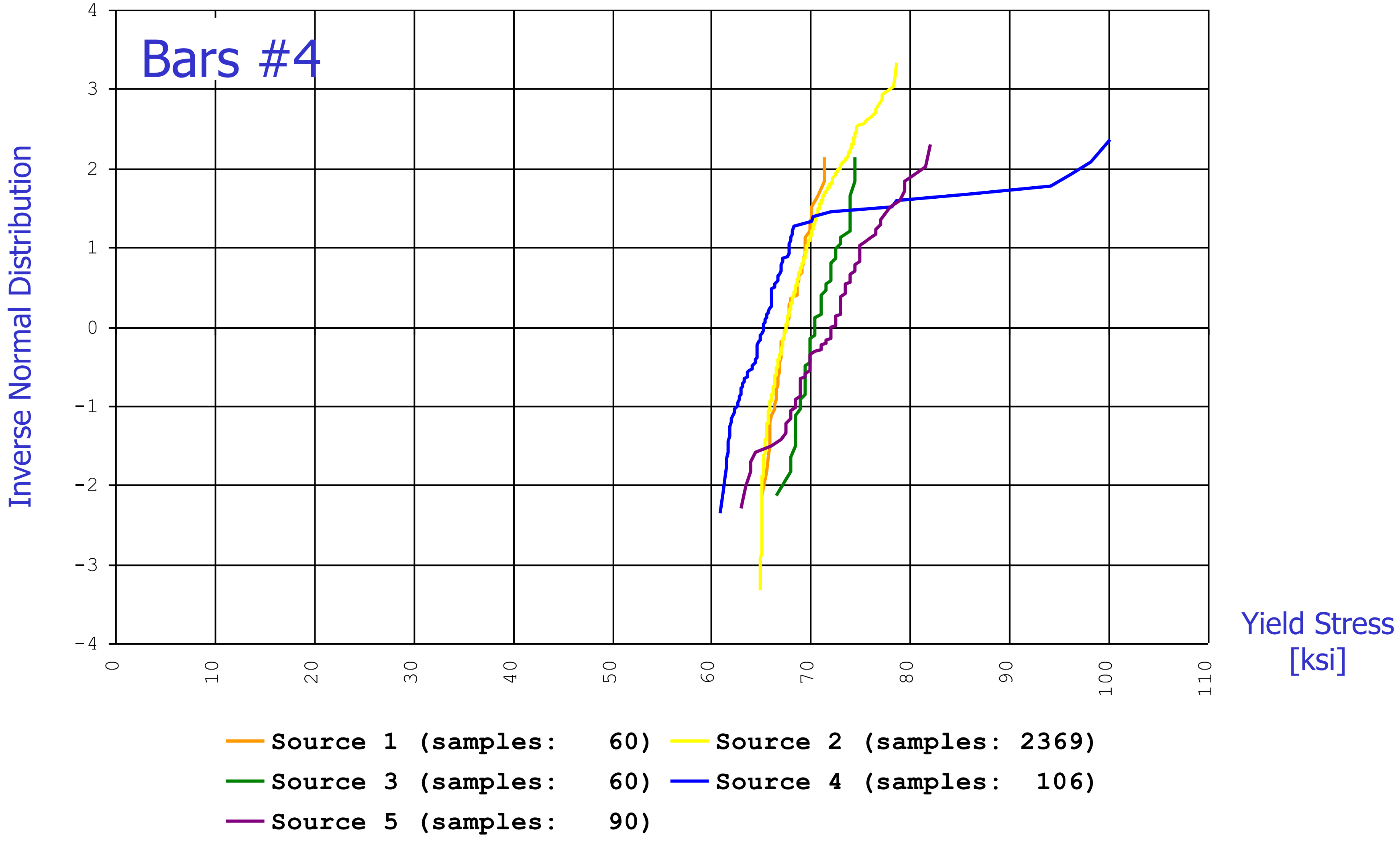
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



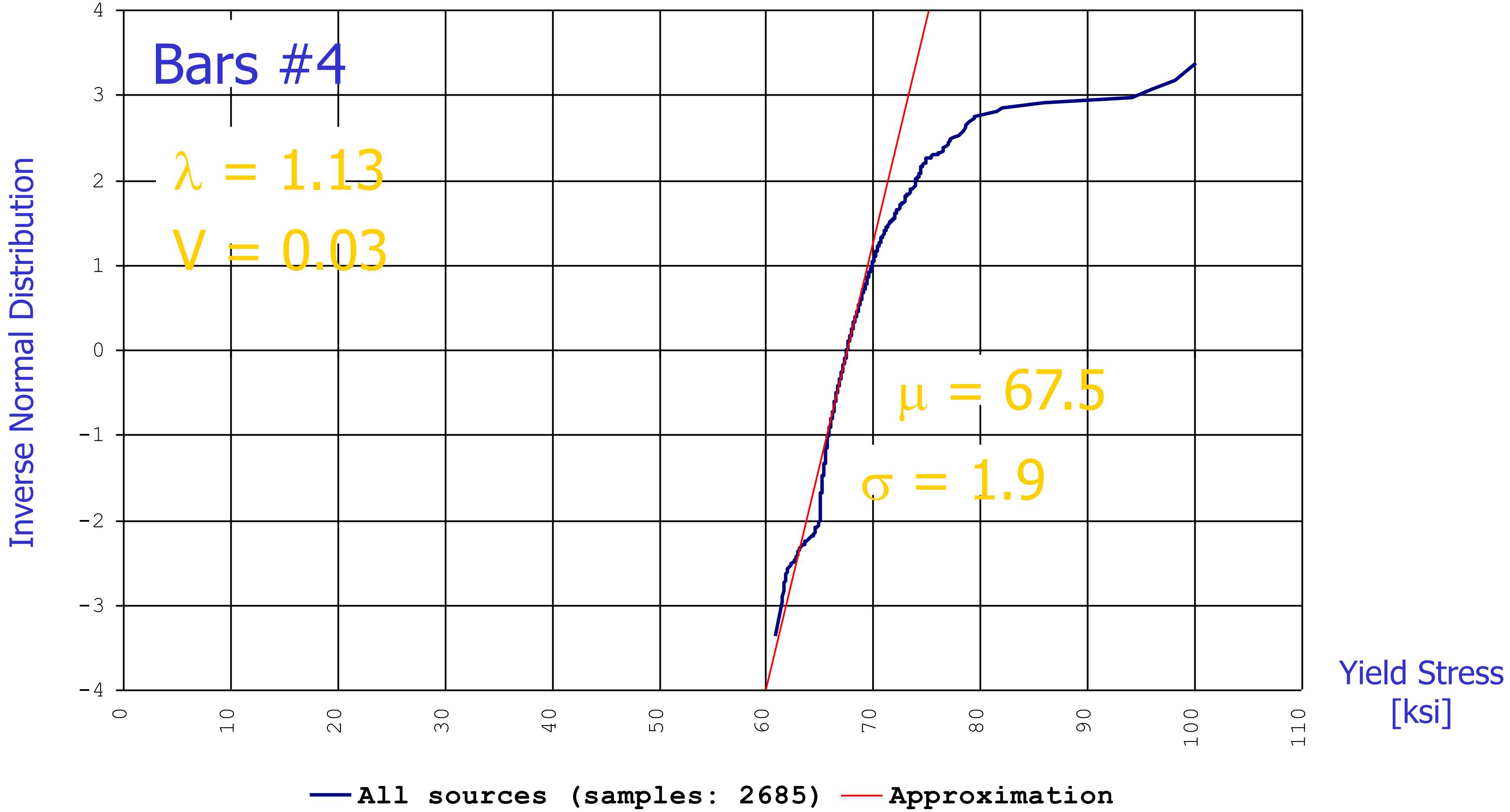
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



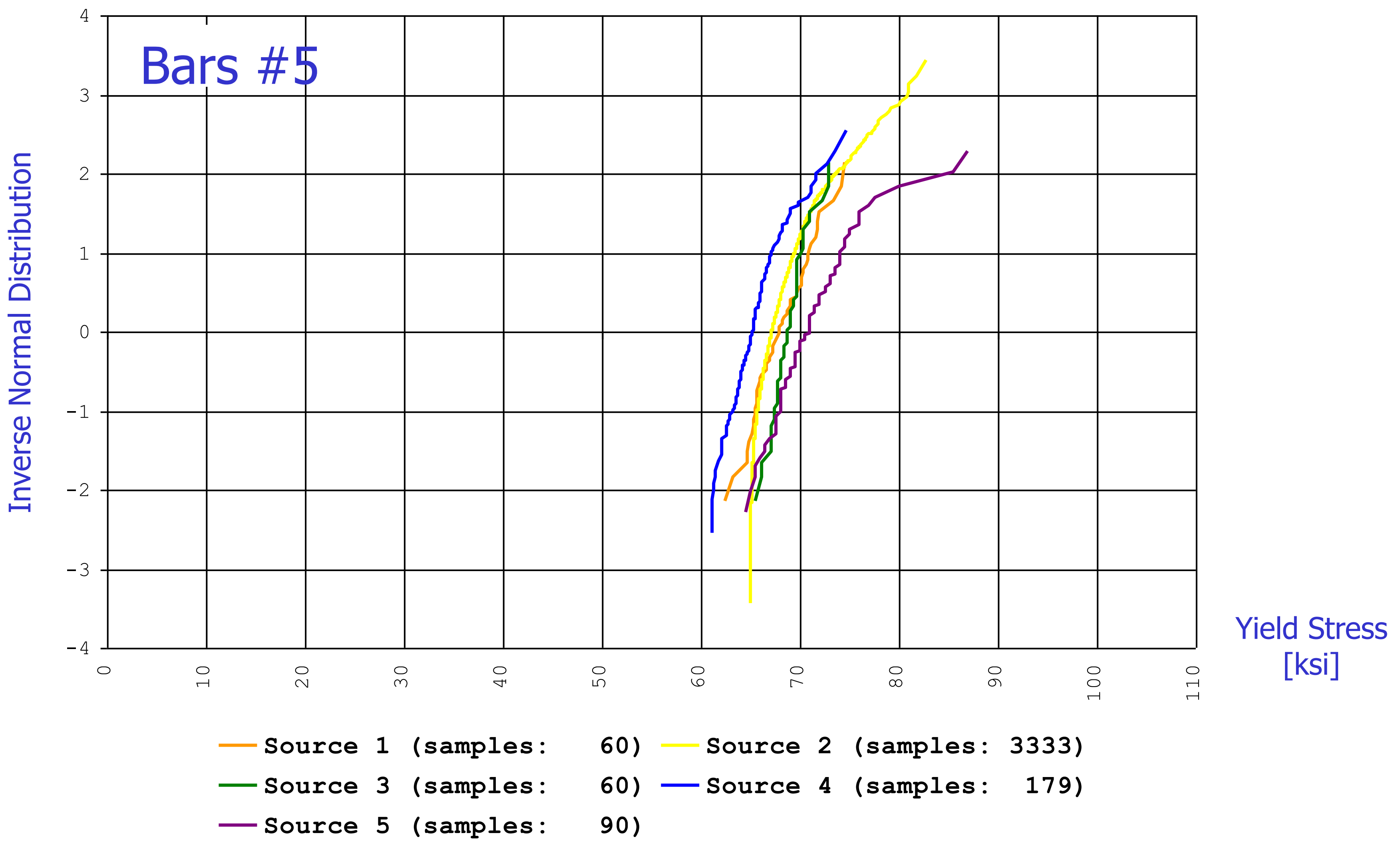
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



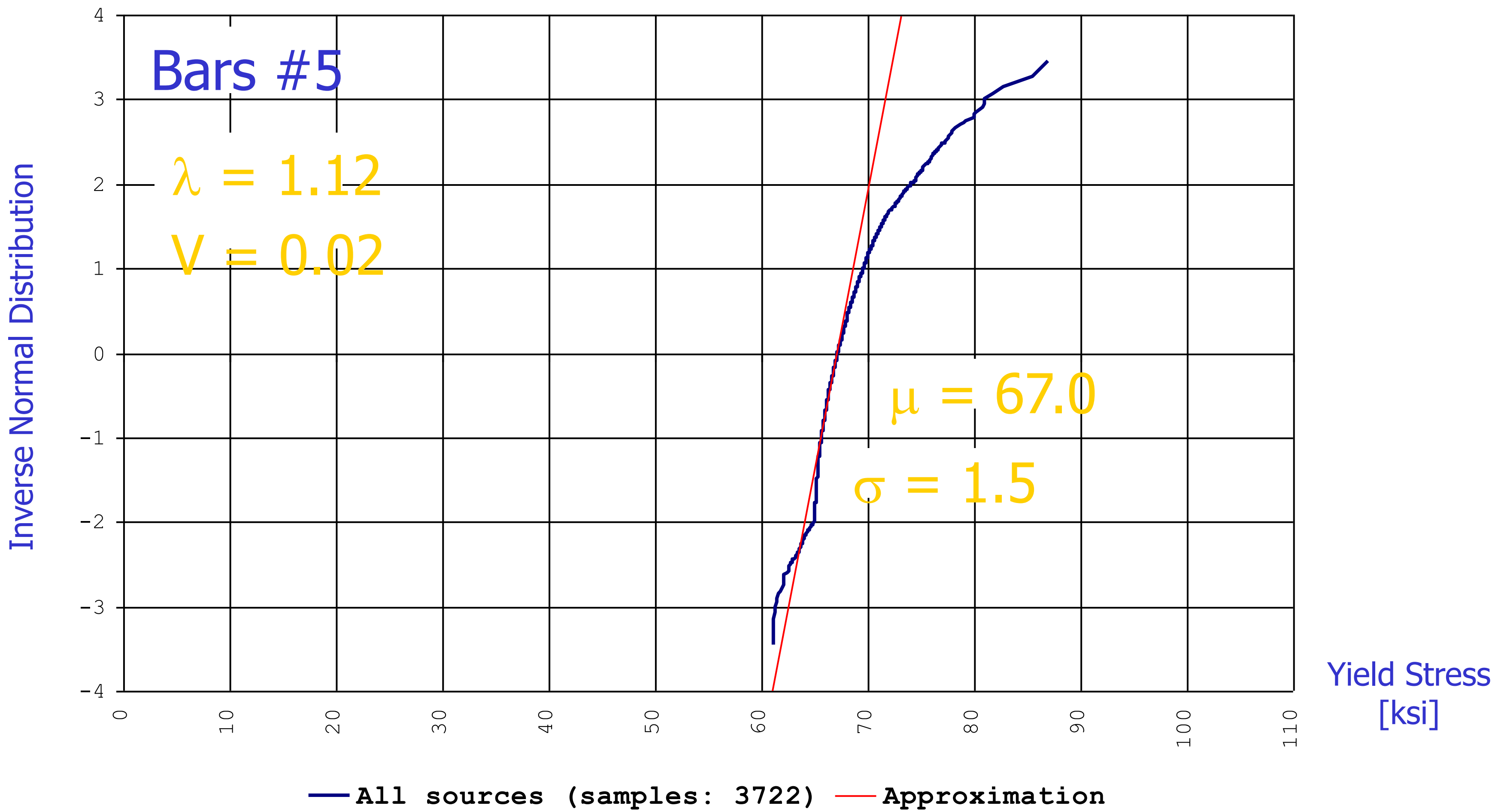
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



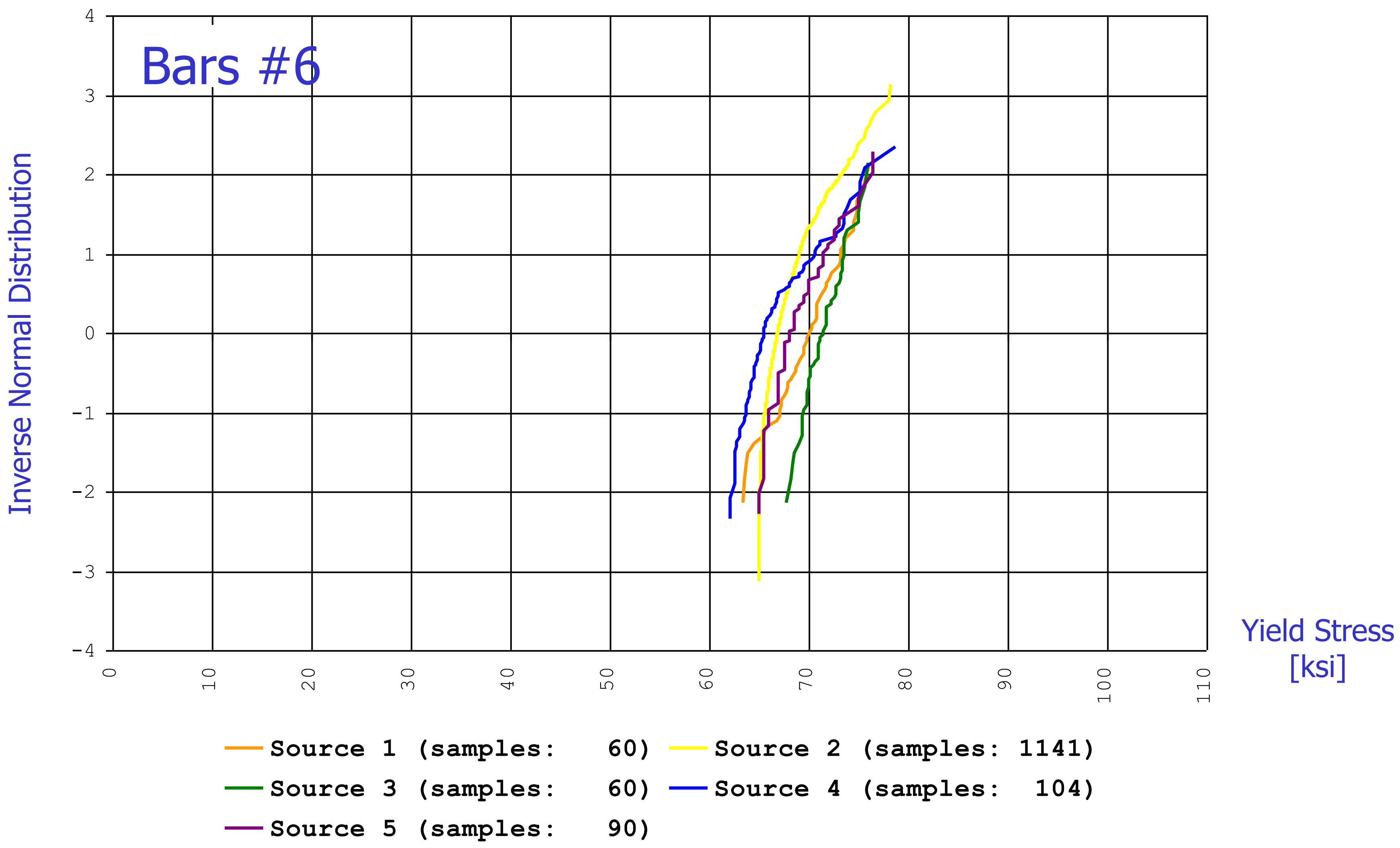
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



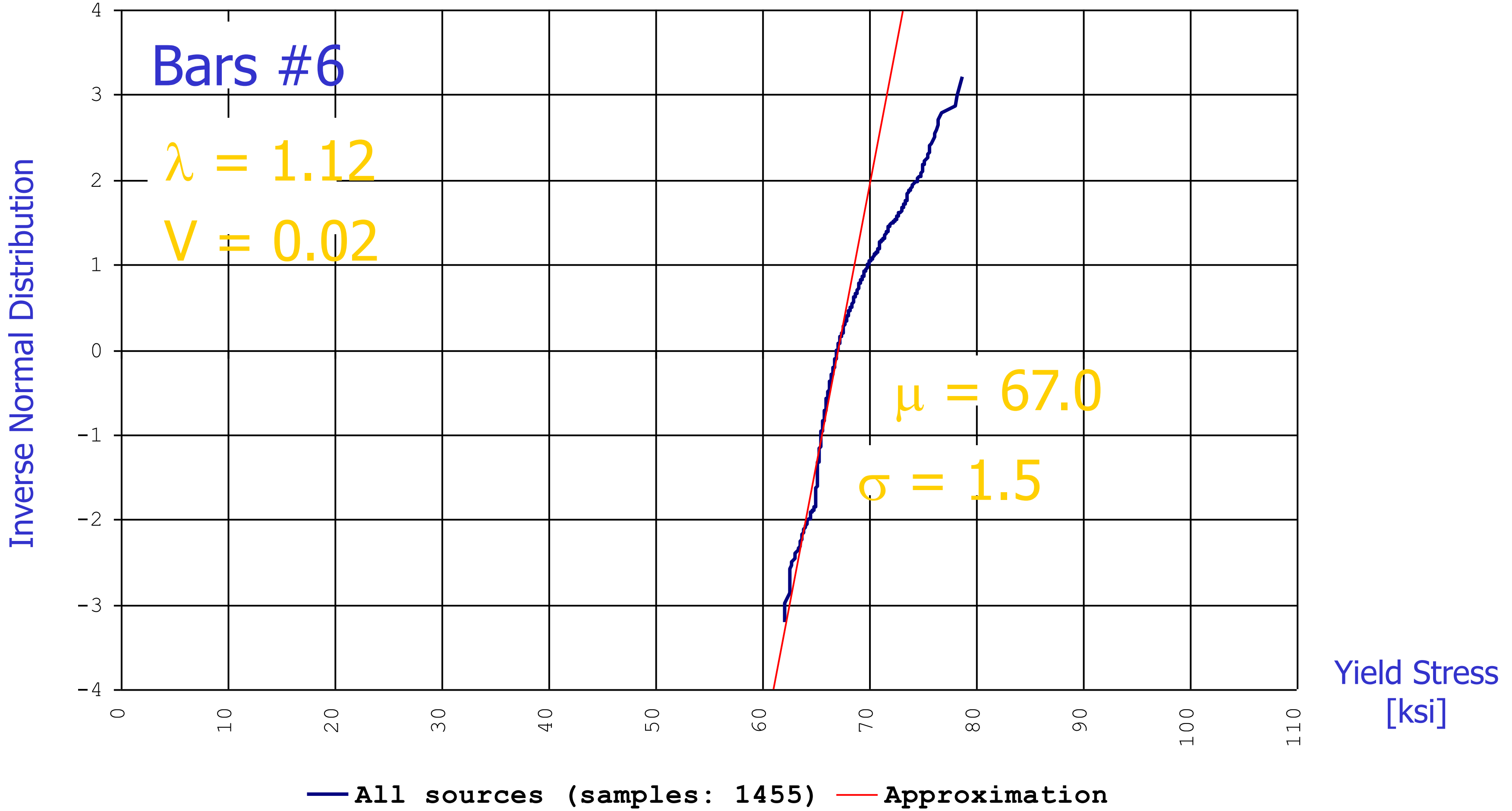
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



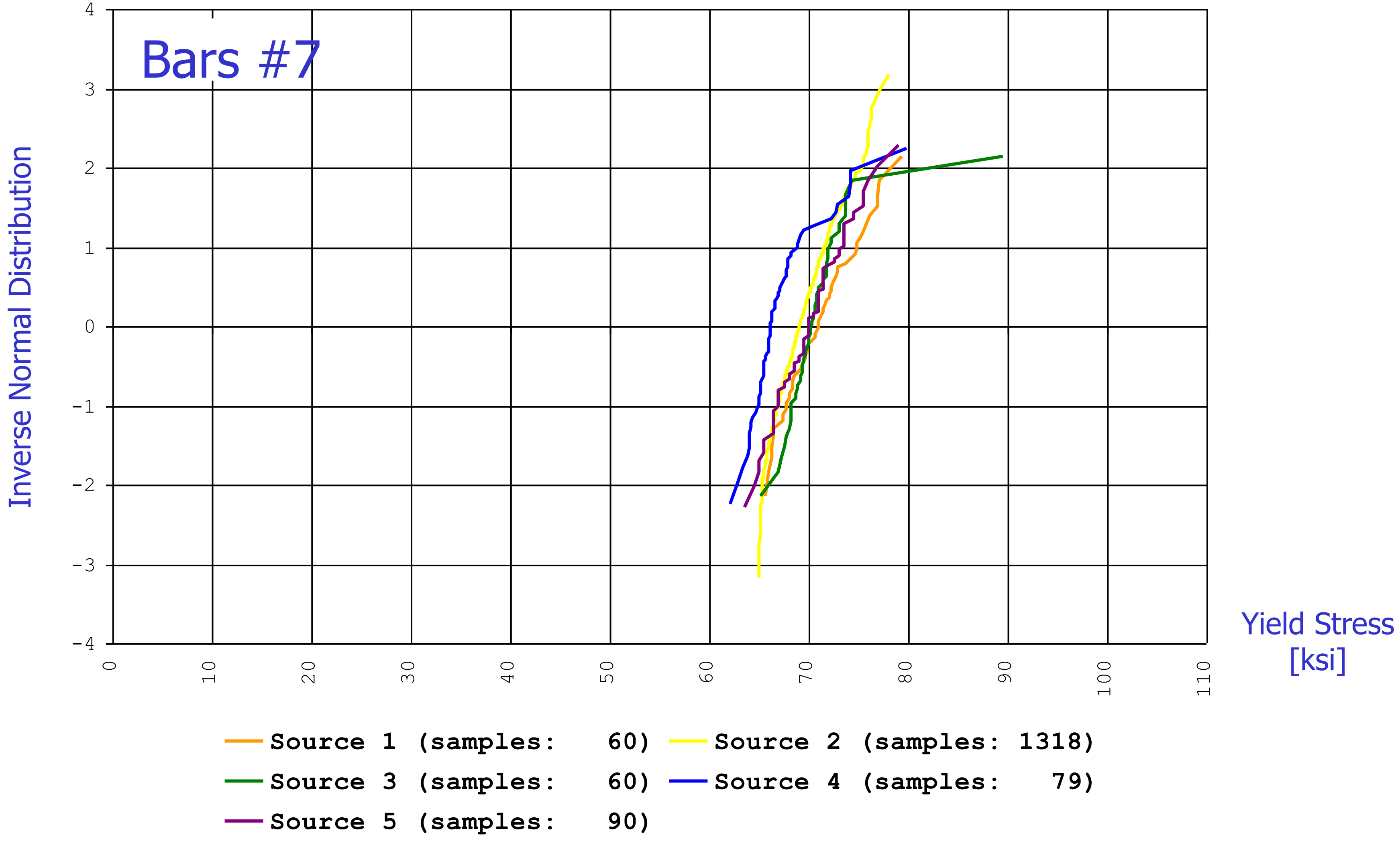
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



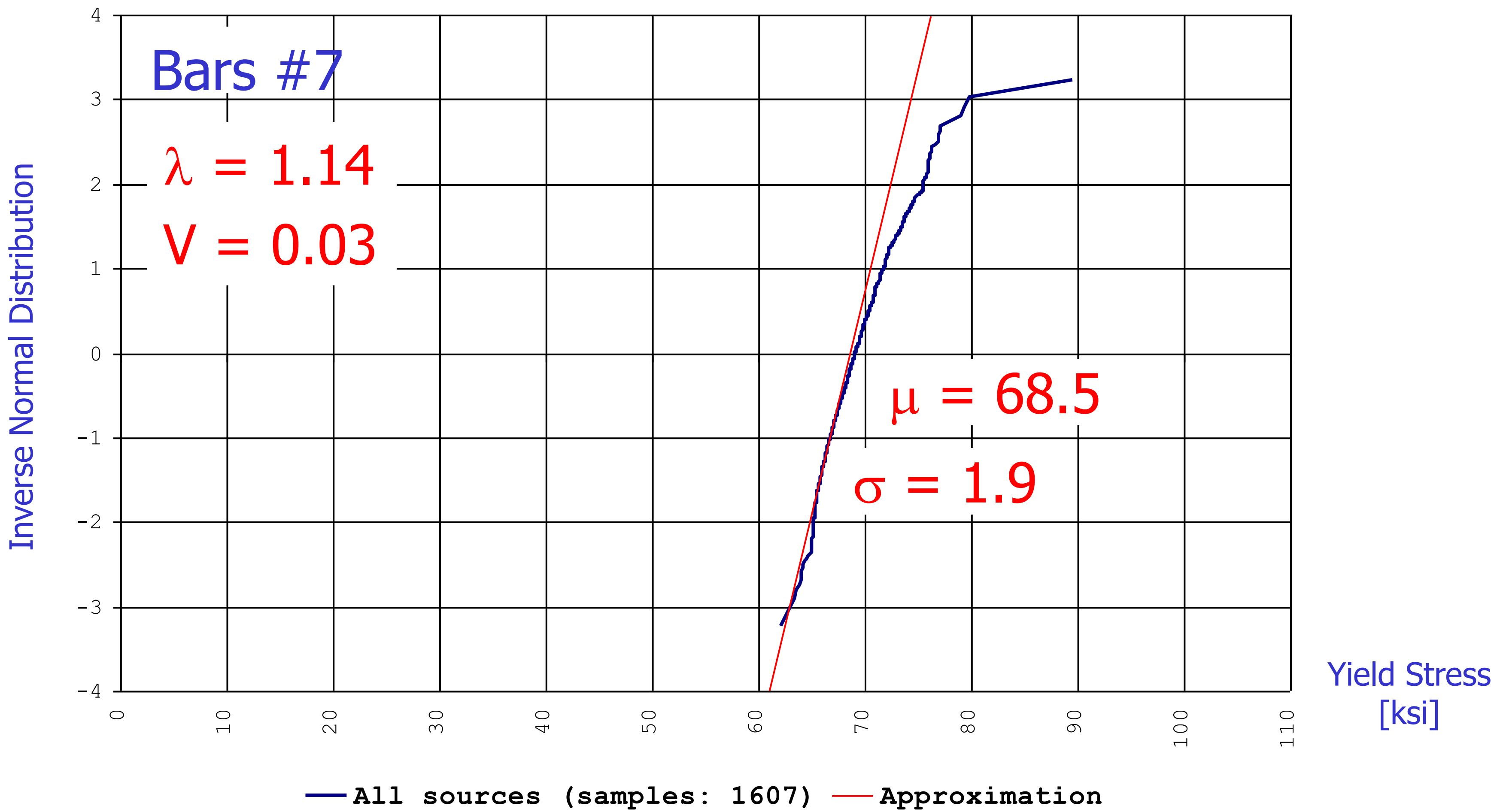
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



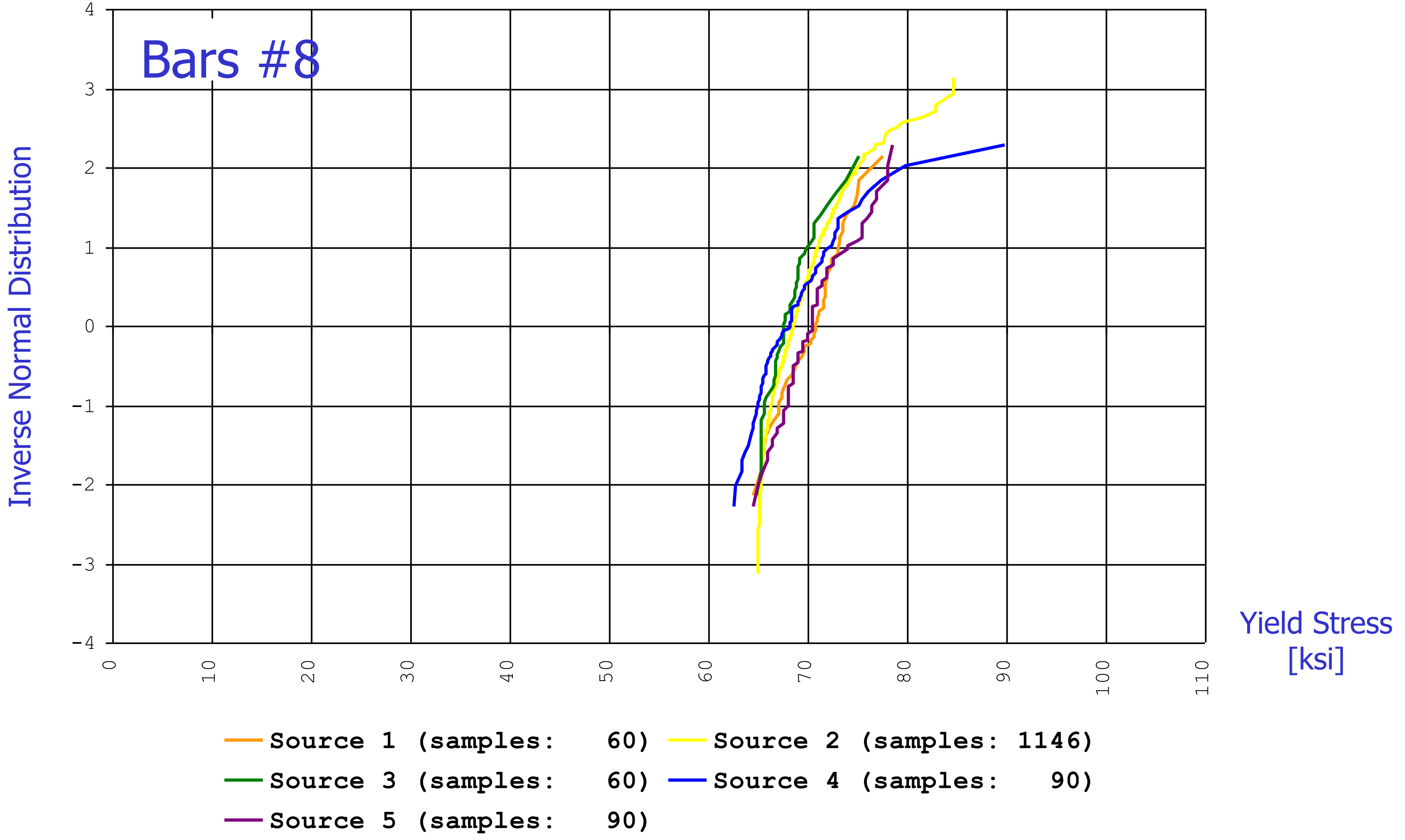
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



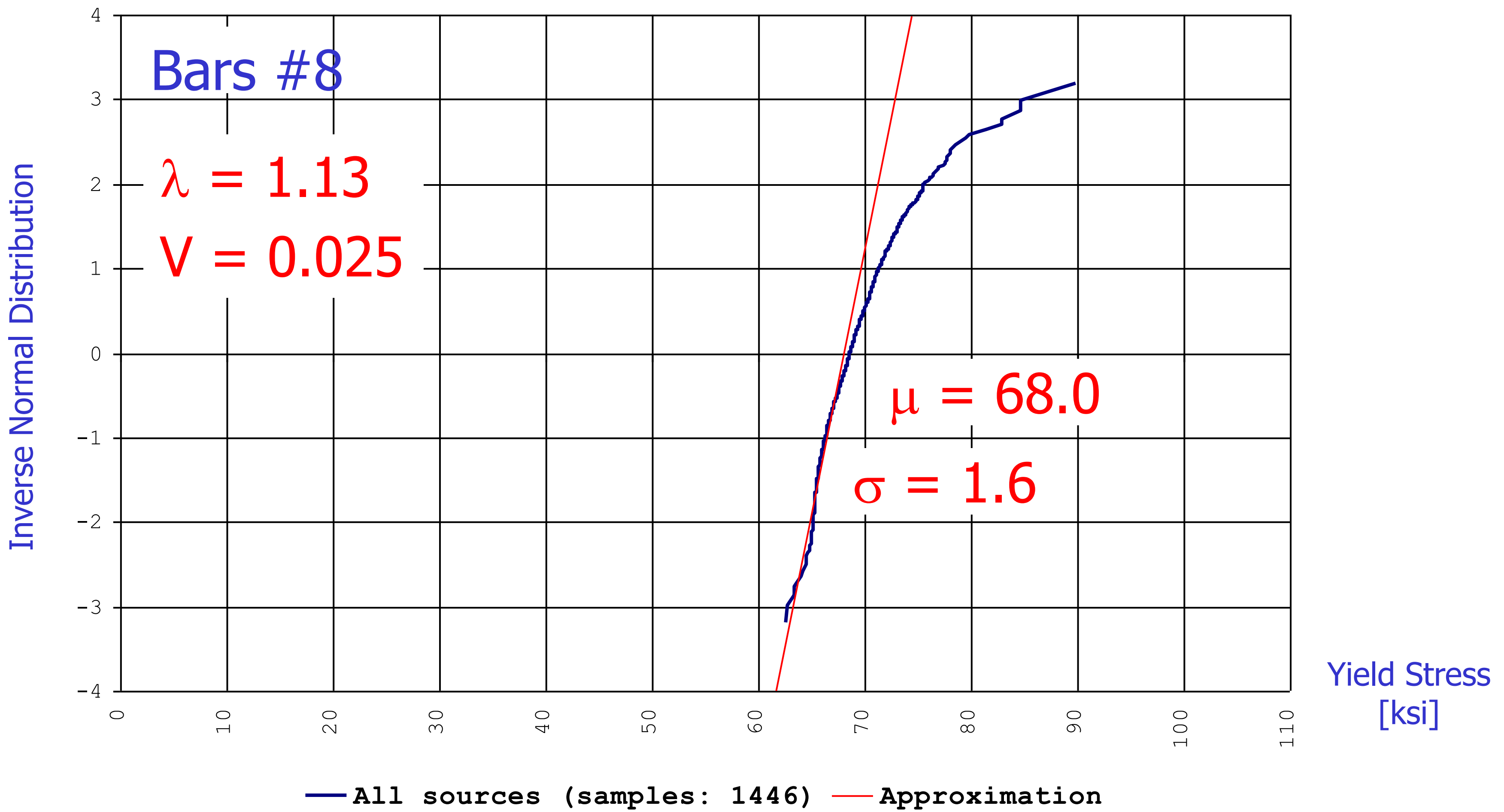
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



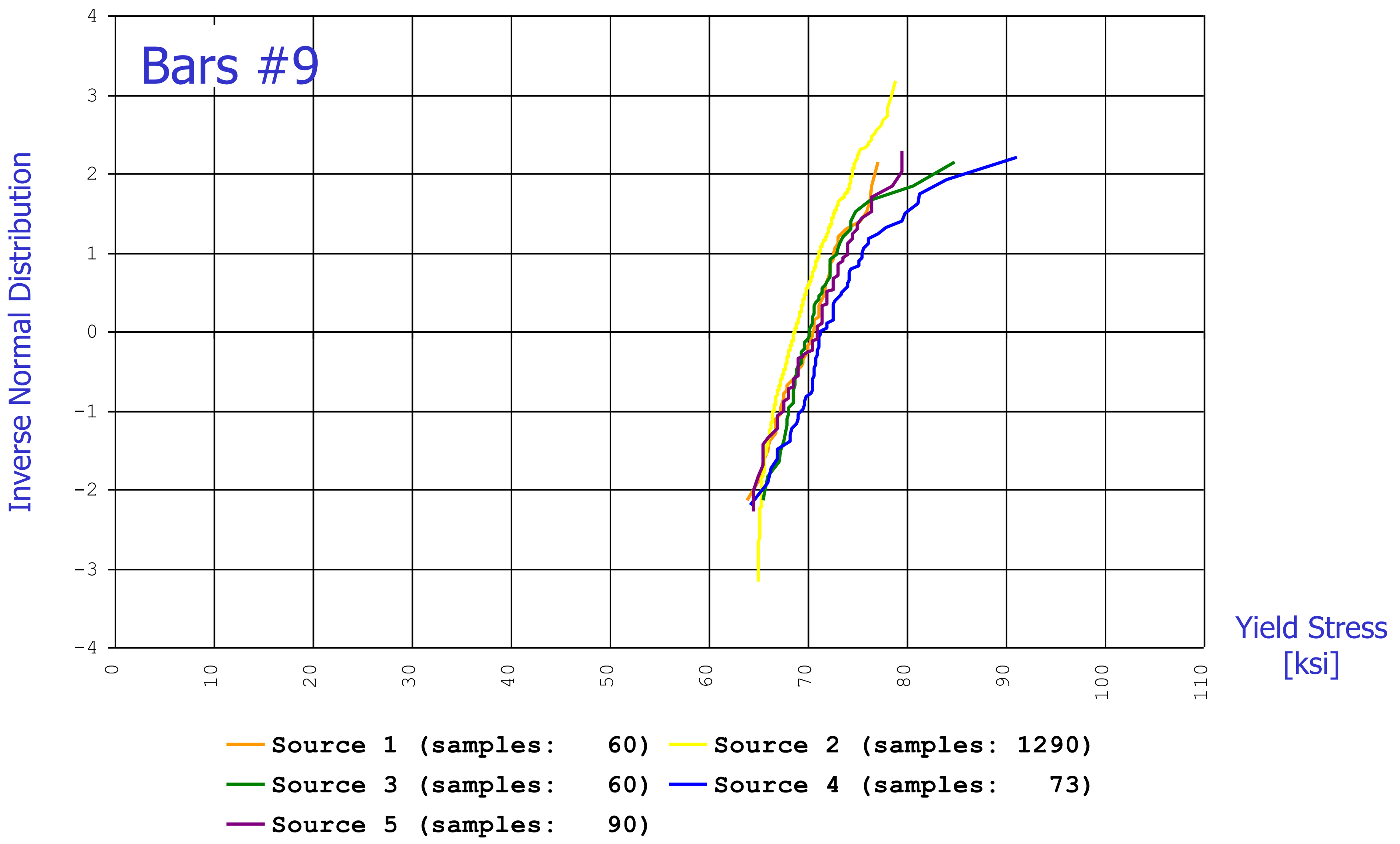
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



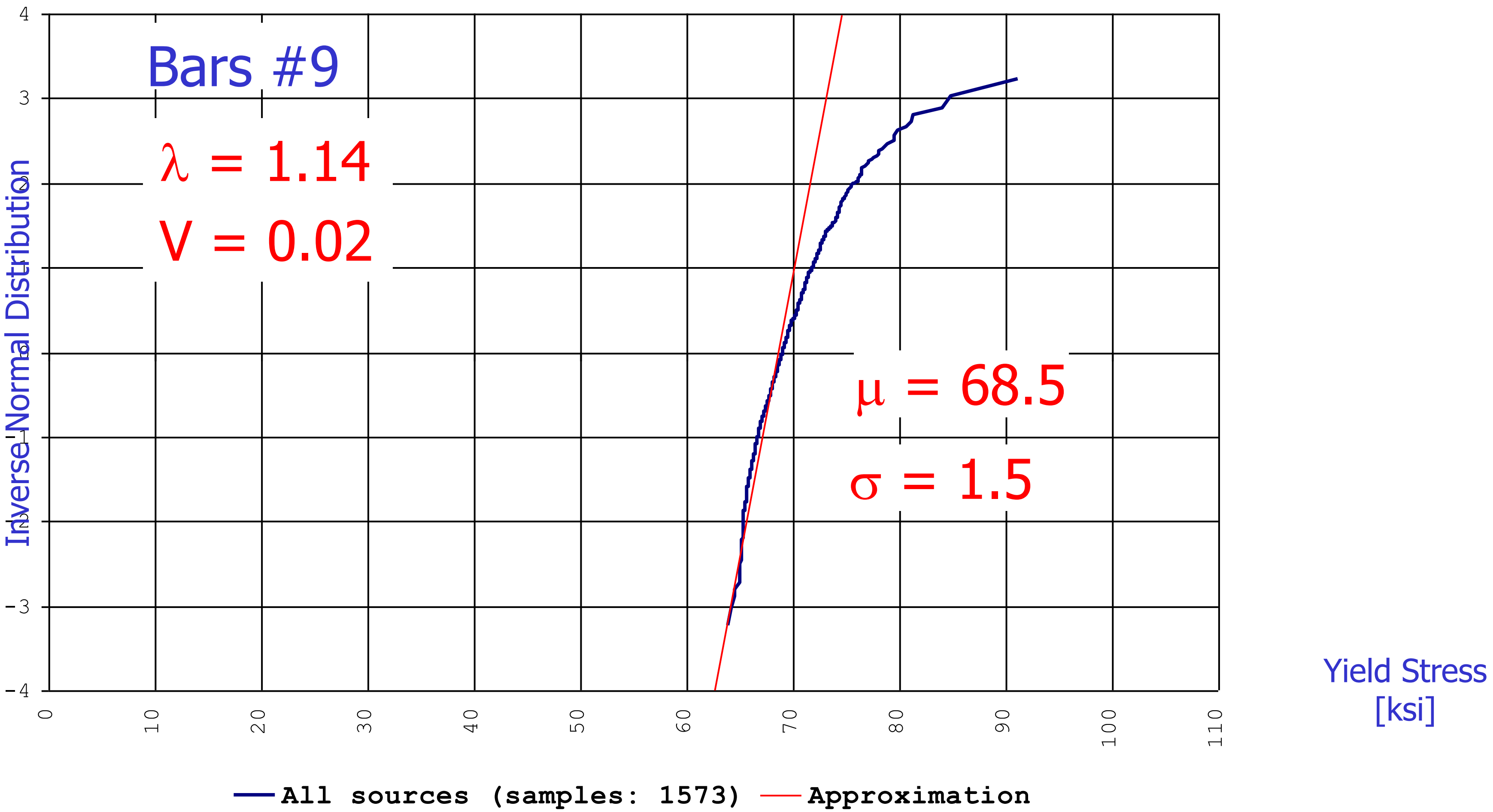
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



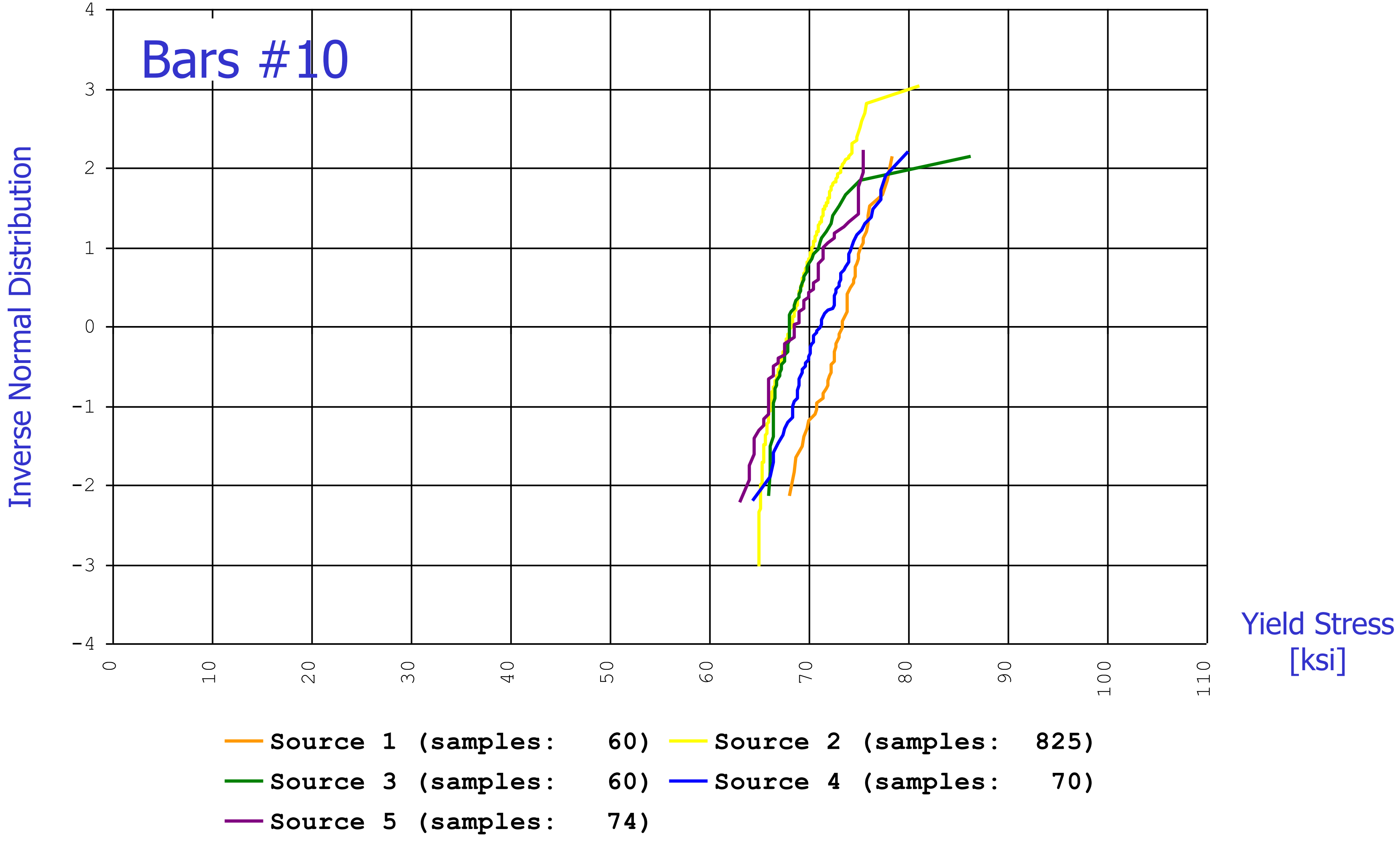
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



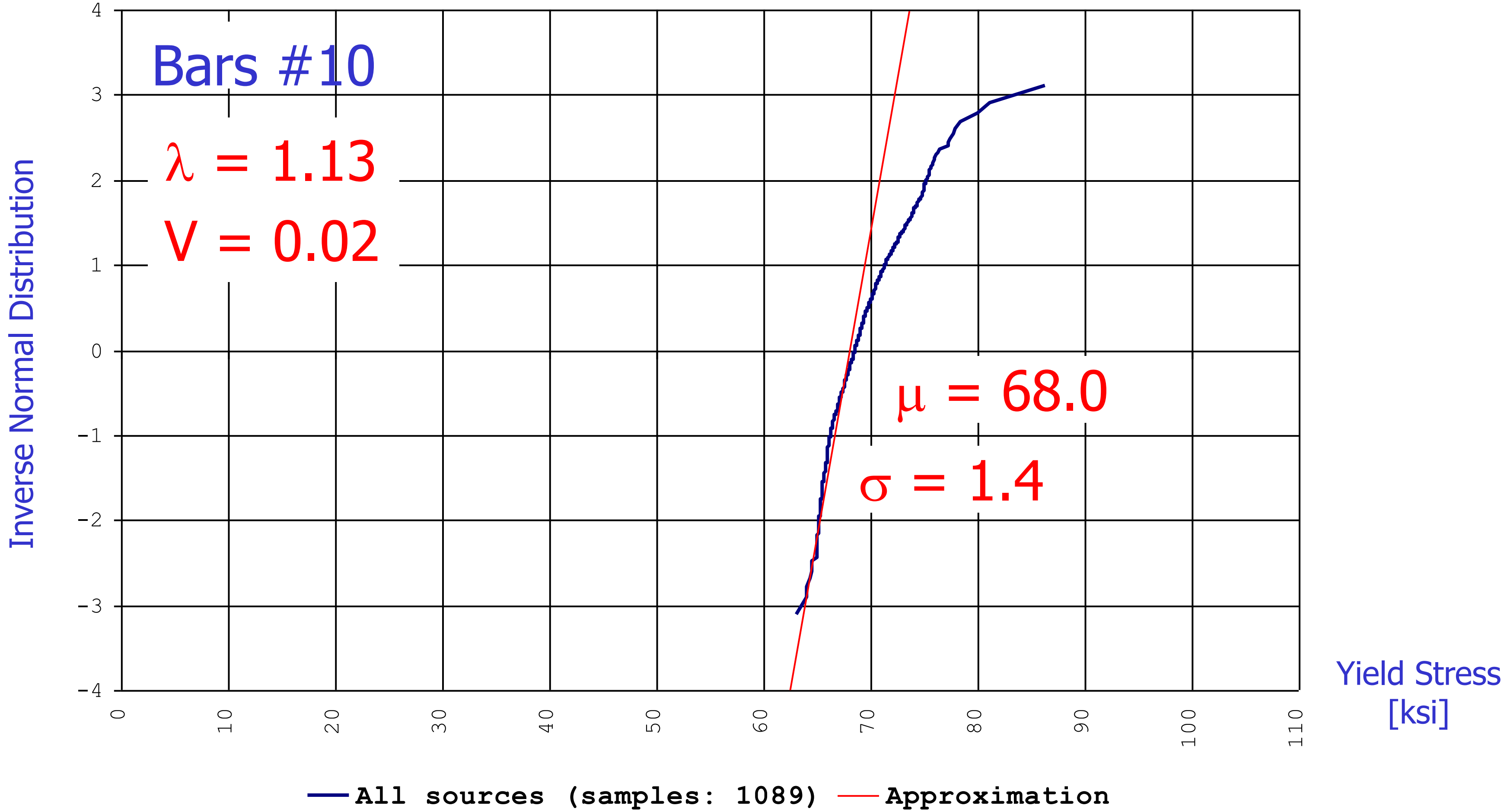
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



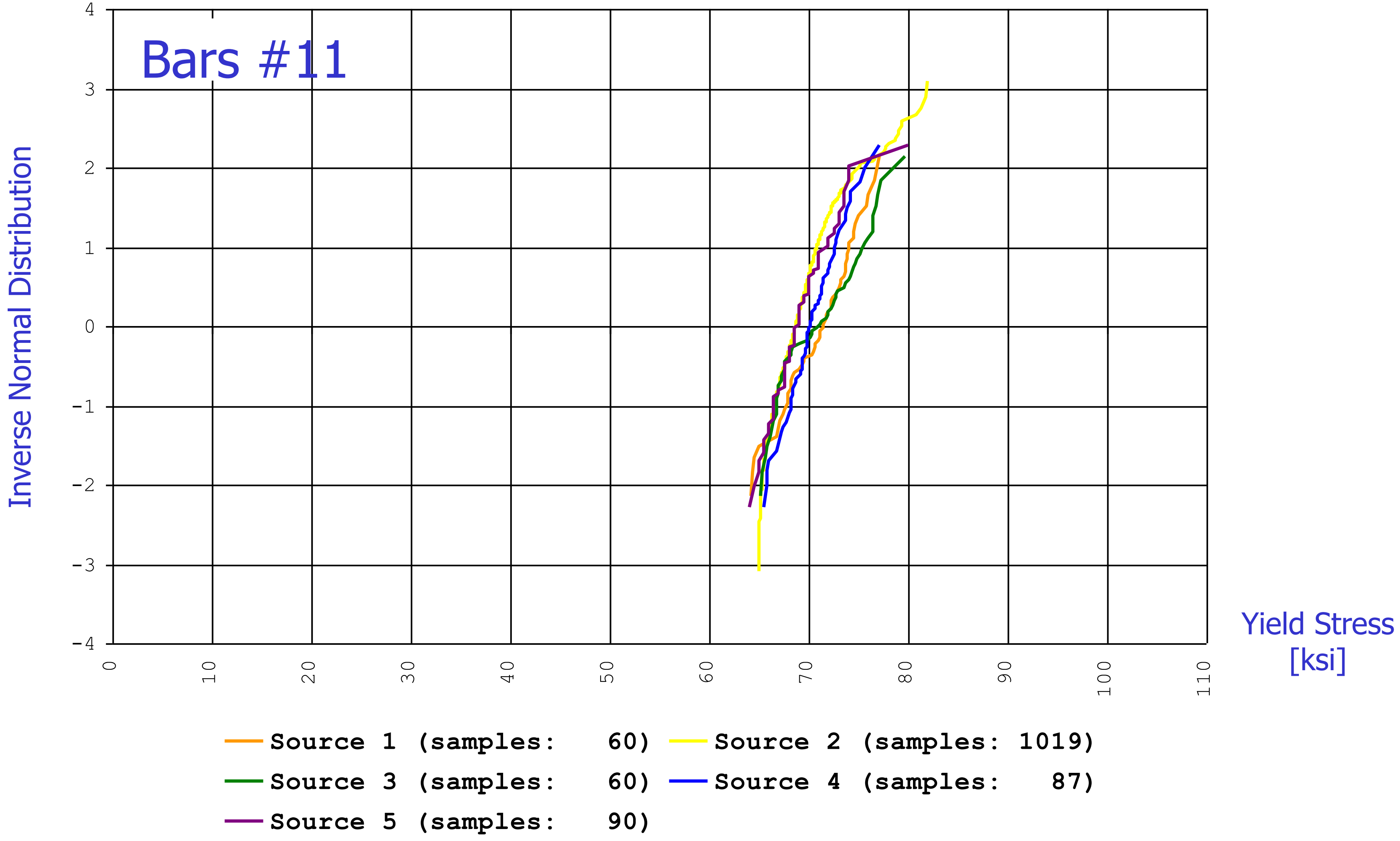
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



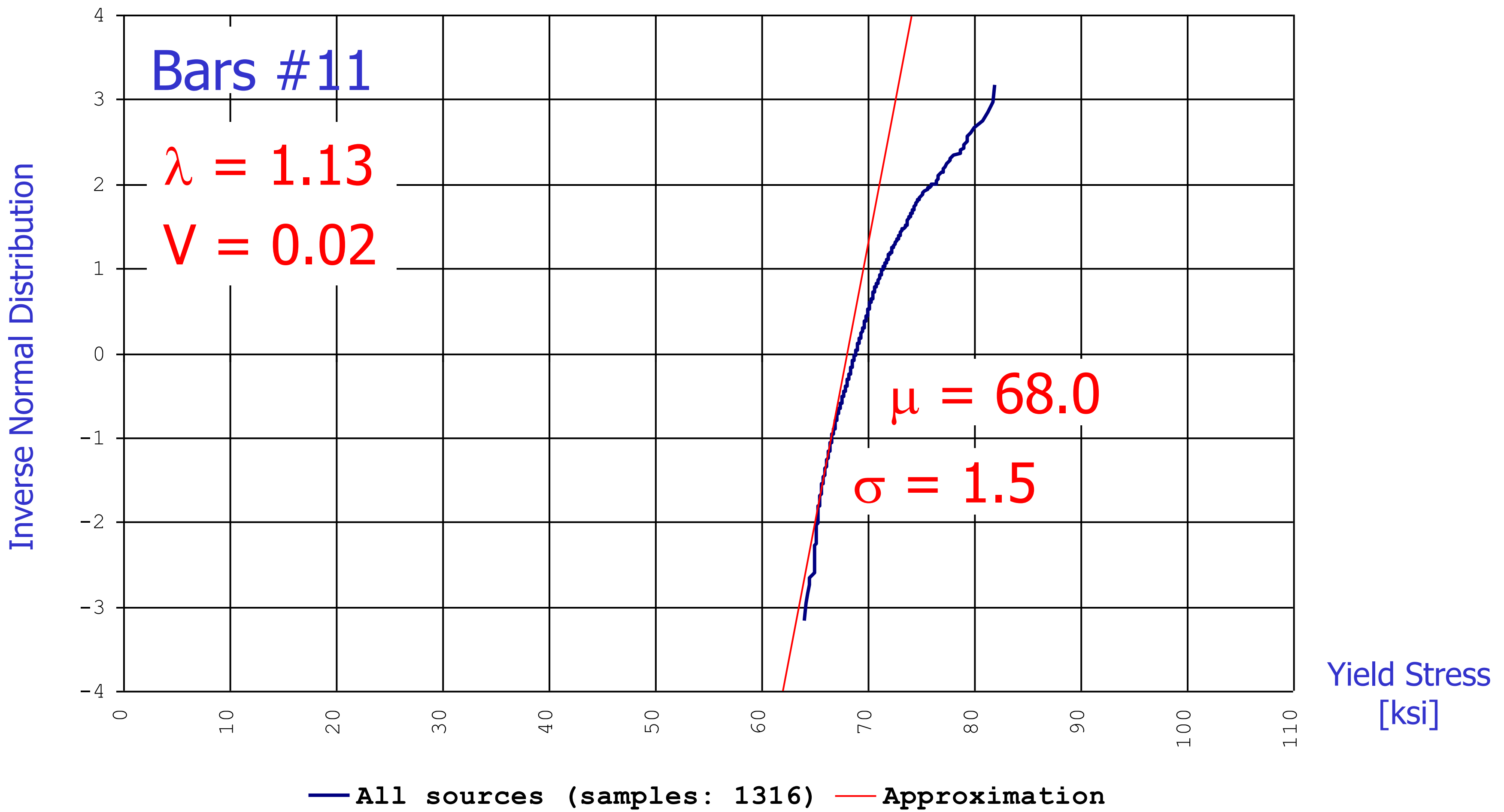
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



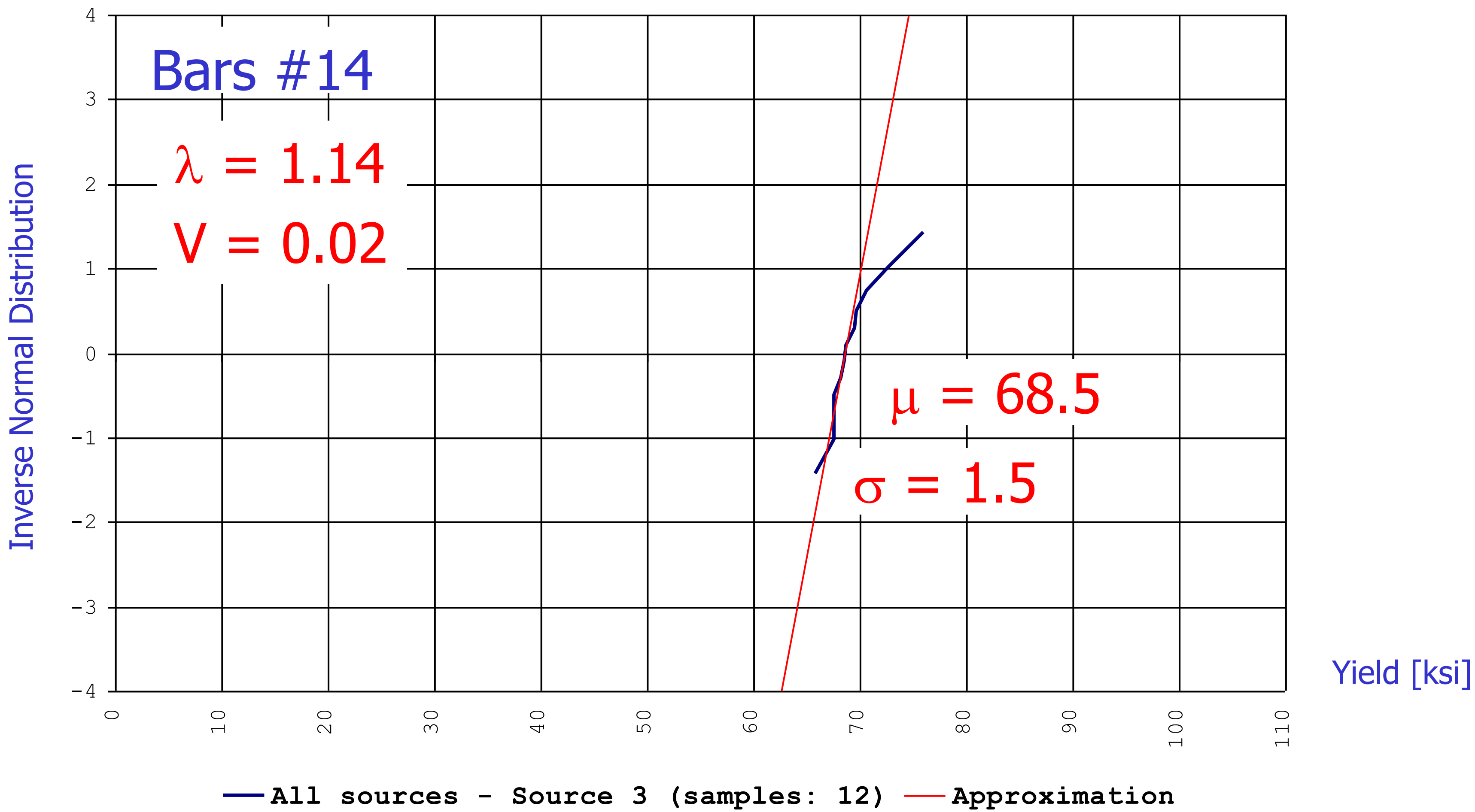
Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



Reinforcing Steel Bars, Grade 60 – CDF of Yield Stress



Reinforcing Steel Bars, Grade 60 – CDF of Yield

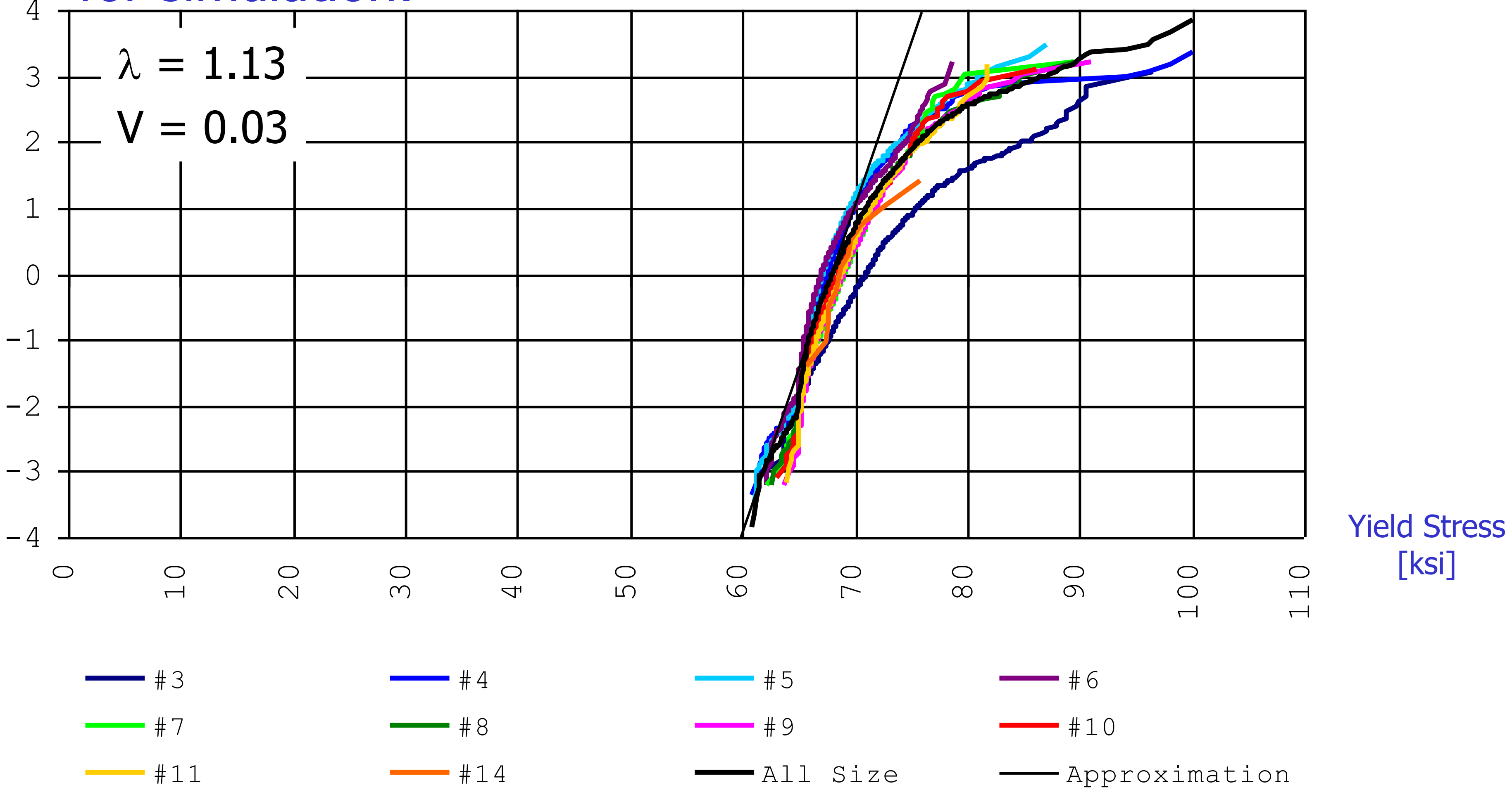


Reinforcing Steel Bars, Grade 60 – Statistical Parameters

Bar Size	λ	V
# 3	1.18	0.04
# 4	1.13	0.03
# 5	1.12	0.02
# 6	1.12	0.02
# 7	1.14	0.03
# 8	1.13	0.025
# 9	1.14	0.02
#10	1.13	0.02
#11	1.13	0.02
#14	1.14	0.02

Statistical Parameters assumed for Monte Carlo Simulations - Reinforcing Steel Bars, Grade 60

for simulation:

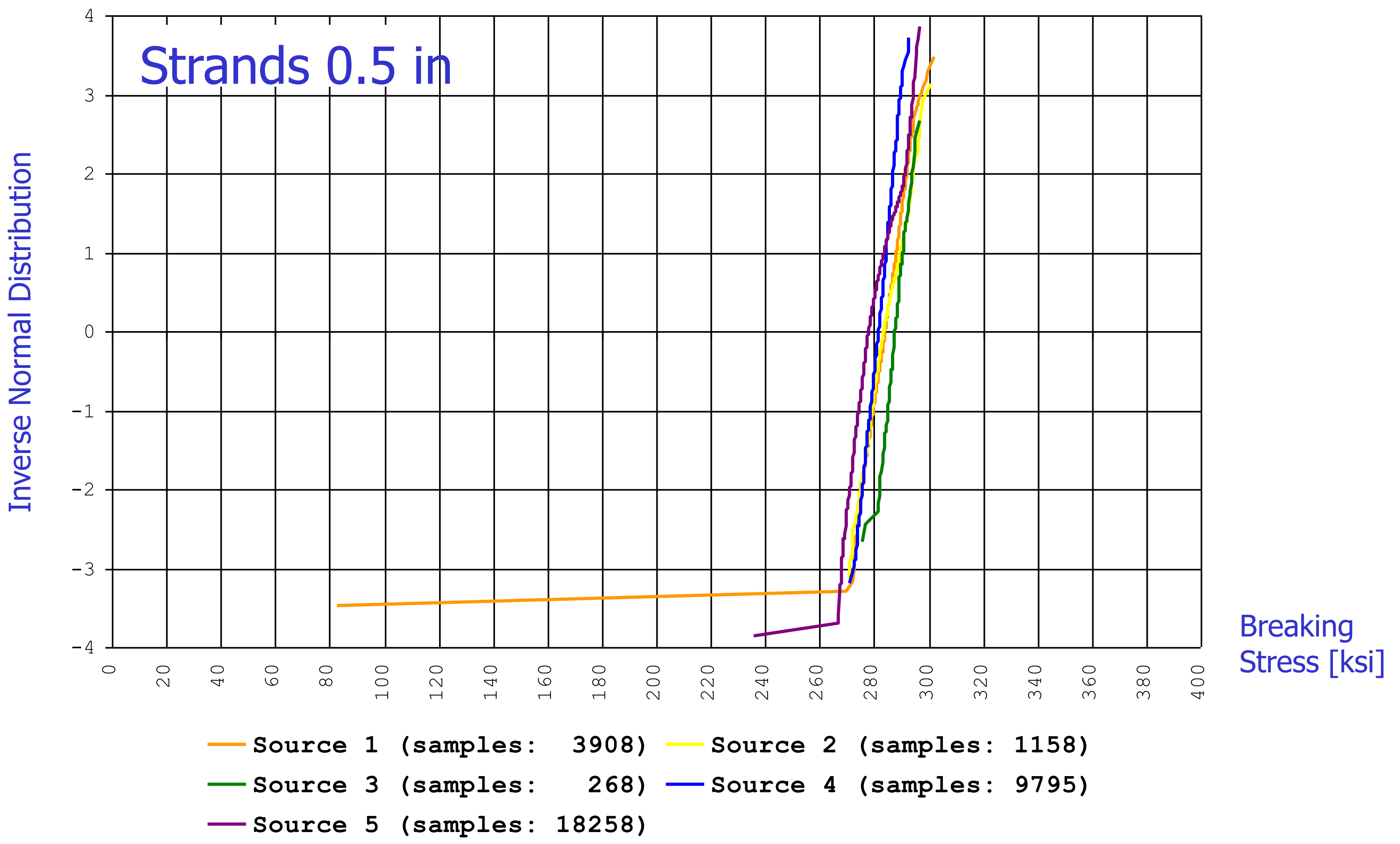


Prestressing Steel (7-wire strands), Grade 270 – Number of Samples

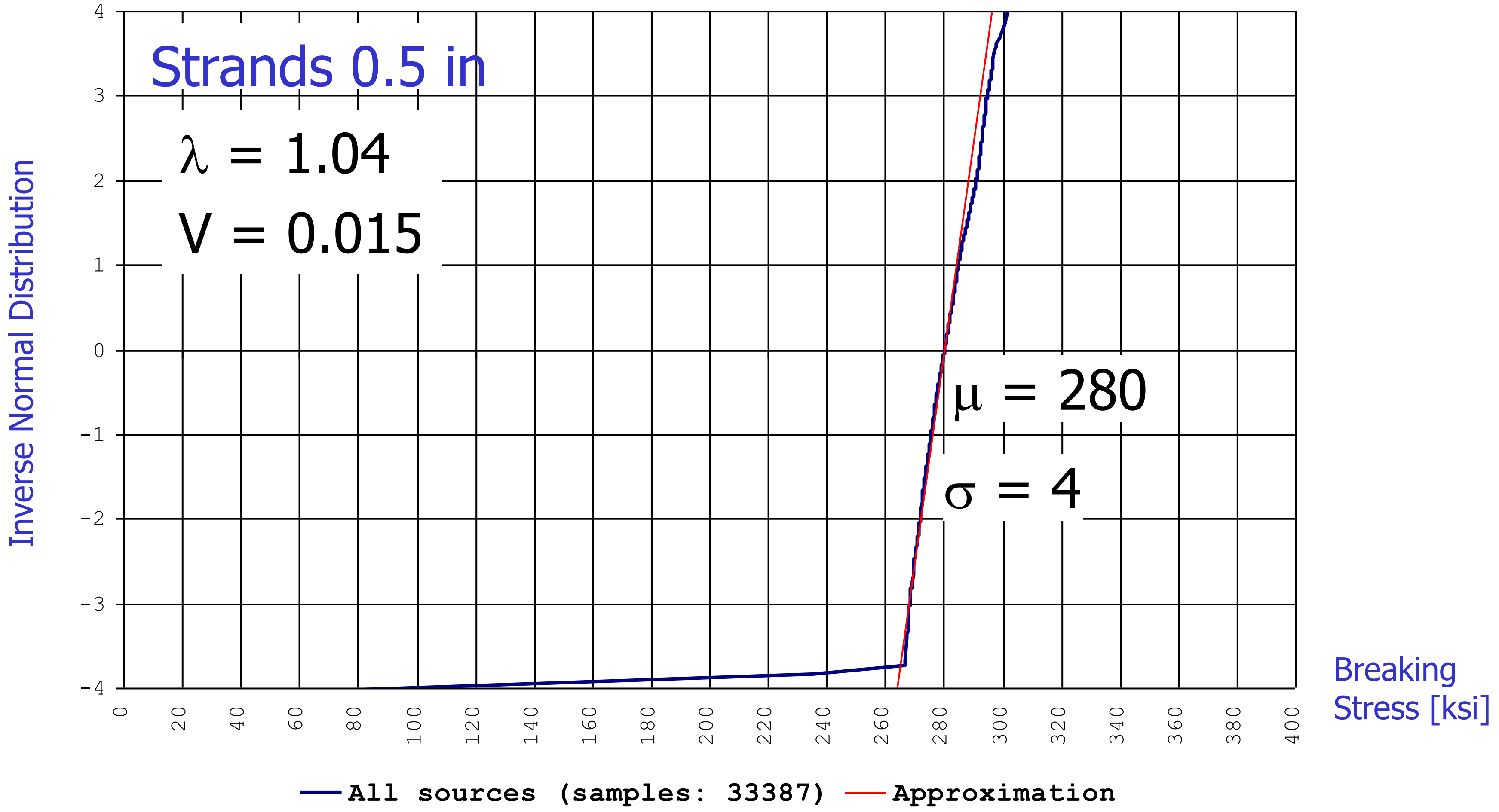
Source:	Nominal Diameter [in]: 1/2 (0.500)	0.600	
1	3908	700	
2	1158	785	
3	268	212	
4	9795	3442	
5	18258	8889	Total:
Total:	33387	14028	47415

Prestressing Steel (7-wire strands), Grade 270

CDF of Breaking Stress

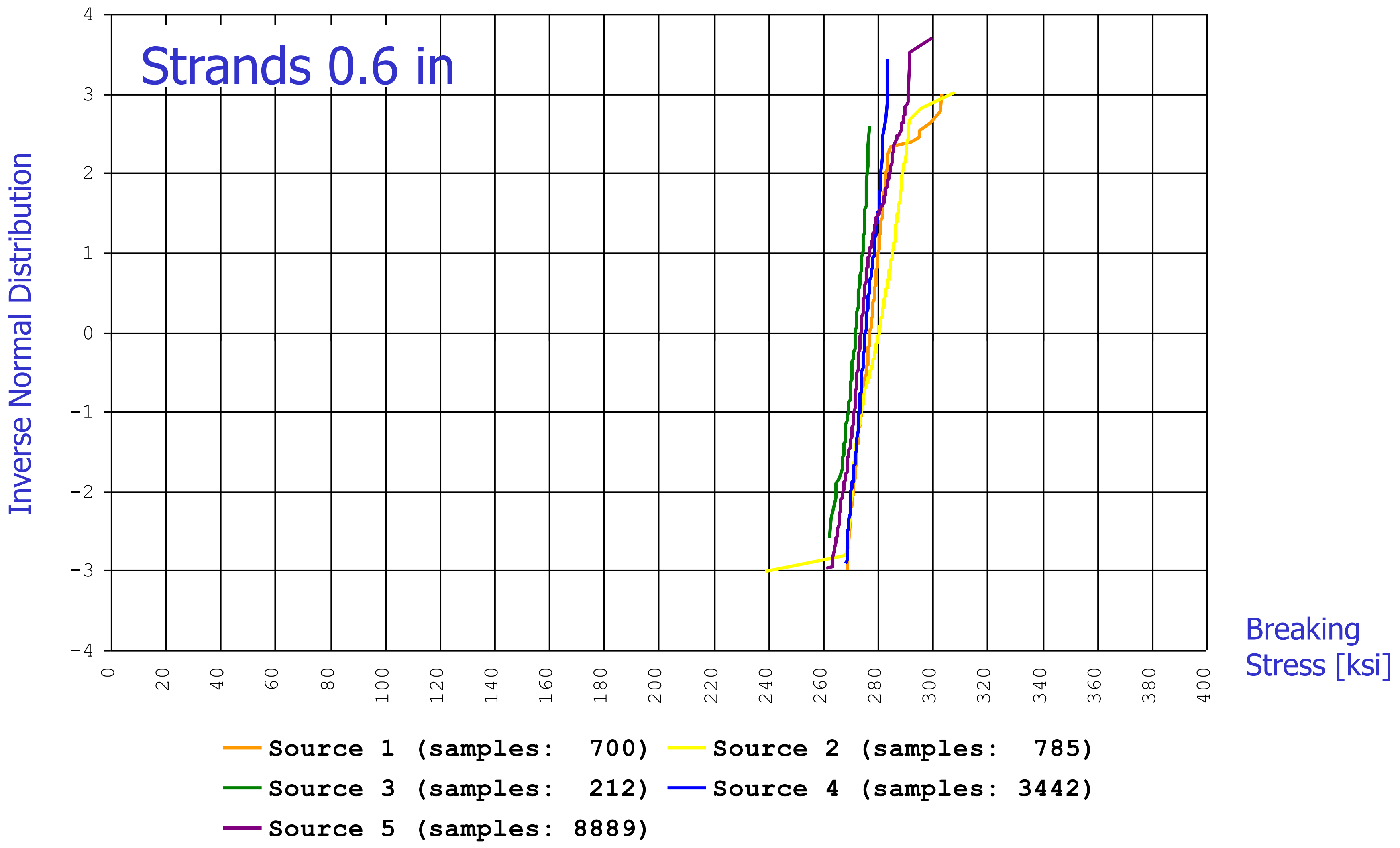


Prestressing Steel (7-wire strands), Grade 270
CDF of Breaking Stress



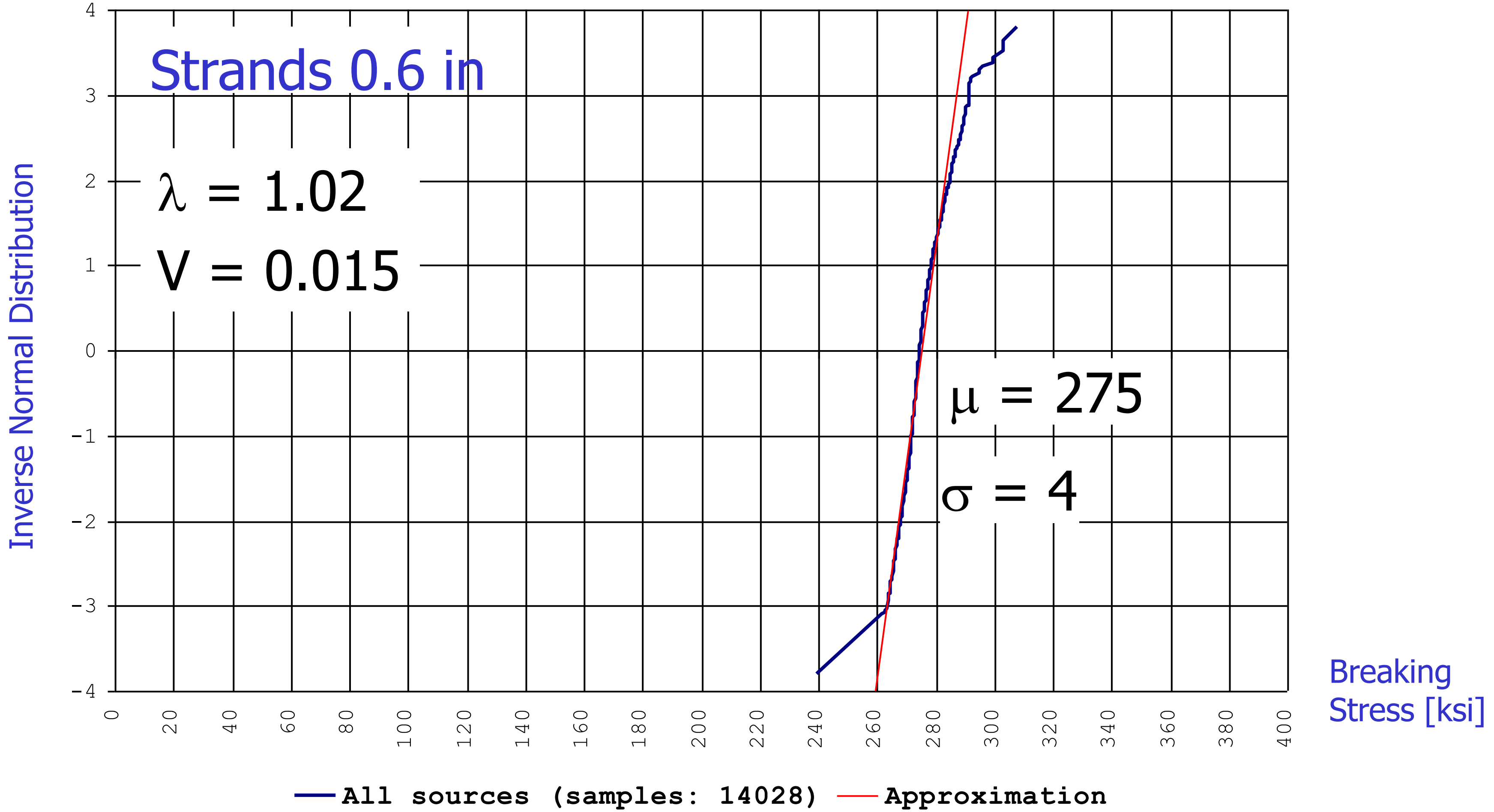
Prestressing Steel (7-wire strands), Grade 270

CDF of Breaking Stress



Prestressing Steel (7-wire strands), Grade 270

CDF of Breaking Stress



Prestressing Steel – Statistical Parameters

Grade	Size	Number of samples	Bias Factor	V
250 ksi	1/4 (6.25 mm) 3/8 (9.5 mm) 7/16(11 mm) 1/2 (12.5 mm)	22 83 114 66	1.07 1.11 1.11 1.12	0.01 0.025 0.01 0.02
270 ksi	3/8 (9.5 mm) 7/16 (11 mm) 1/2 (12.5 mm) 0.6 (15 mm)	54 16 33570 14028	1.04 1.07 1.04 1.02	0.02 0.02 0.015 0.015

Compressive strength in structure vs. in cylinder test, f_c'

- It is assumed that the actual compressive strength of concrete in structure is 10% lower than the compressive strength obtained in a cylinder test (f_c')
- It is assumed that the actual shear strength in structure is 5% lower than the shear strength obtained from the cylinder test results

Statistical Parameters of Fabrication Factor (Ellingwood, Galambos, MacGregor, Cornell)

		λ	V	
width of beam, b		1.01	0.04	
effective depth of beam, d			0.99	0.04
effective depth of one-way slab, d			0.92	0.12
effective depth of two-way slab, d				
	d = 4 in	1.03	0.09	
	d = 6 in	1.02	0.06	
	d = 8 in	1.015	0.04	
depth and width of column, b_1, b_2			1.005	0.04
area of reinforcement, A_s, A_v		1.00	0.015	
spacing of shear reinforcement, s			1.00	0.04

Statistical Parameters of Professional Factor (Ellingwood, Galambos, MacGregor, Cornell)

	λ	V
R/C beams - flexure	1.02	0.06
R/C beams - shear without stirrups	1.16	0.11
R/C beams - shear with stirrups	1.075	0.10
Axially loaded columns, tied	1.00	0.08
Axially loaded columns, spiral	1.05	0.06
One way slabs - flexure	1.02	0.06
One way slabs - shear	1.16	0.11
Two way slabs - shear	1.16	0.11
Bearing strength	1.02	0.06

Bending Resistance

$$R = \left(A_s \times f_y\right)\left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s . f_y}{0.85 f'_c . b}$$

for beams ρ = 0.6 and 1.6%,

for slabs ρ = 0.30%.

Shear Resistance of Flexural Members

$$R = V_n = V_c + V_s$$

$$V_C = 2\sqrt{f'_c}\, b_w . d$$

$$V_s = \frac{A_v . f_y . d}{s}$$

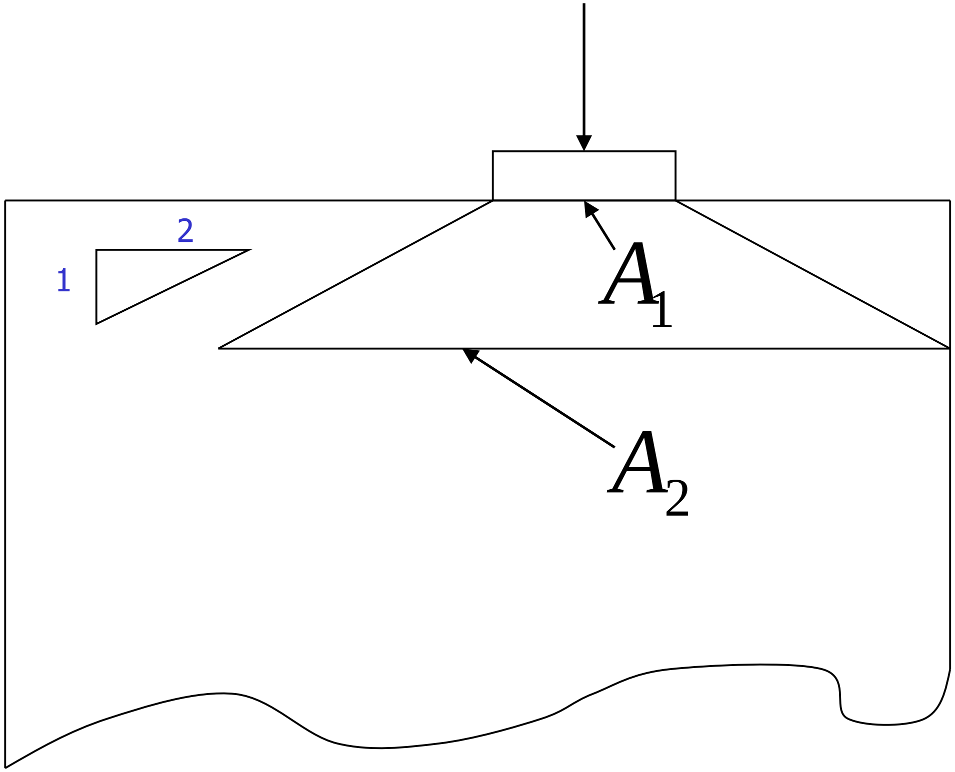
Shear Resistance of Slabs in Two-Way Shear

$$R = \min\left\{ \left(1 + \frac{2}{\beta_c}\right) \cdot 2\sqrt{f_c^{'}}\, b_0 d\,, \quad \left(\frac{\alpha_s d}{2b_0} + 1\right) \cdot 2\sqrt{f_c^{'}}\, b_0 d\,, \quad 4 \cdot \sqrt{f_c^{'}}\, b_0 d \right\}$$

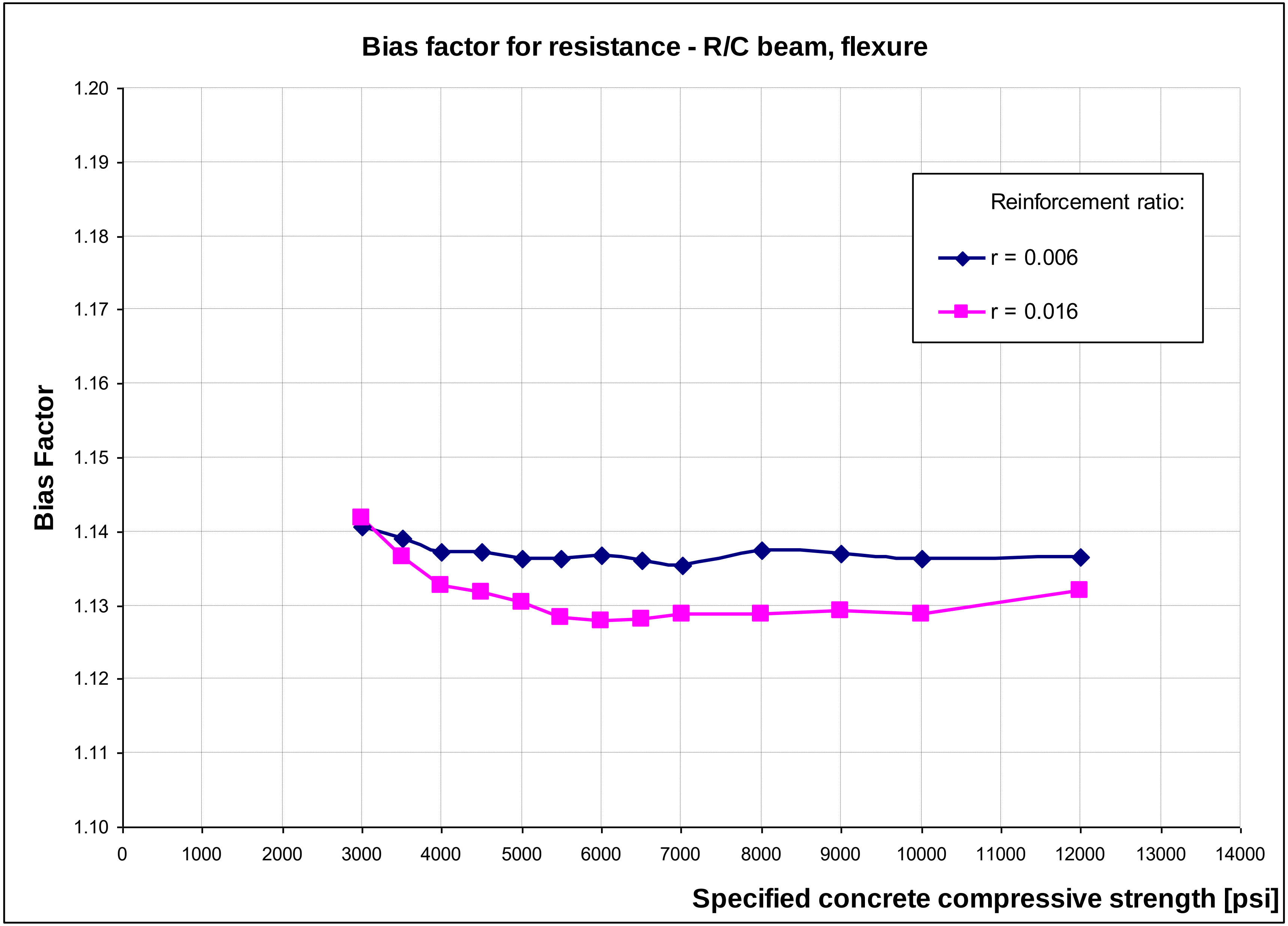
$$\left(\sqrt{f_c^{'}}\right)_{simulations} = 0.95\left(\sqrt{f_c^{'}}\right)_{\mathrm{nominal}}$$

Bearing Resistance of Concrete

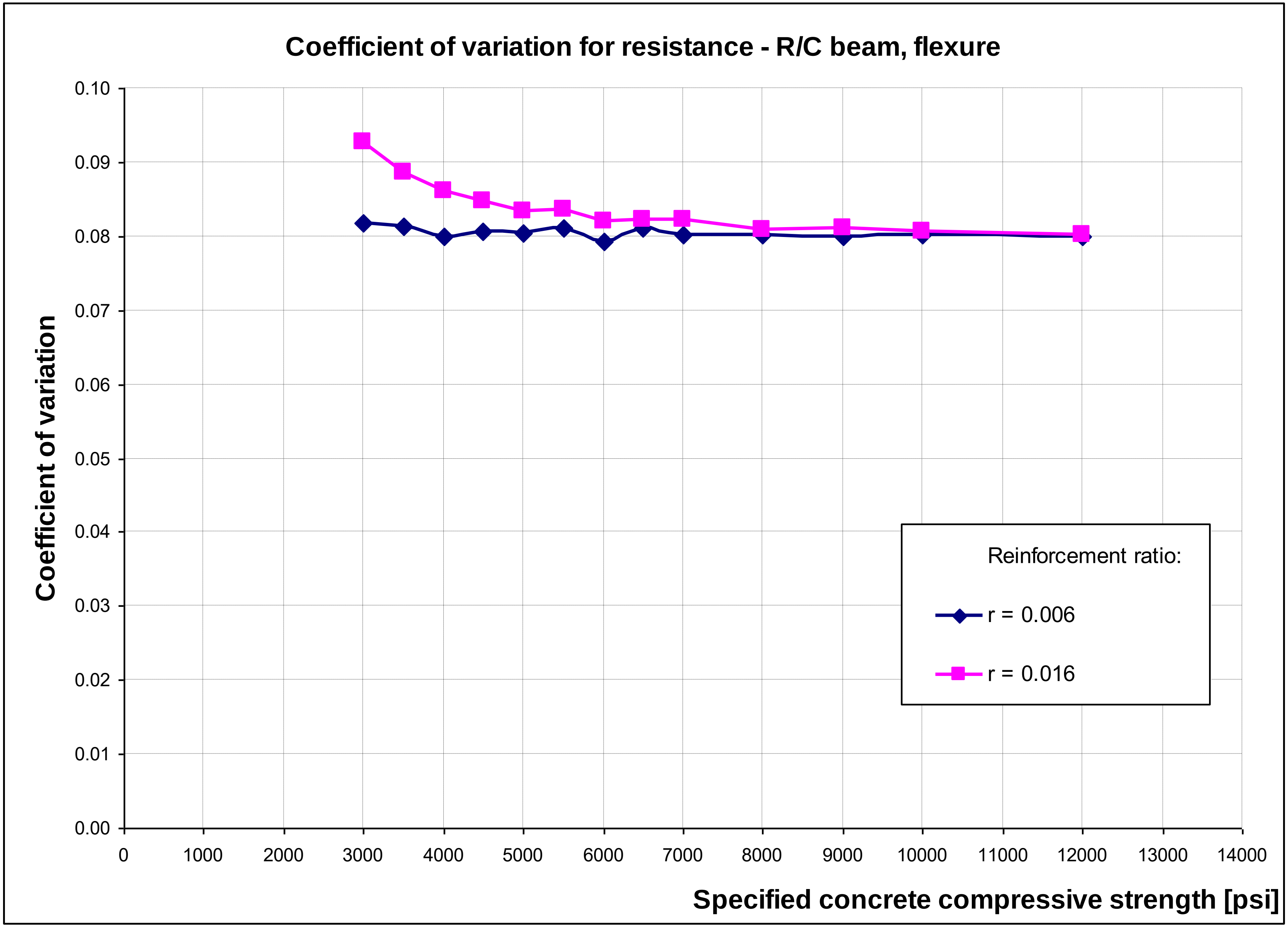
$$R = 0.85 f'_c A_1 \sqrt{\frac{A_2}{A_1}}$$



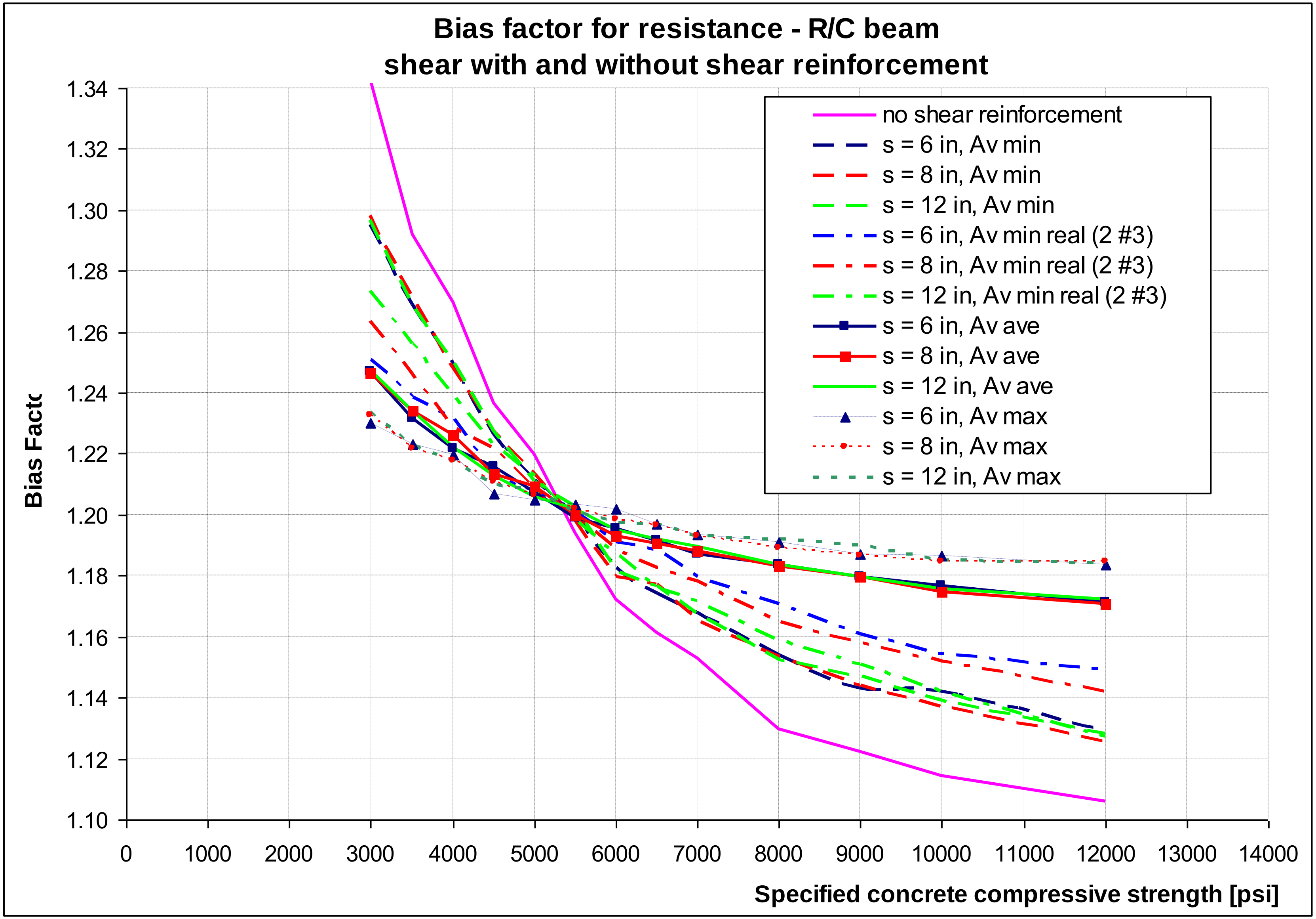
Bias Factor of Resistance for Beams, Flexure



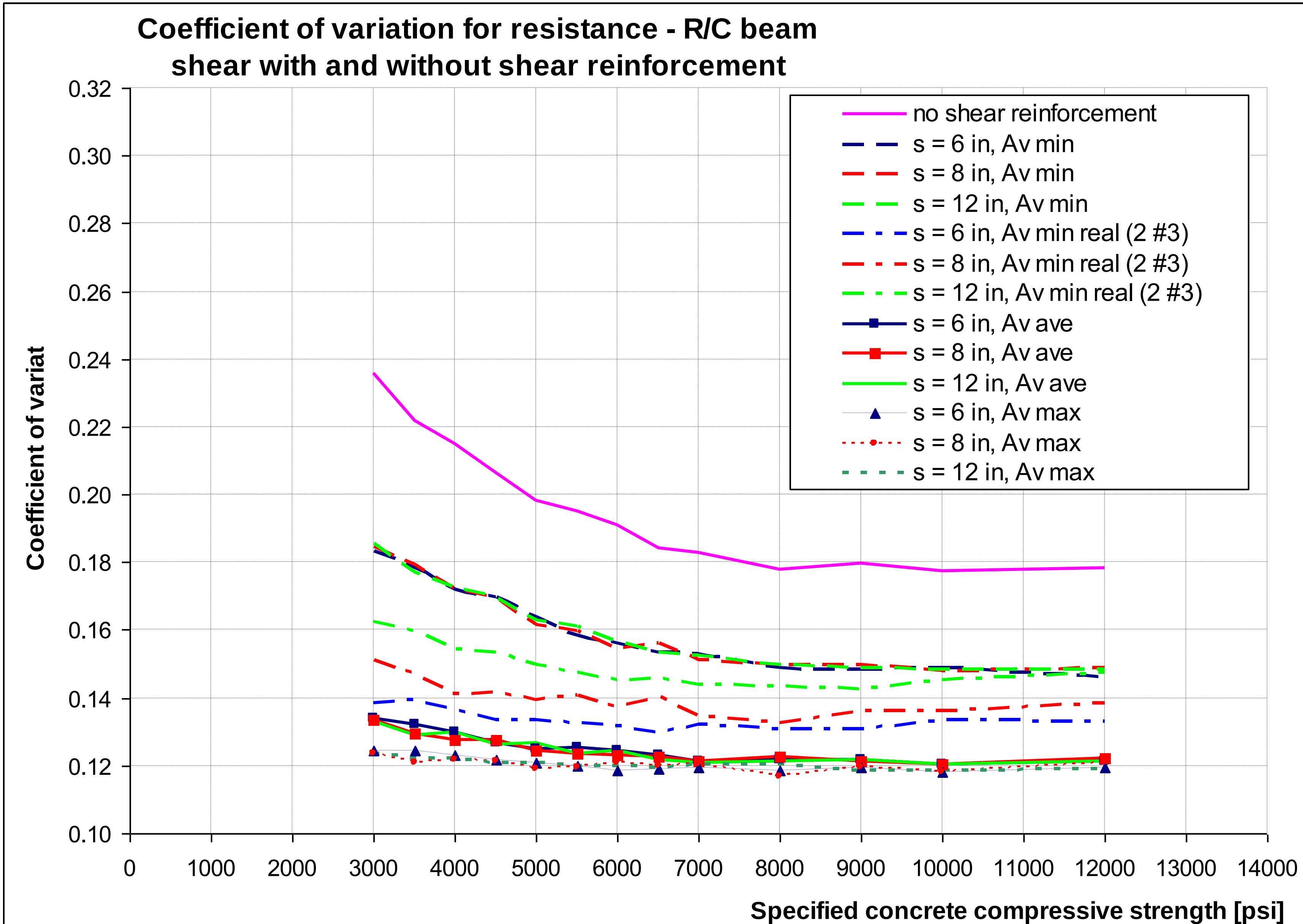
Coefficient of Variation of Resistance for Beams, Flexure



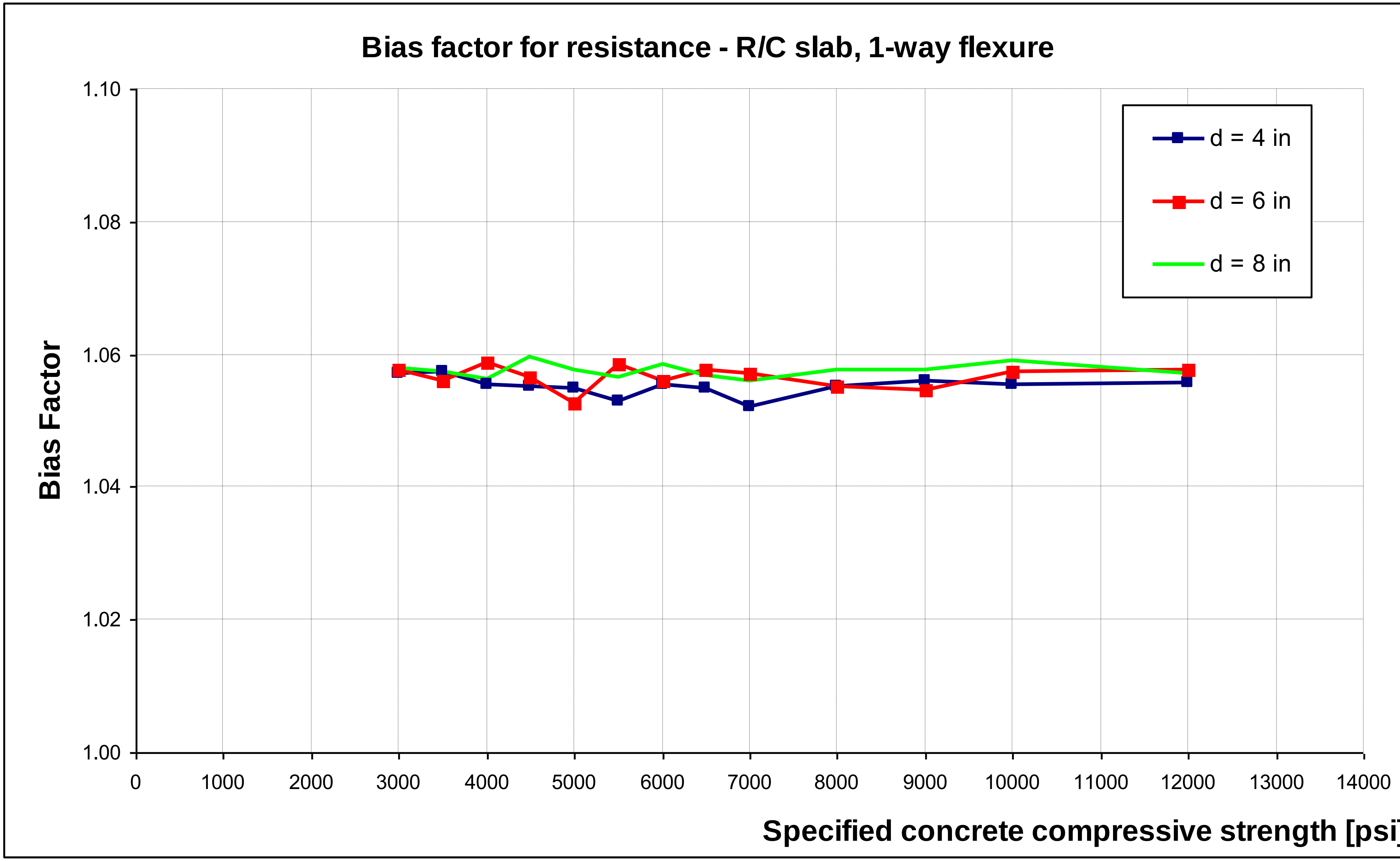
Bias Factor of Resistance for Beams, Shear



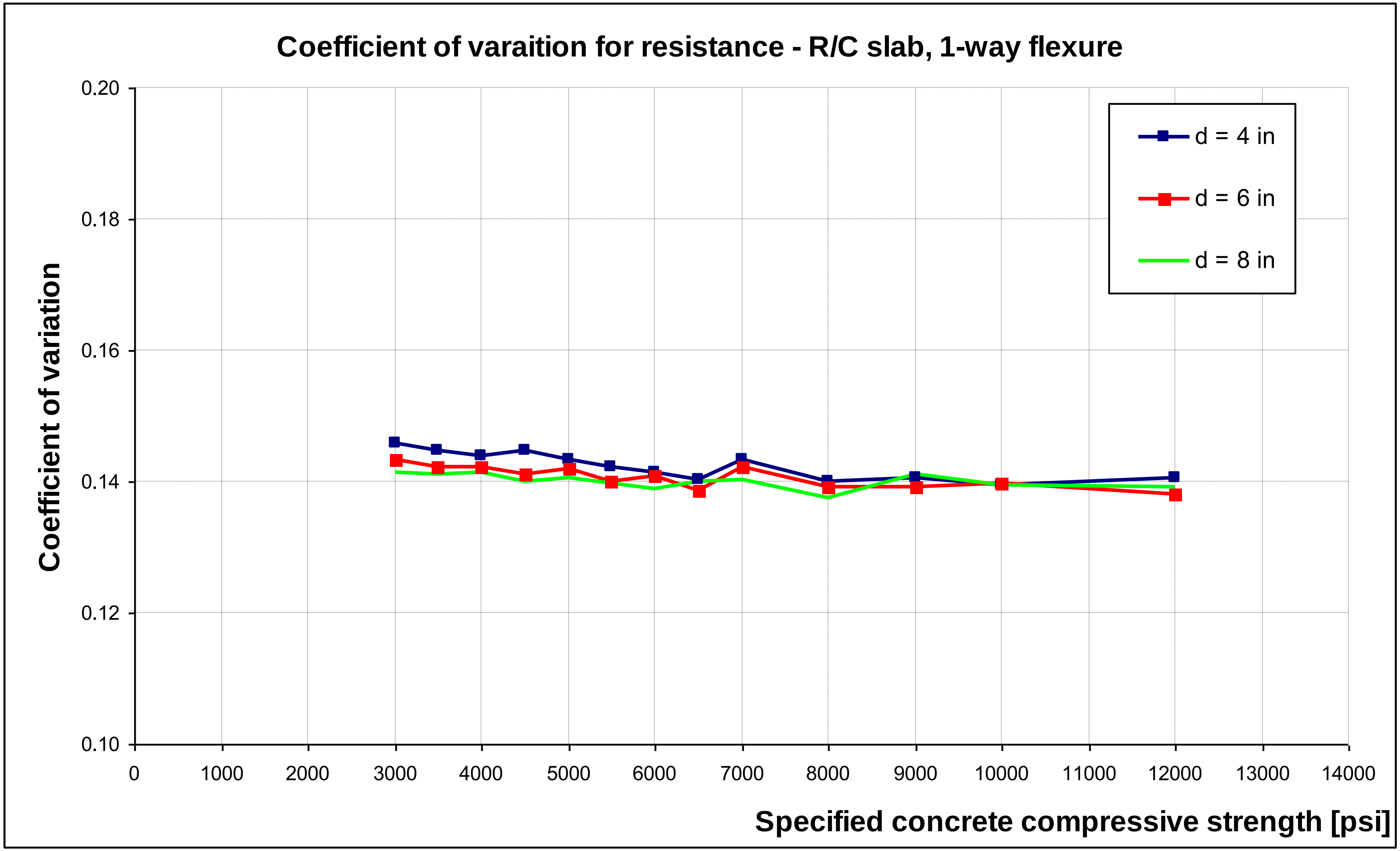
Coefficient of Variation of Resistance for Beams, Shear



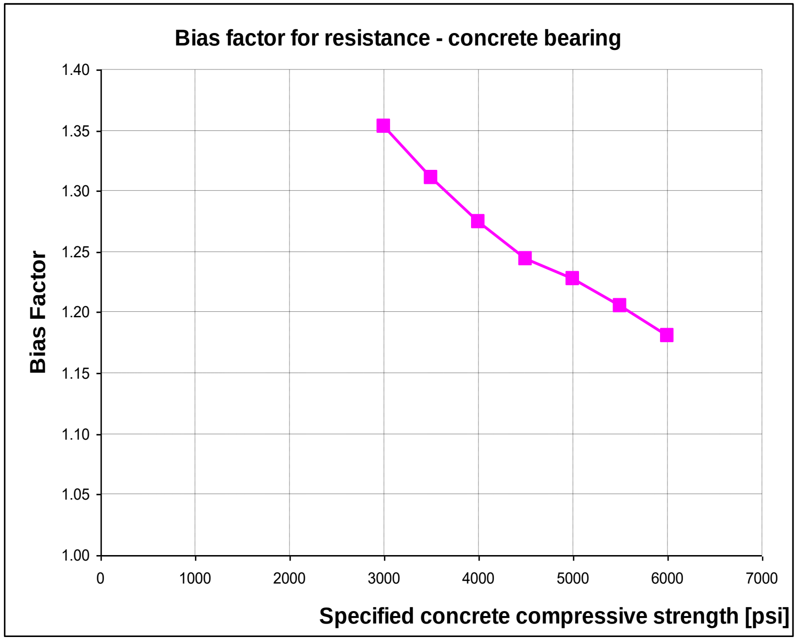
Bias Factor of Resistance for One way Slab, Flexure

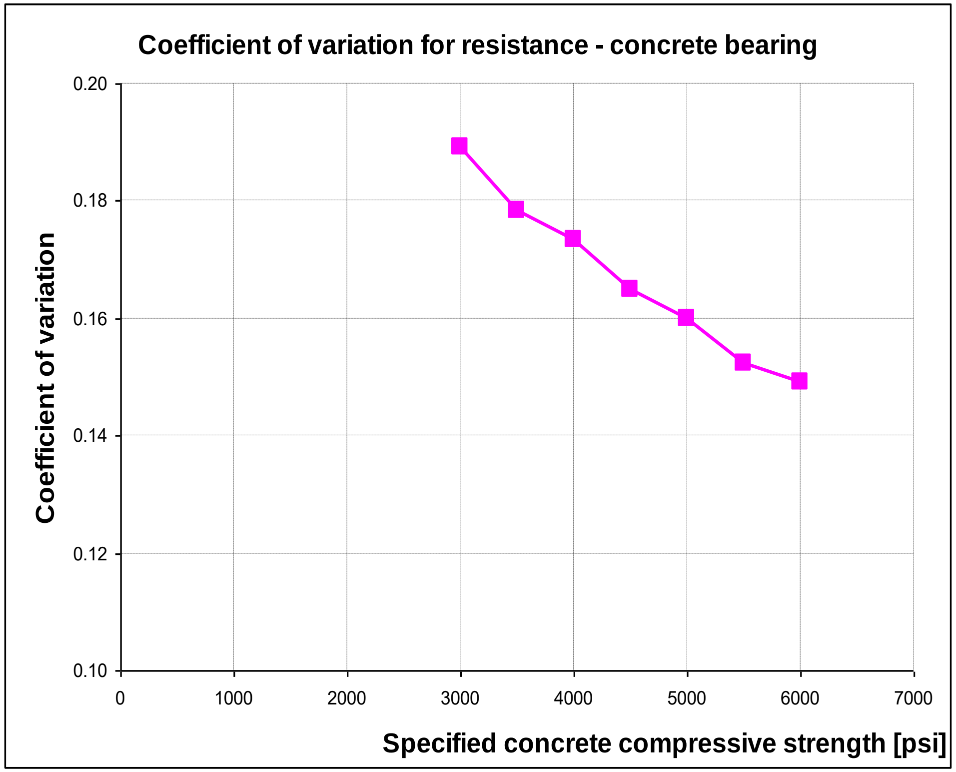


Coefficient of Variation of Resistance for One way slab, Flexure



Bias Factor and Coefficient of Variation of Resistance for Concrete Bearing Strength





Statistical Parameters of Resistance, R, Beams

Structural type and limit state	Statistical parameters							
	Old material data		New material data					
			Ordinary concrete		High strength concrete		Light weight concrete	
	Bias	V	Bias	V	Bias	V	Bias	V
R/C beam cast-in-place, flexure	1.114	0.119	1.190	0.089	1.16	0.09	1.18	0.09
R/C beam plant cast, flexure	1.128	0.133	1.205	0.081				
R/C beam cast-in-place, shear	1.159	0.120	1.230	0.109	1.19	0.11	1.23	0.11
R/C beam plant cast, shear	1.170	0.116	1.242	0.105				
P/S beam plant cast, flexure	1.034	0.081	1.084	0.073				
P/S beam plant cast, shear	1.130	0.105	1.194	0.103				

Statistical Parameters of Resistance, R, Slabs

Structural type and limit state	Statistical parameters							
	Old material data		New material data					
			Ordinary concrete		High strength concrete		Light weight concrete	
	Bias	V	Bias	V	Bias	V	Bias	V
R/C slab cast-in-place	1.052	0.169	1.077	0.146	1.070	0.145	1.080	0.150
R/C slab plant cast	1.146	0.116	1.174	0.082				
P/S slab plant cast	1.053	0.070	1.075	0.070				
Post-tensioned slab cast-in-place	0.961	0.146	0.982	0.145	1.030	0.110		

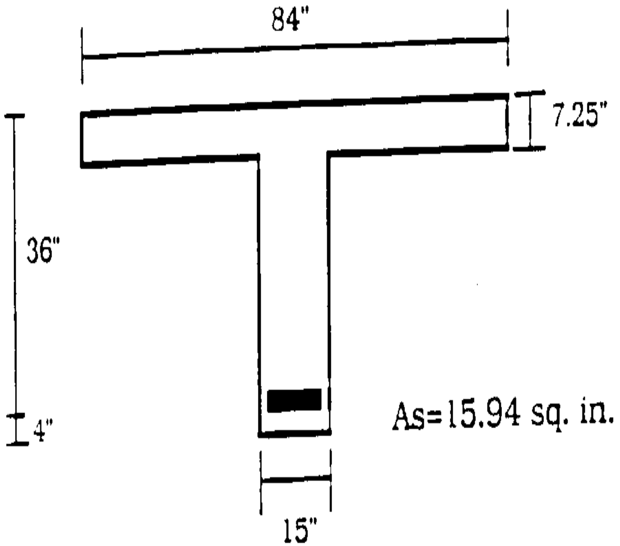
Statistical Parameters of Resistance, R, Columns and Plain Concrete Elements

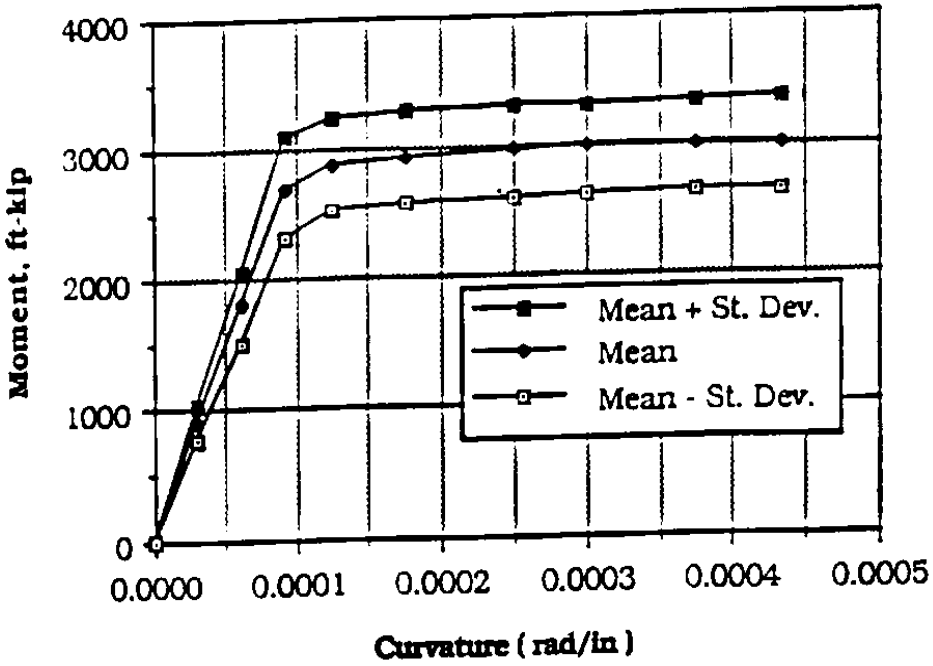
Structural type and limit state	Statistical parameters							
	Old material data		New material data					
			Ordinary concrete		High strength concrete		Light weight concrete	
	Bias	V	Bias	V	Bias	V	Bias	V
R/C column cast-in-place, tied	1.107	0.136	1.260	0.107	1.20	0.12	1.26	0.13
R/C column plant cast, tied	1.102	0.134	1.252	0.103				
R/C column cast-in-place, spiral	1.163	0.124	1.316	0.097	1.26	0.11	1.33	0.12
R/C column plant cast, spiral	1.156	0.122	1.323	0.091				
P/S column plant cast, tied	1.017	0.094	1.080	0.090				
P/S column plant cast, spiral	1.068	0.076	1.133	0.071				
Plain concrete, flexure, shear	1.004	0.082	1.105	0.082	1.24	0.08	1.40	0.08

Moment Capacity of Reinforced Concrete T-Beams in Bridges

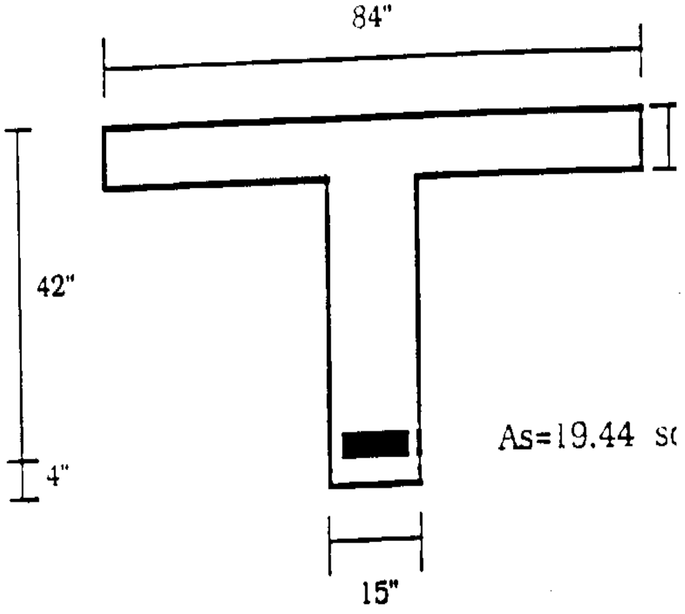
- The statistical parameters of bending resistance were derived by Nowak, Yamani, and Tabsh (1994).
- Three sections are considered
  - a flange width = 7 ft (2.1 m)
  - slab thickness= 7.25 inches (180 mm).
  - spans ranging from 40 to 80 ft (12 to 24 m).
- The bias factor and coefficient of variation of MF (materials and fabrication) for lightly reinforced concrete T-beams were assumed to be 1.12 and 0.12, respectively. The parameters for analysis factors were taken as $\lambda_P = 1.00$ and $V_P = 0.06$.
- The resistance parameters were $\boldsymbol{\lambda_R = 1.12}$ and $\mathbf{V_R = 0.13}$**.**

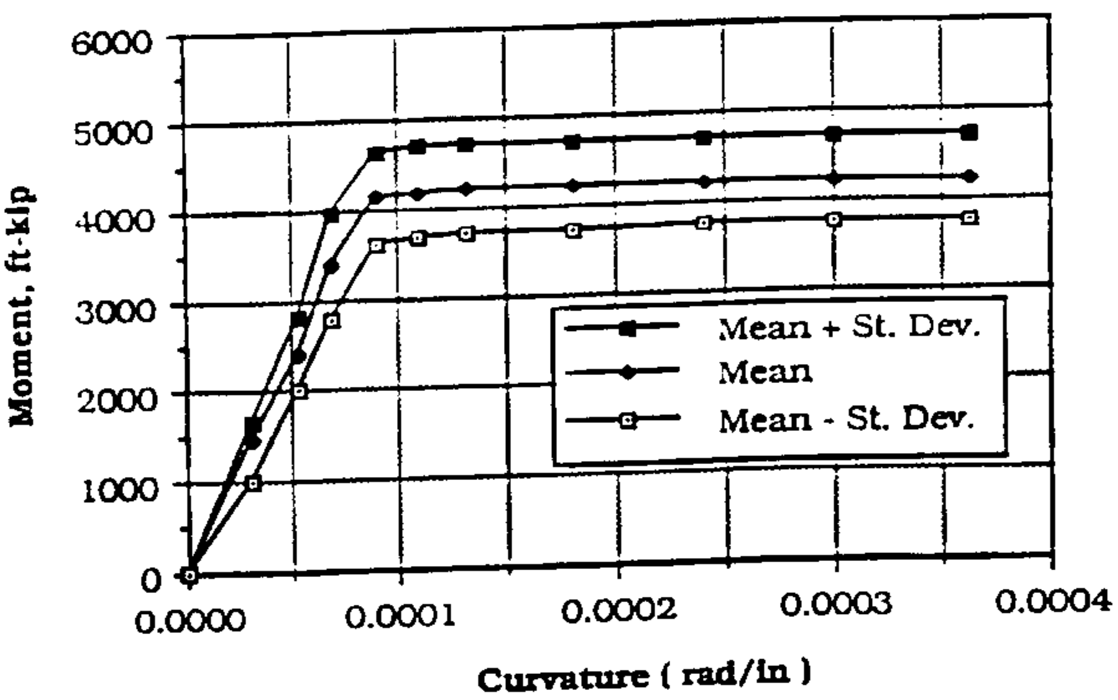
Moment Capacity of Reinforced Concrete T-Beams



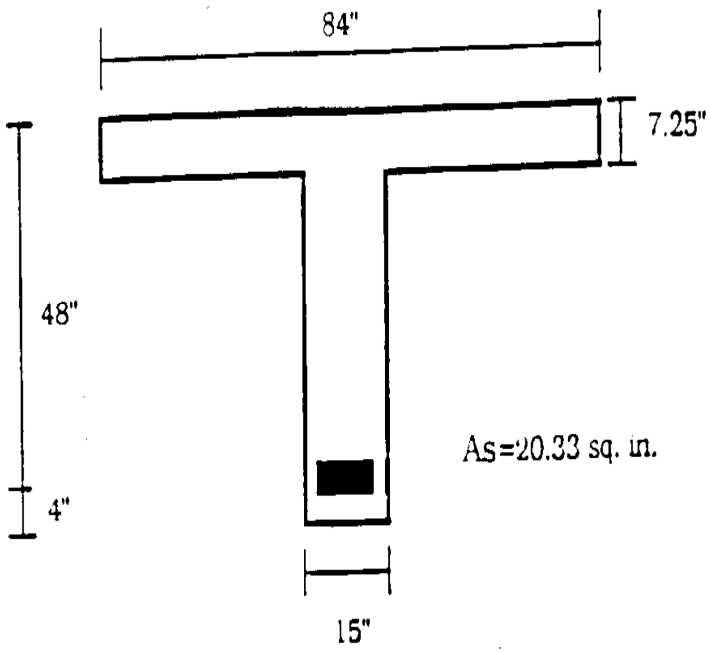


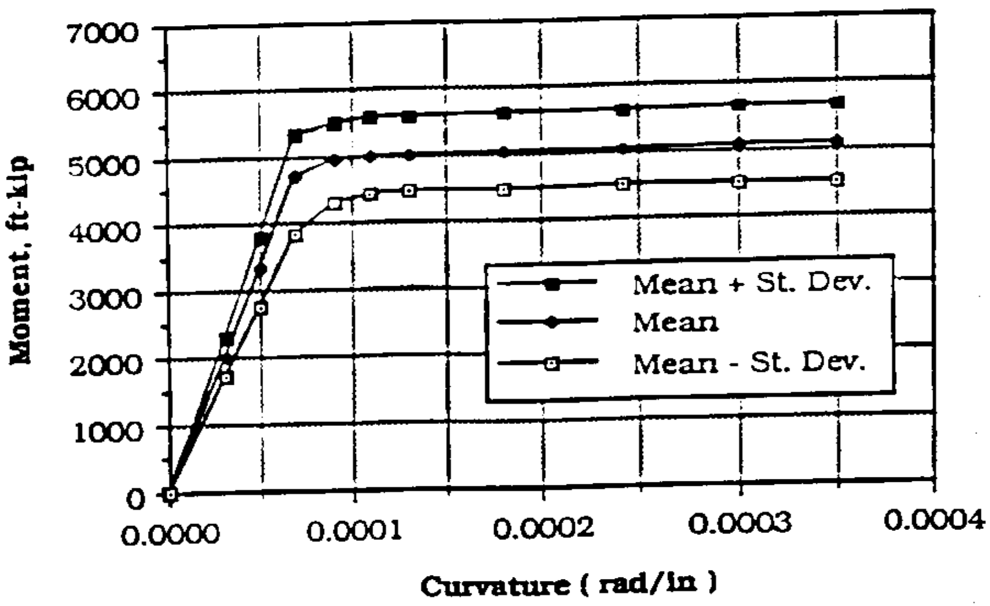
Moment Capacity of Reinforced Concrete T-Beams





Moment Capacity of Reinforced Concrete T-Beams

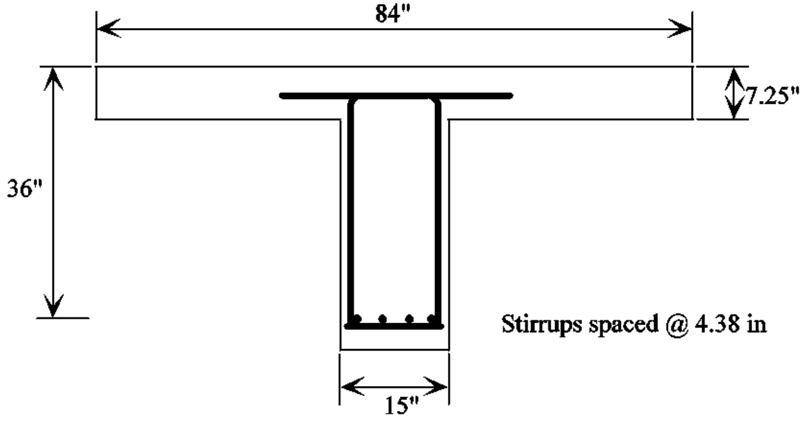


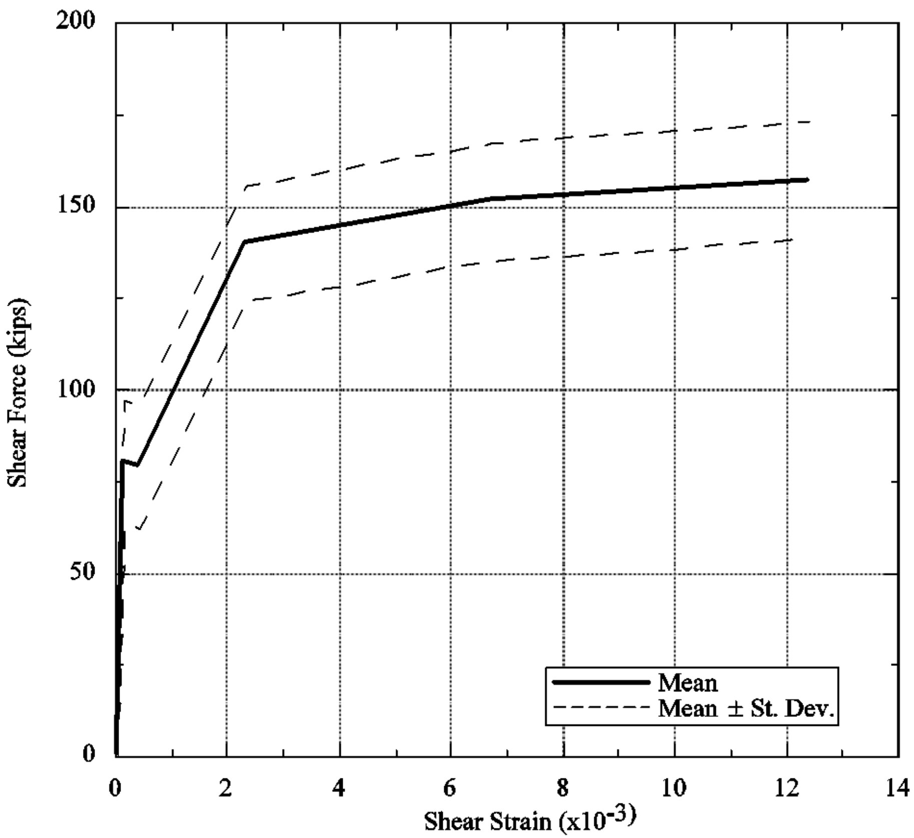


Shear Capacity of Reinforced Concrete T-Beams

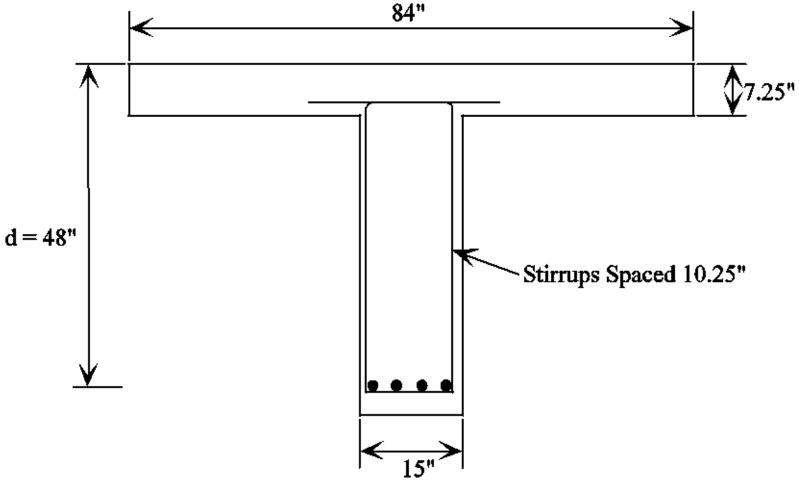
- The shear capacity of reinforced concrete girders can be calculated using the Modified Compression Field Theory (Vecchio and Collins, 1982 & 1986).
- The statistical parameters were determined on the basis of simulations performed by Yamani (1992).
- The relationship between shear force and shear strain was established for representative T-beams.
- The nominal (design) value of shear capacity was calculated according to current AASHTO provisions.

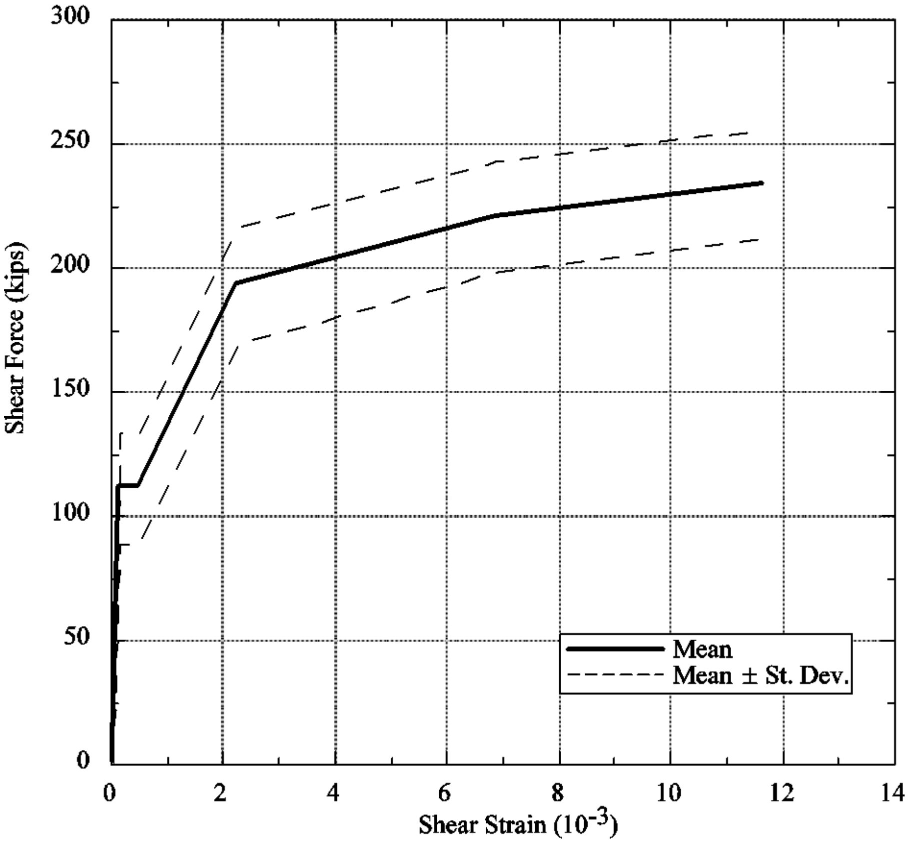
Shear Capacity of Reinforced Concrete T-Beams



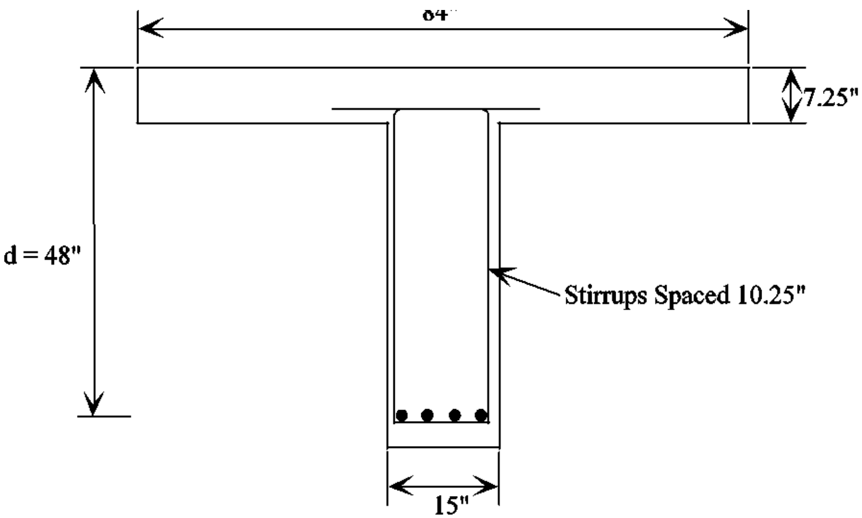


Shear Capacity of Reinforced Concrete T-Beams





Shear Capacity of Reinforced Concrete T-Beams



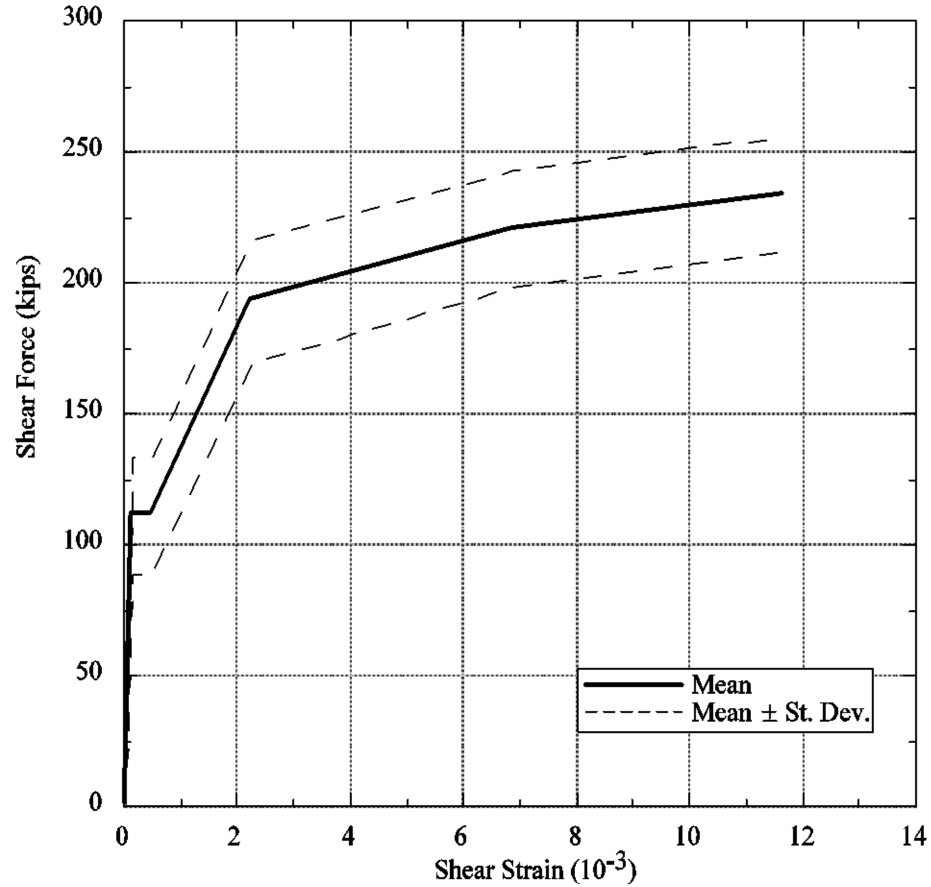


Figure 7-19 Shear Force-Shear Strain Curves for R/C Section C

MOMENT CAPACITY OF PRESTRESSED CONCRETE AASHTO GIRDERS IN BRIDGES

- The parameters of resistance for prestressed concrete bridge girders are derived on the basis of the statistical data from Ellingwood, Galambos, MacGregor and Cornell (1980) and Siriaksorn and Naaman (1980).

- The strains are assumed to be linearly distributed.

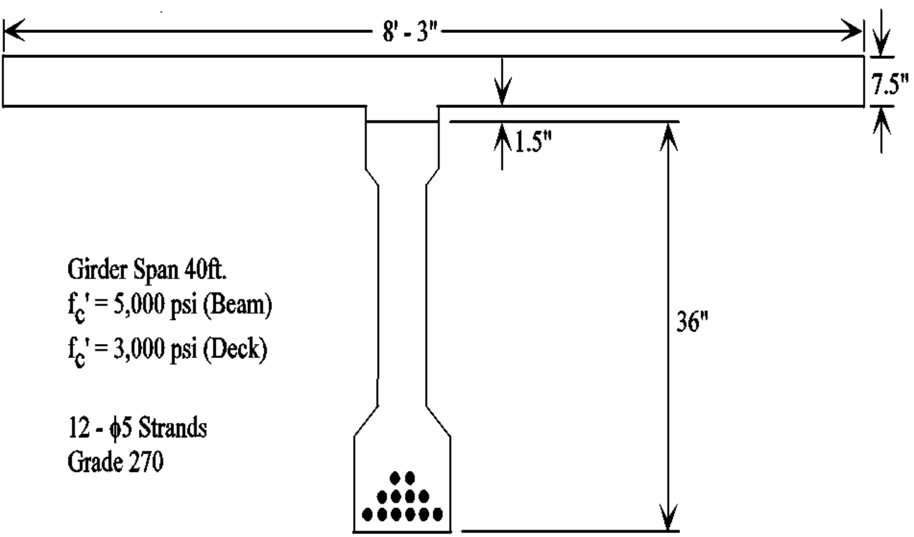
- Uncracked and cracked sections are considered.

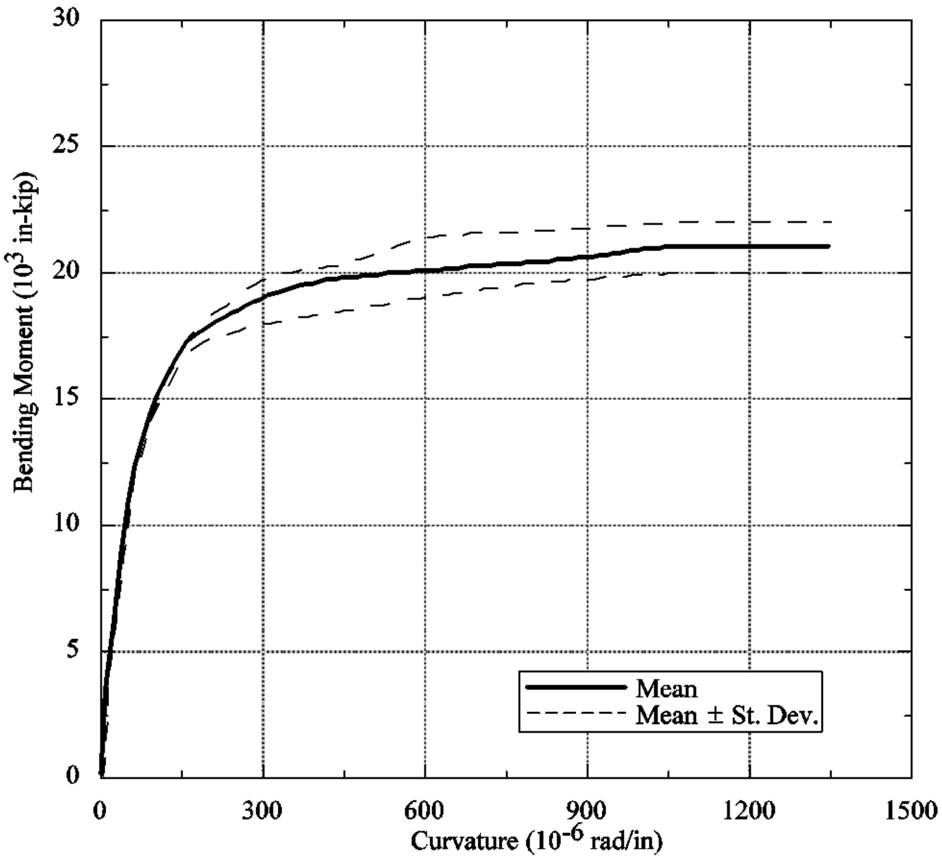
- Representative moment-curvature relationships for typical AASHTO girders are shown in the following figures. The solid line corresponds to the average, whereas the dashed lines correspond to the average plus one and minus one standard deviation.

- Bias factor for the product MF λ_{MF} = 1.04, coefficient of variation of MF = 0.045
  For the analysis factor bias, λ_P = 1.01 and V_P = 0.06

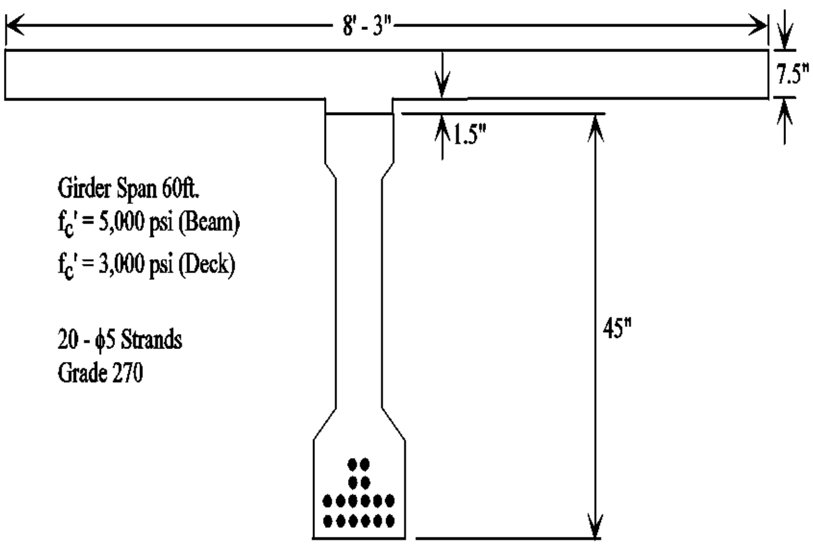
- The resistance parameters are **λ_R= 1.05 V_R= 0.075**

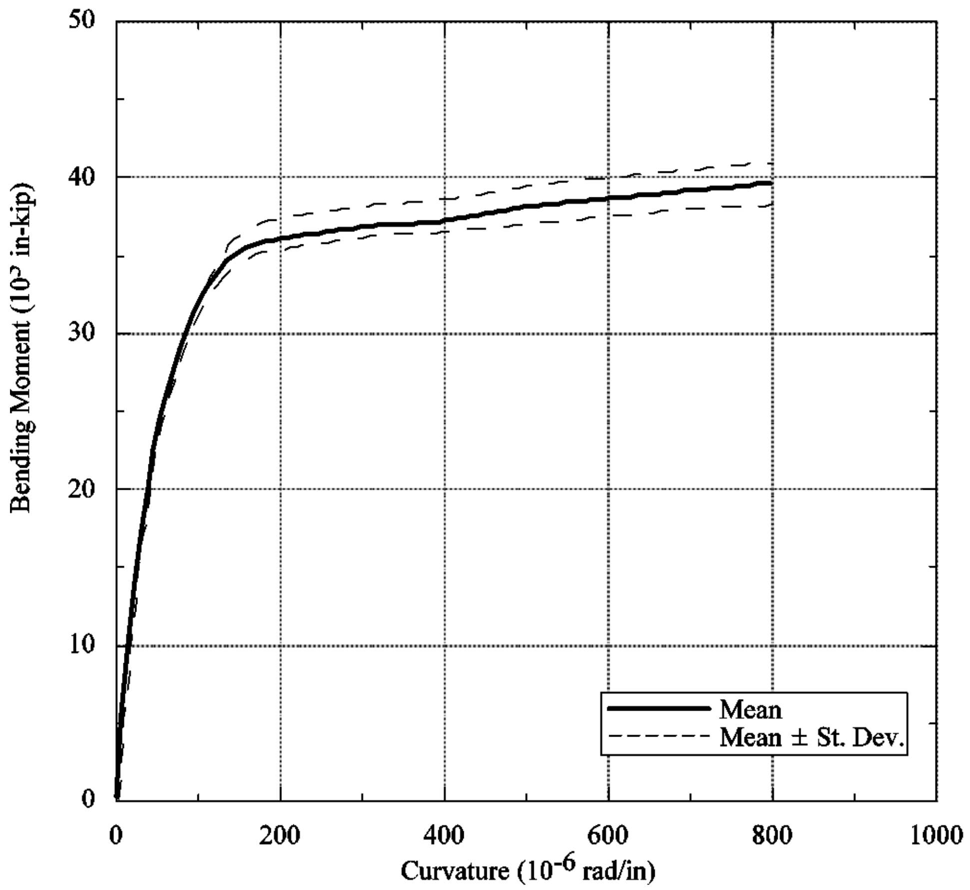
Moment Capacity (P/C)



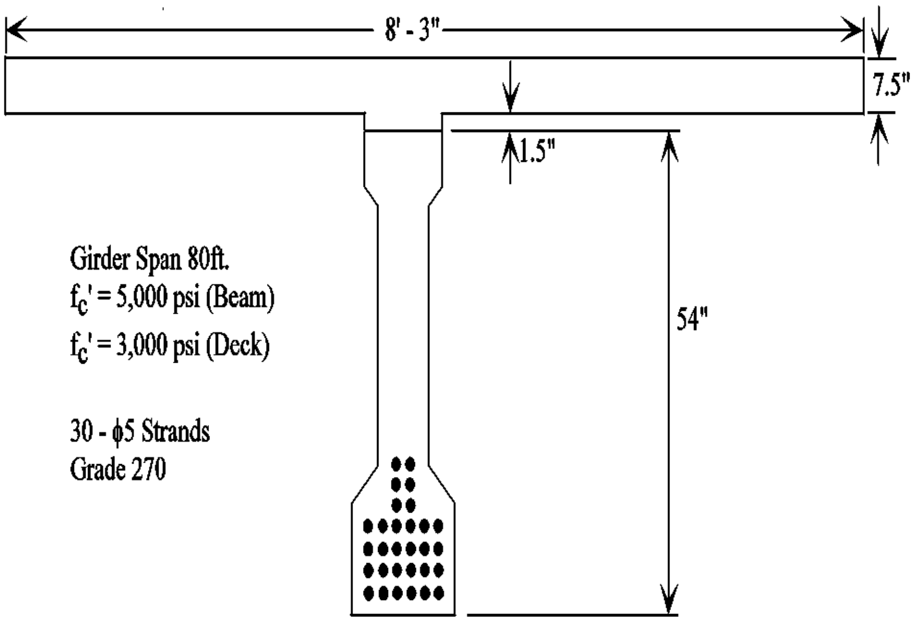


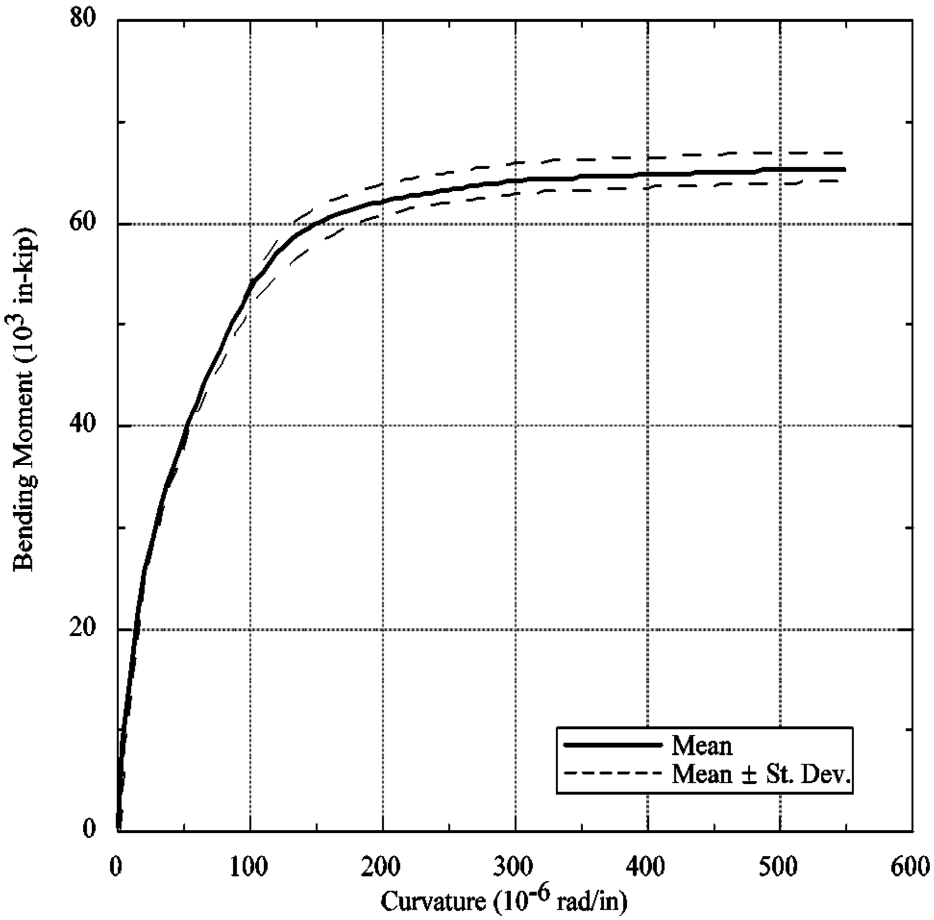
Moment Capacity (P/C)





Moment Capacity (P/C)





Shear Capacity (P/C)

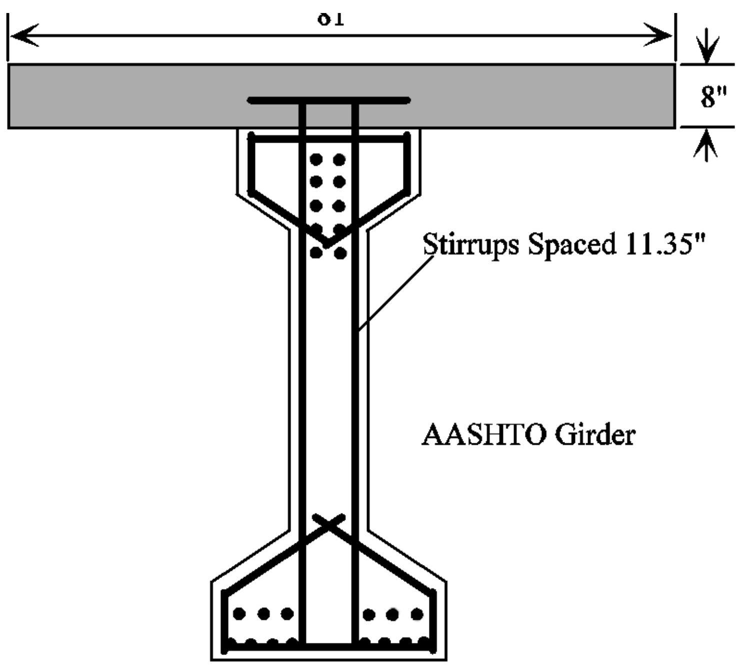
- **The shear capacity of reinforced concrete girders** can be calculated using the Modified Compression Field Theory (Vecchio and Collins, 1982 & 1986).

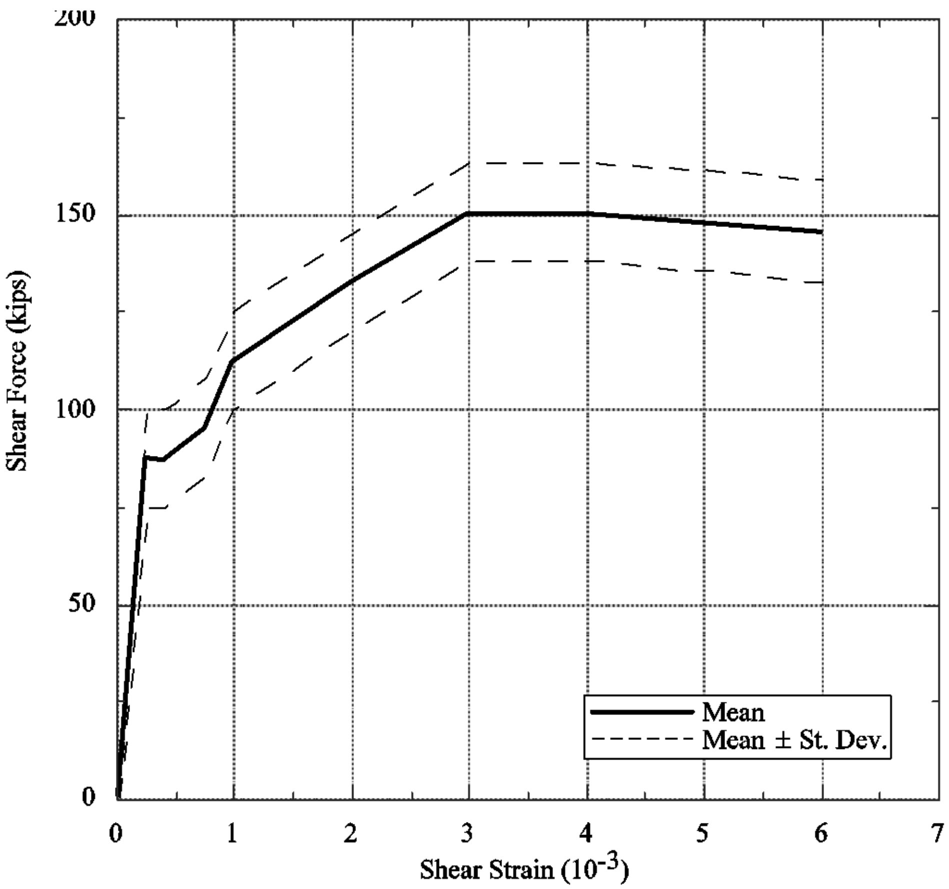
- **The statistical parameters** were determined on the basis of simulations performed by Yamani (1992).

- **The relationship between shear force and shear strain** was established for representative T-beams.

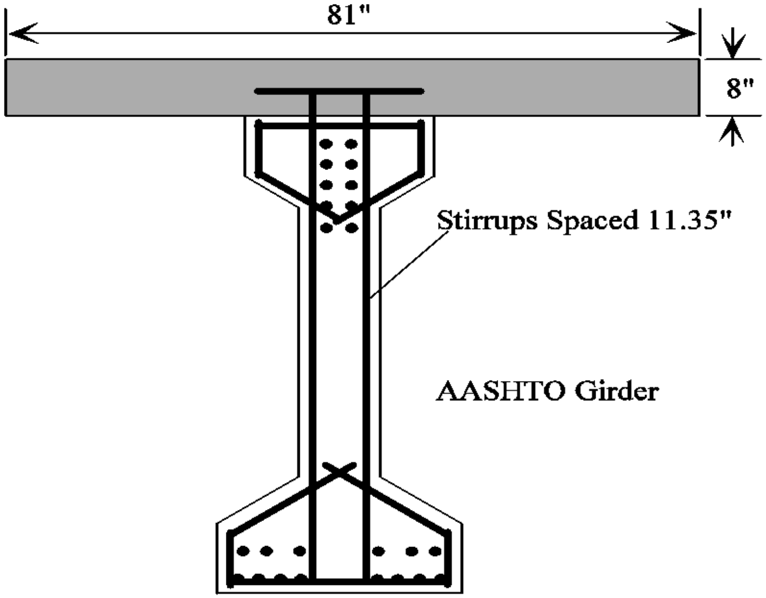
- **The nominal (design) value of shear capacity** was calculated according to current AASHTO provisions
- FM : $\lambda_{MF} = 1.07$ and $V_{MF} = 0.10$.
- Analysis factor P: $\lambda_P = 1.075$ and $V_P = 0.10$.
- Shear resistance : λ_R **= 1.15 and V_R = 0.14.**

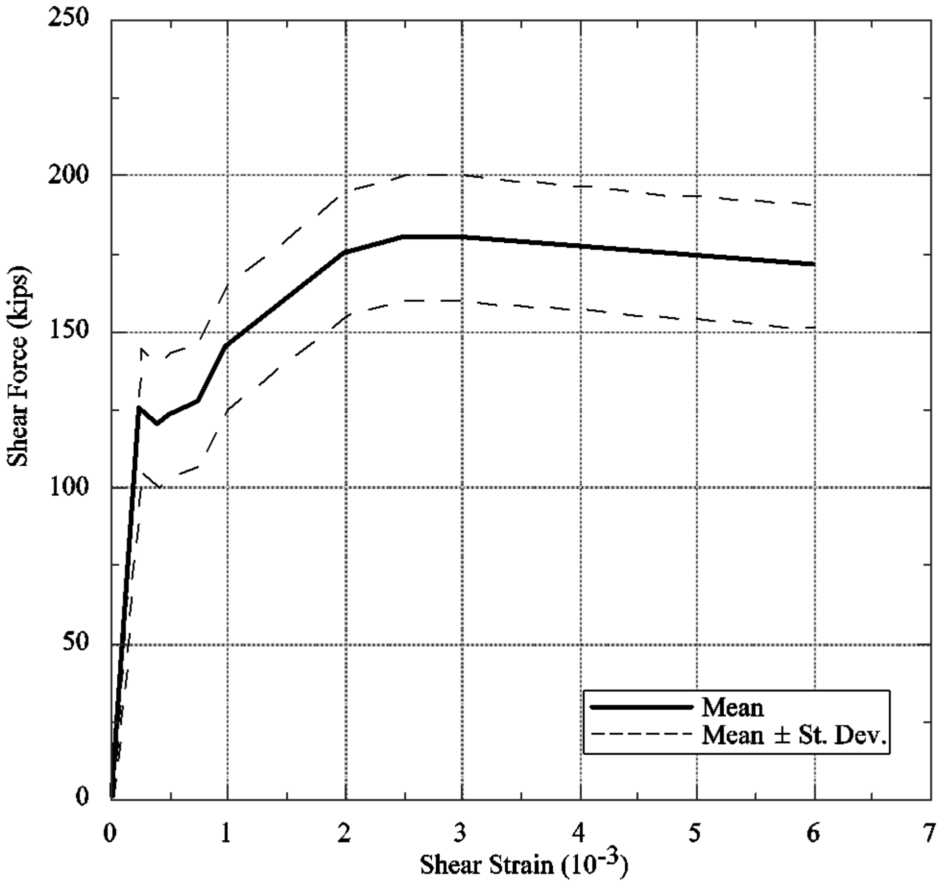
Shear Capacity (P/C)



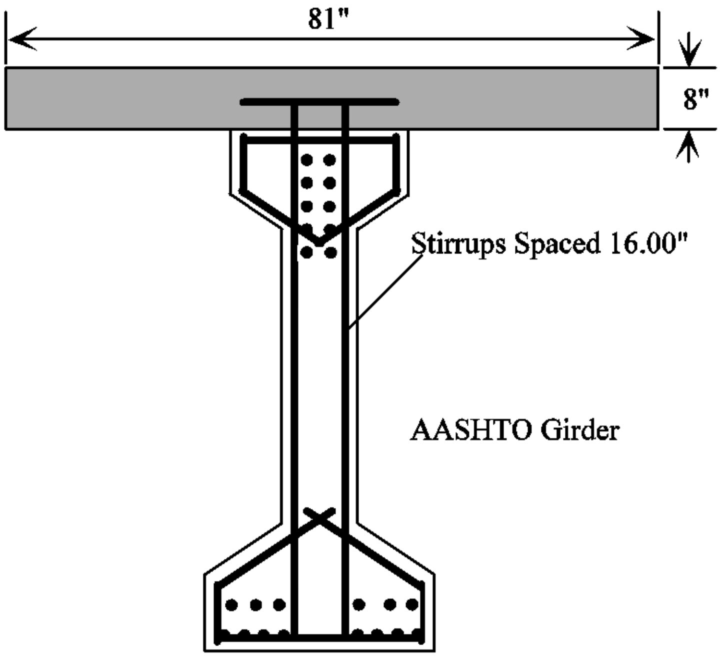


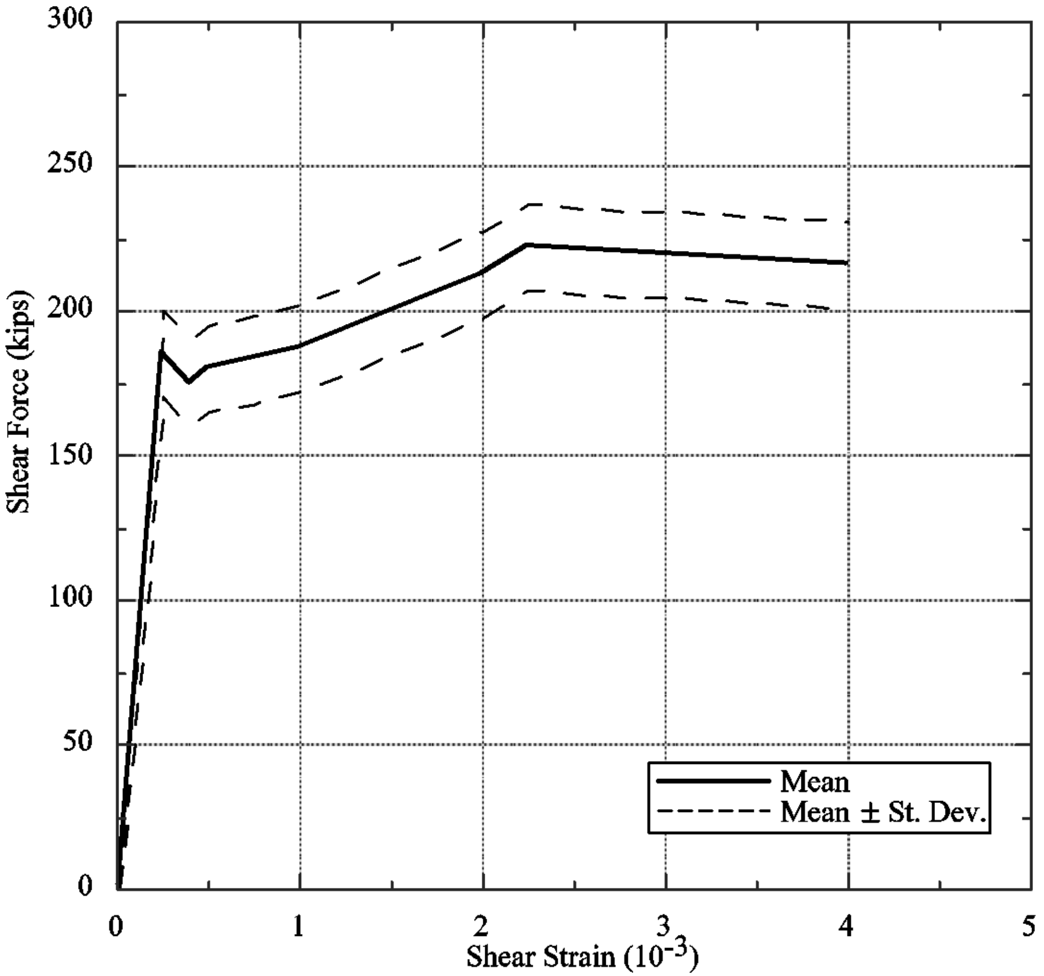
Shear Capacity (P/C)





Shear Capacity (P/C)





Summary of Resistance Parameters for Bridges

Material	Limit State	Bias Factor	Coefficient of Variation
Steel	Moment	1.12	0.10
	Shear	1.14	0.105
Reinforced Concrete	Moment	1.14	0.13
	Shear	1.20	0.155
Prestressed Concrete	Moment	1.05	0.075
	Shear	1.15	0.14

Resistance of Components with High Strength Prestressing Bars

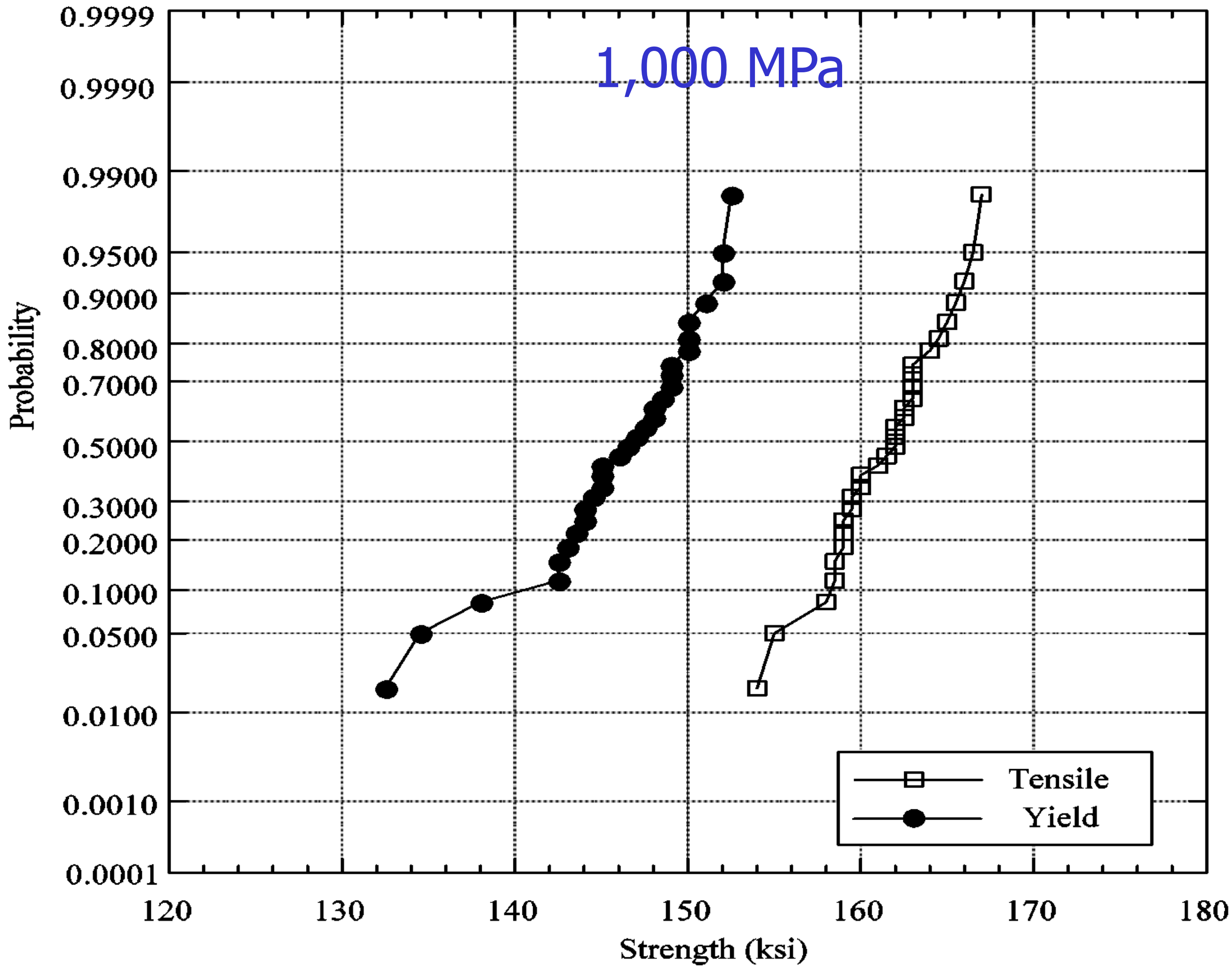
- The resistance of components made with high-strength prestressing bars is determined by the mechanical properties of the prestressing bars.

- Next figure presents the results of tests on 30 samples conducted by Dywidag Systems International.

- The tests were conducted to determine the yield stress, **F_y**, and the tensile strength (ultimate stress), **F_u**.

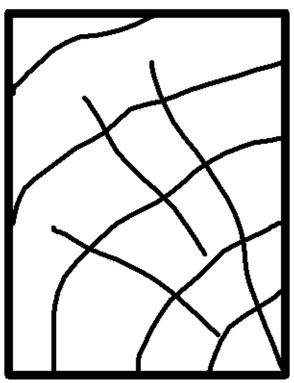
- The calculated coefficients of variation are **0.03 for F_y** and **0.01 for F_u**. However, the lower tails of the CDF's show a higher variation, which is important in reliability analysis. The statistical parameters of the resistance can be assumed to be the same as for reinforced concrete T-beams

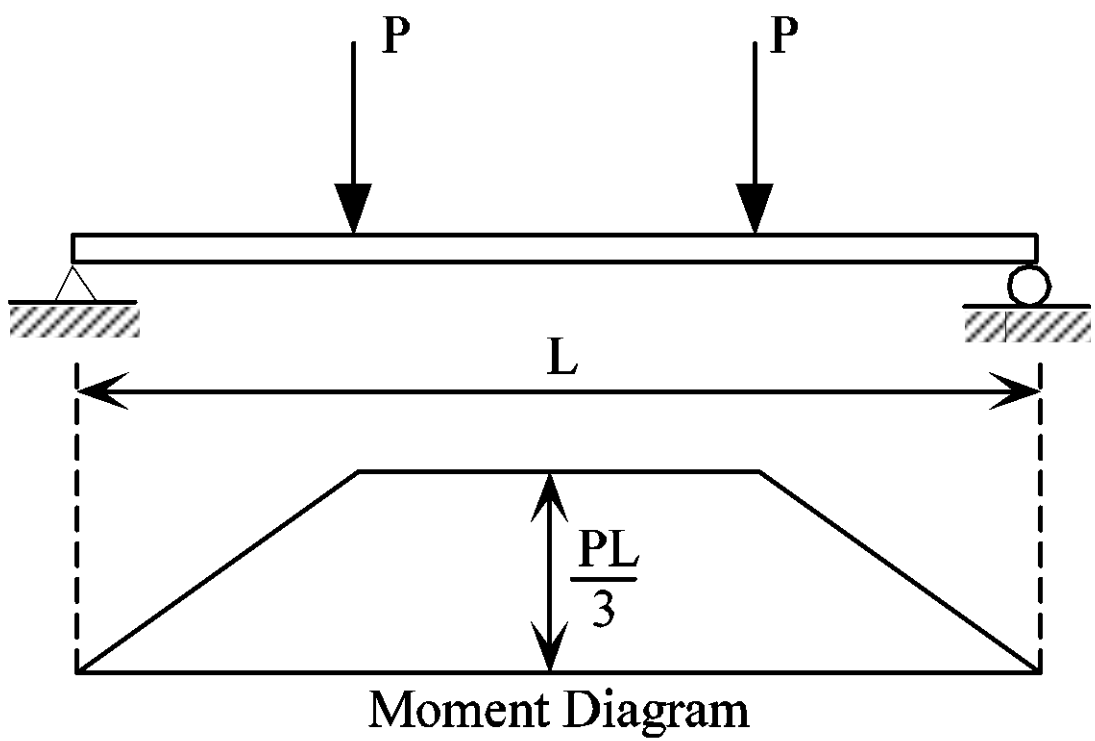
Resistance of High Strength Prestressing Bars

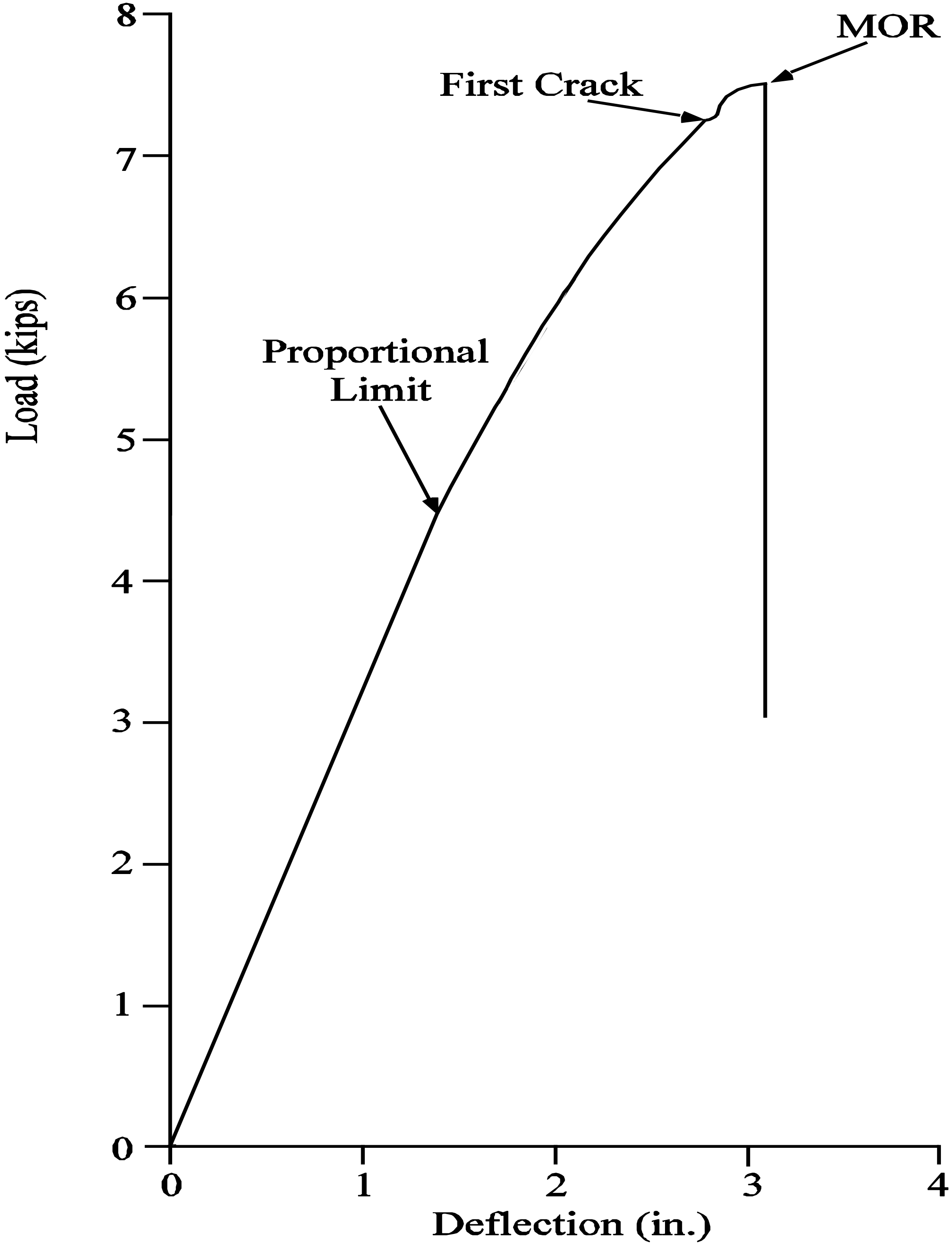


WOOD COMPONENTS

The major material parameters governing the resistance of structural members made of wood are the modulus of rupture (MOR) and the modulus of elasticity (MOE).

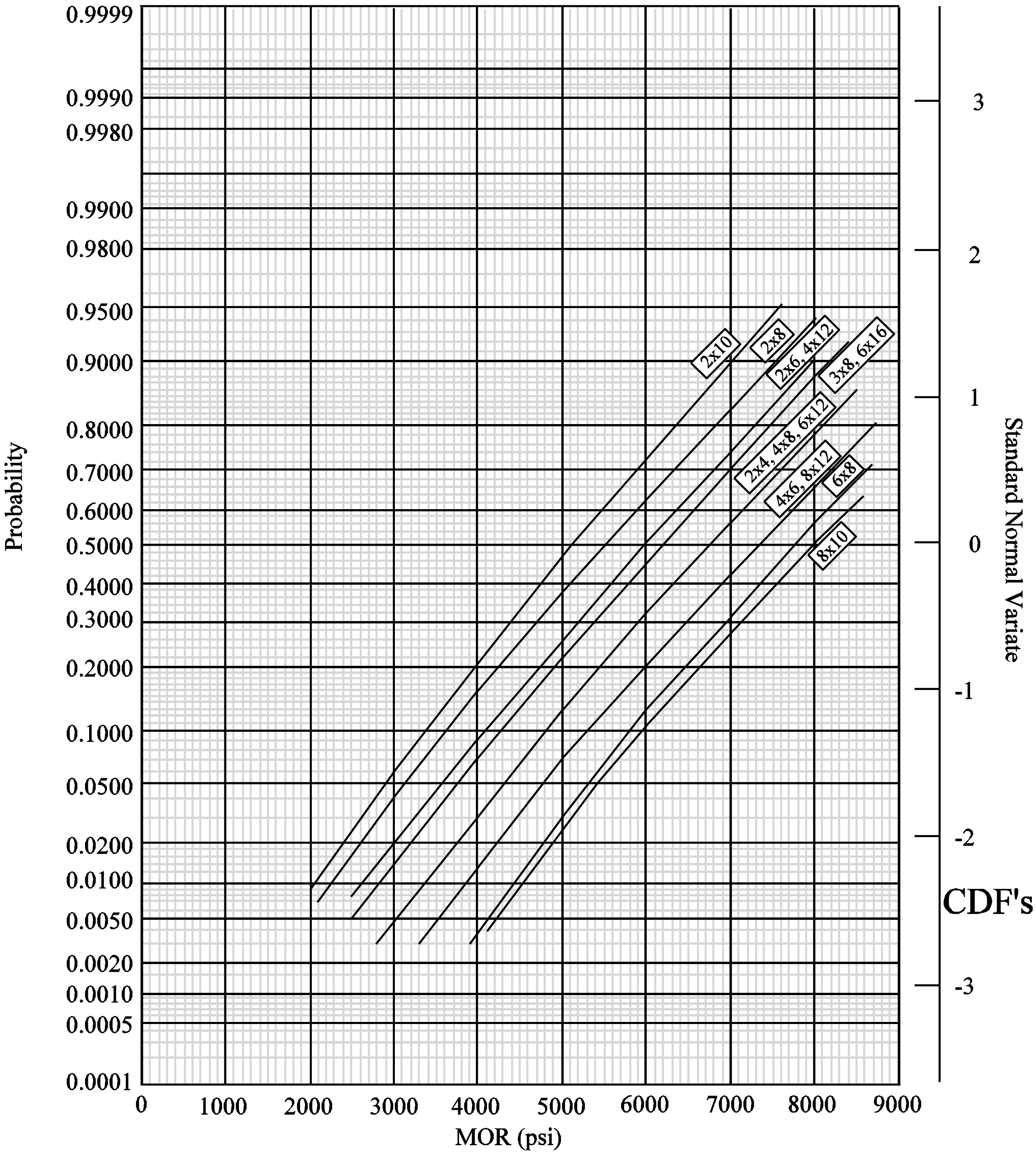






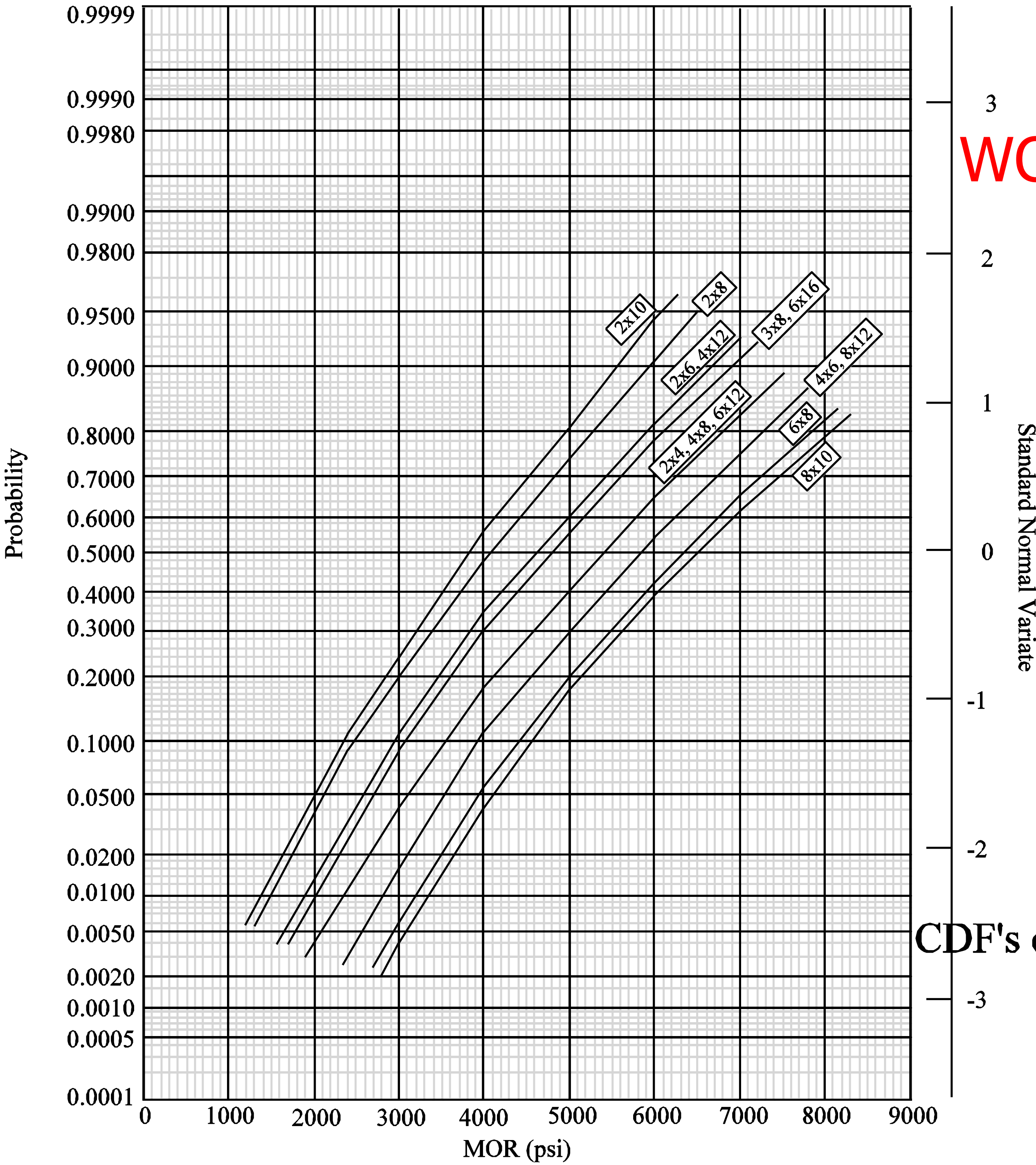
WOOD COMPONENTS, Load-Deflection Curve

WOOD COMPONENTS, MOR



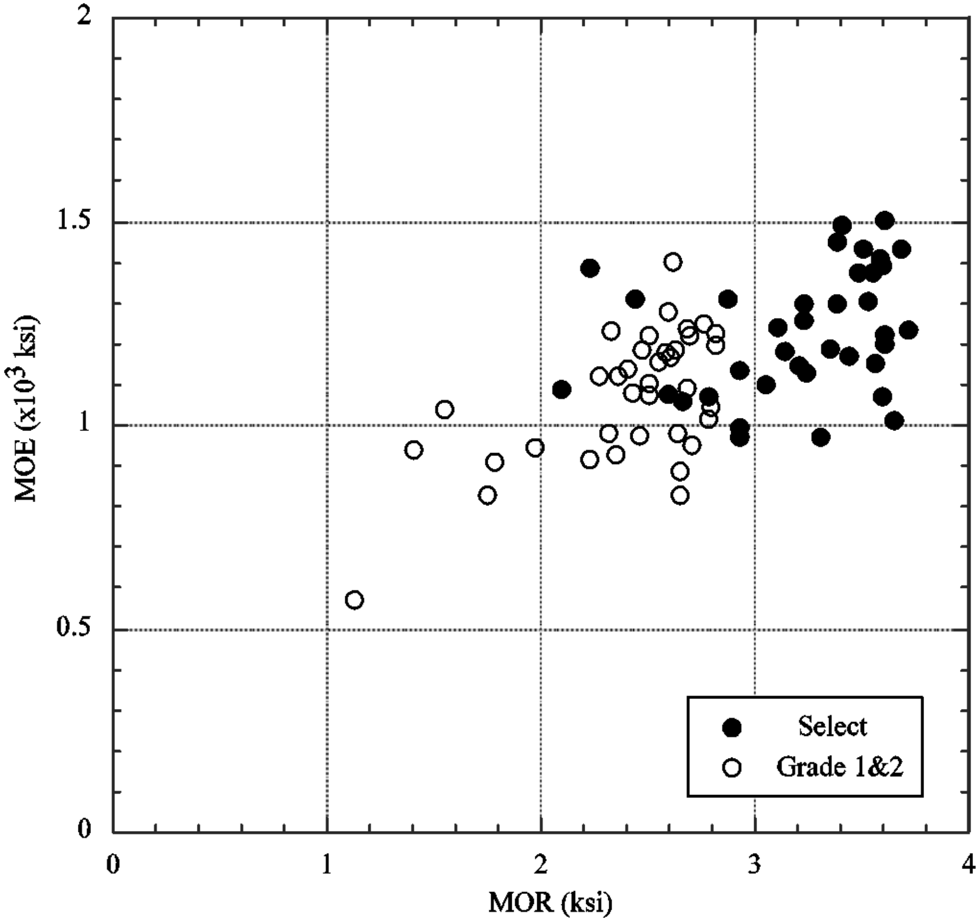
CDF's of MOR for D-F, Select Structural.

WOOD COMPONENTS MOR



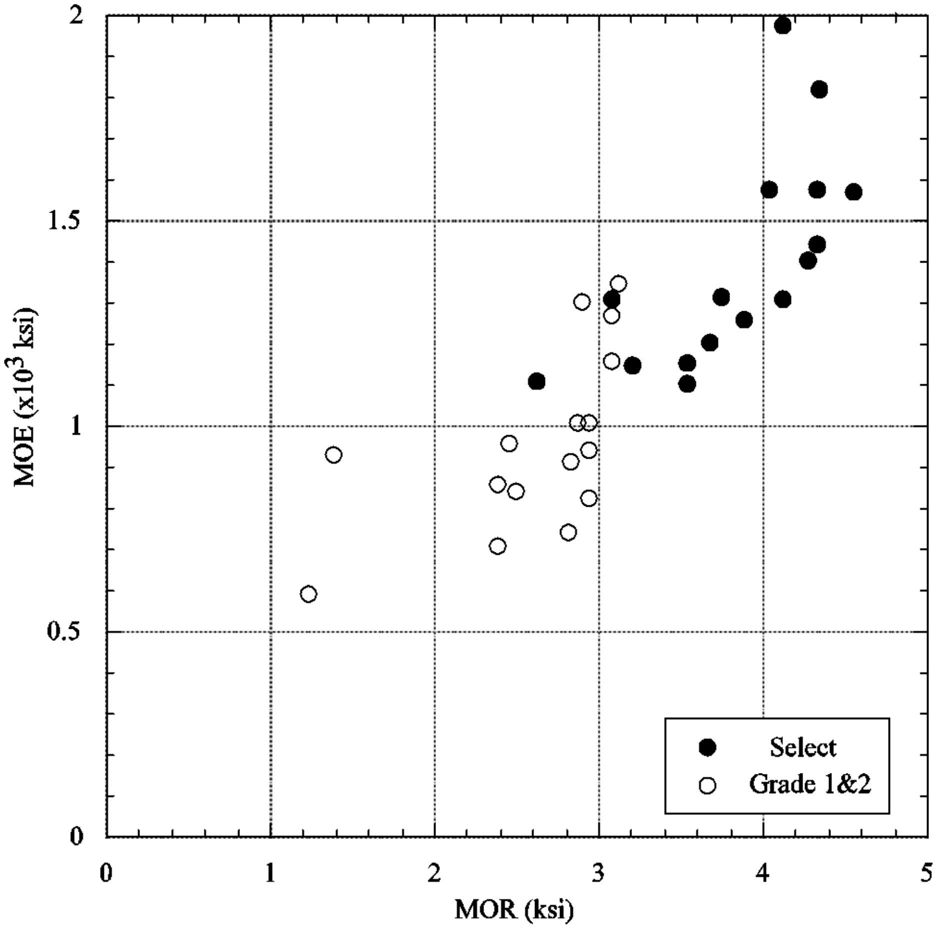
CDF's of MOR for D-F, Grade 1 & 2.

WOOD COMPONENTS, MOR vs. MOE



Scatter Plot of MOR vs. MOE for D-F, size 2x8.

WOOD COMPONENTS, MOR vs. MOE



Scatter Plot of MOR vs. MOE for D-F, size 3x8.

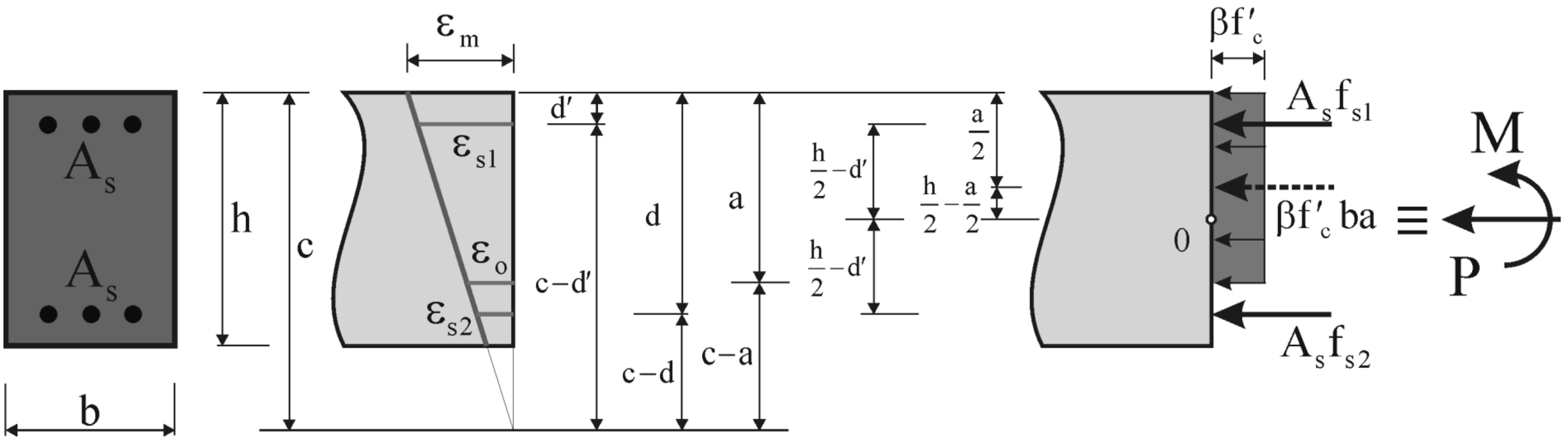
Reliability-Based Sensitivity Analysis of RC Column Resistance

Outline

- Types
- Axial load vs. Moment formulas
- Monte Carlo simulations
- Reliability Analysis

Eccentrically Loaded Columns

1. Basic Assumptions



Eccentrically Loaded Columns

Assumption of plane cross section

$$\frac{\varepsilon_m}{c} = \frac{\varepsilon_{s2}}{c-d} \qquad \varepsilon_{s2} = \varepsilon_m\left(1-\frac{d}{c}\right)$$

$$\frac{\varepsilon_m}{c} = \frac{\varepsilon_{s1}}{c-d'} \qquad \varepsilon_{s1} = \varepsilon_m\left(1-\frac{d'}{c}\right)$$

Eccentrically Loaded Columns

Reduction factor of compression zone in concrete

$$a = \begin{cases} \beta_1 c & \text{for} \quad c \le \dfrac{h}{\beta_1} \\ h & \text{for} \quad c > \dfrac{h}{\beta_1}. \end{cases}$$

Eccentrically Loaded Columns

Mechanical behavior of reinforcement steel

$$f_s = \begin{cases} -f_y & \text{for} \quad \varepsilon_s < -\varepsilon_y \\ E_s \varepsilon_s & \text{for} \quad -\varepsilon_y \le \varepsilon_s \le \varepsilon_y \\ f_y & \text{for} \quad \varepsilon_y < \varepsilon_s < \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_m < \varepsilon_s \ , \end{cases}$$

$$E_s = \frac{f_y}{\varepsilon_y}$$

Eccentrically Loaded Columns

Mechanical behavior of concrete

$$f_c = \begin{cases} 0 & \text{for} \quad \varepsilon_c < \varepsilon_o \\ \beta f'_c & \text{for} \quad \varepsilon_o \le \varepsilon_c \le \varepsilon_m \\ 0 & \text{for} \quad \varepsilon_m < \varepsilon_c \,. \end{cases}$$

$$\varepsilon_o = \varepsilon_m (1 - \beta_1)$$

Eccentrically Loaded Columns

Force and moment caring capacity

$$P = A_s f_{s1} + A_s f_{s2} + \beta f'_c \, ba$$

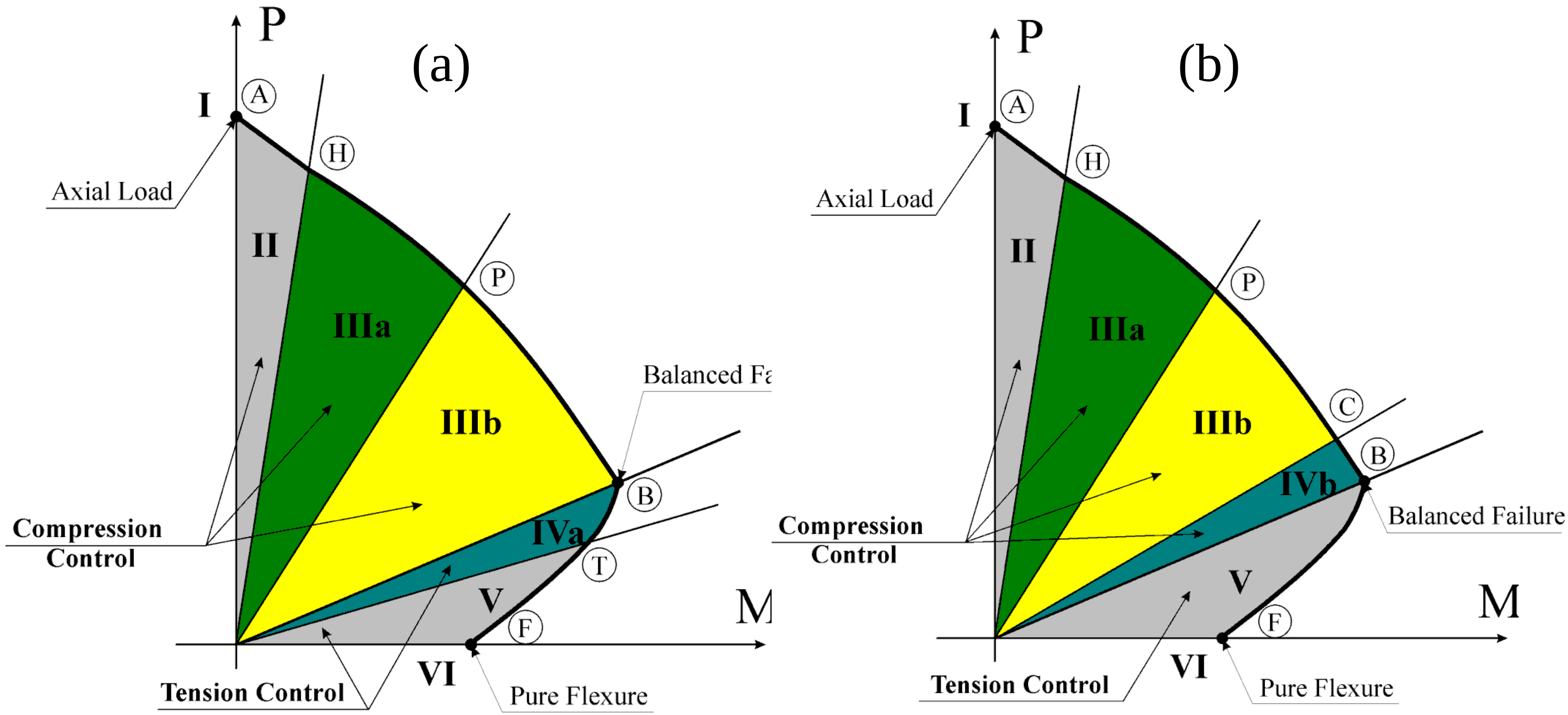
$$M = A_s f_{s1}\left(\frac{h}{2} - d'\right) - A_s f_{s2}\left(\frac{h}{2} - d'\right) + \beta f'_c \, ba\left(\frac{h}{2} - \frac{a}{2}\right)$$

Eccentrically Loaded Columns

Eccentricity of force

$$e = \frac{M}{P}$$

Analysis of Possible Cases of Cross Section Behavior



Interaction Diagram for Eccentrically Compressed Columns;
(a) Cross Sections Type I, (b) Cross Sections Type II.

Types of Cross Section

For **Type I of cross sections**

$$\varepsilon_{s_1} \geq \varepsilon_y$$

$$\frac{d'}{d} \leq \frac{9}{49}$$

For **Type II of cross sections**

$$\varepsilon_{s_1} \leq \varepsilon_y$$

$$\frac{d'}{d} \geq \frac{9}{49}$$

Type I of Cross Section

Case I (Axial Load)

$$P = 2A_s f_y + \beta f'_c bh$$

$$M = 0$$

$$e = 0$$

Case II (Whole Concrete Cross Section in Compression)

$$P = A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right) + \beta f'_c\, bh$$

$$M = A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{d}{c}\right)\left(\frac{h}{2} - d'\right)$$

$$c = \frac{A_s E_s \varepsilon_m d (2e + h - 2d')}{A_s f_y (2e - h + 2d') + A_s E_s \varepsilon_m (2e + h - 2d') + 2e\beta f'_c\, bh}$$

$$e_H = \frac{A_s f_{s1} \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right)\left(\frac{h}{2} - d'\right)}{A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{h}\right) + \beta f'_c\, bh}$$

Case III (Part of Concrete Cross Section in Compression – Compression Control)

$$P = A_s f_y + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right) + \beta f'_c \, ba$$

$$M = A_s f_y \left(\frac{h}{2} - d'\right) - A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d}{a}\right)\left(\frac{h}{2} - d'\right) + \frac{1}{2} \beta f'_c \, ba(h - a)$$

Case III (Part of Concrete Cross Section in Compression – Compression Control)

$$a = 2\sqrt{p}\,\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{q}{\sqrt{p^3}}\right)\right] - t \qquad \text{for} \qquad p > 0$$

$$a = \sqrt[3]{2q} - t \qquad \text{for} \qquad p = 0$$

$$a = 2\sqrt{-p}\,\sinh\left[\frac{1}{3}\operatorname{arcsinh}\left(\frac{q}{\sqrt{-p^3}}\right)\right] - t \qquad \text{for} \qquad p < 0$$

p,q,t - depending on geometry and material properties

•Case III (Part of Concrete Cross Section in Compression Compression Control)

$$e_P = \frac{h}{2} + \frac{3A_s\left(f_y + E_s\varepsilon_m\right) - \sqrt{\left[3A_s\left(f_y + E_s\varepsilon_m\right)\right]^2 + 24\beta f'_c\, bA_s\left[E_s\varepsilon_m h + \left(f_y - E_s\varepsilon_m\right)d'\right]}}{4\beta f'_c\, b}$$

$$e_B = \frac{A_s f_y\left(\varepsilon_m + \varepsilon_y\right)\left(h - 2d'\right)}{\beta f'_c\, b\varepsilon_m\beta_1 d} + \frac{1}{2}\left(h - \frac{\varepsilon_m\beta_1 d}{\varepsilon_m + \varepsilon_y}\right)$$

Note: Case IIIa is for $e_P < e \leq e_B$, and **Case IIIb** is for . $e_H \leq e < e_P$

⑩ Case IVa (Part of Concrete Cross Section in Compression – Tension Control)

$$P = \beta f'_c \, ba$$

$$M = A_s f_y \left(h - 2d'\right) + \frac{1}{2} \beta f'_c \, ba \left(h - a\right)$$

$$e_T = \frac{A_s f_y \left(\varepsilon_m - \varepsilon_y\right)\left(h - 2d'\right)}{\beta f'_c \, b \varepsilon_m \beta_1 d'} + \frac{1}{2}\left(h - \frac{\varepsilon_m \beta_1 d'}{\varepsilon_m - \varepsilon_y}\right)$$

Case V (Part of Concrete Cross Section in Compression – Tension Control)

$$P = A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d'}{a}\right) - A_s f_y + \beta f'_c\, ba$$

$$M = A_s f_y \left(\frac{h}{2} - d'\right) + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d'}{a}\right)\left(\frac{h}{2} - d'\right) + \frac{1}{2}\beta f'_c\, ba(h - a)$$

$$a = 2\sqrt{p}\cos\left[\frac{1}{3}\arccos\left(\frac{q}{\sqrt{p^3}}\right)\right] - t$$

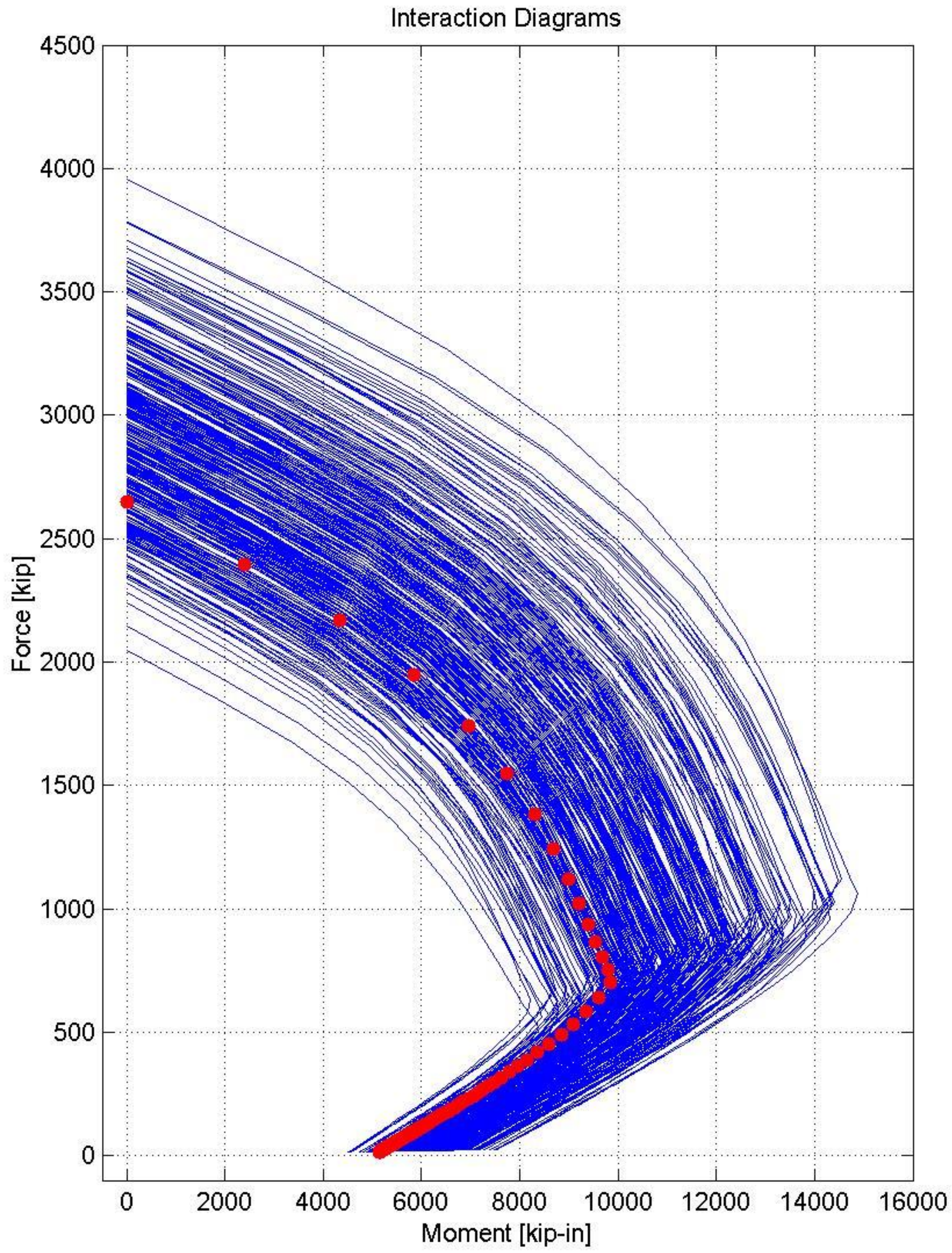
p,q,t - depending on geometry and material properties

For the **pure flexure (case VI)** is

$$P = 0$$

$$M = A_s f_y \left(\frac{h}{2} - d' \right) + A_s E_s \varepsilon_m \left(1 - \frac{\beta_1 d'}{a_M} \right) \left(\frac{h}{2} - d' \right) + \frac{1}{2} \beta f'_c \, b a_M \left(h - a_M \right)$$

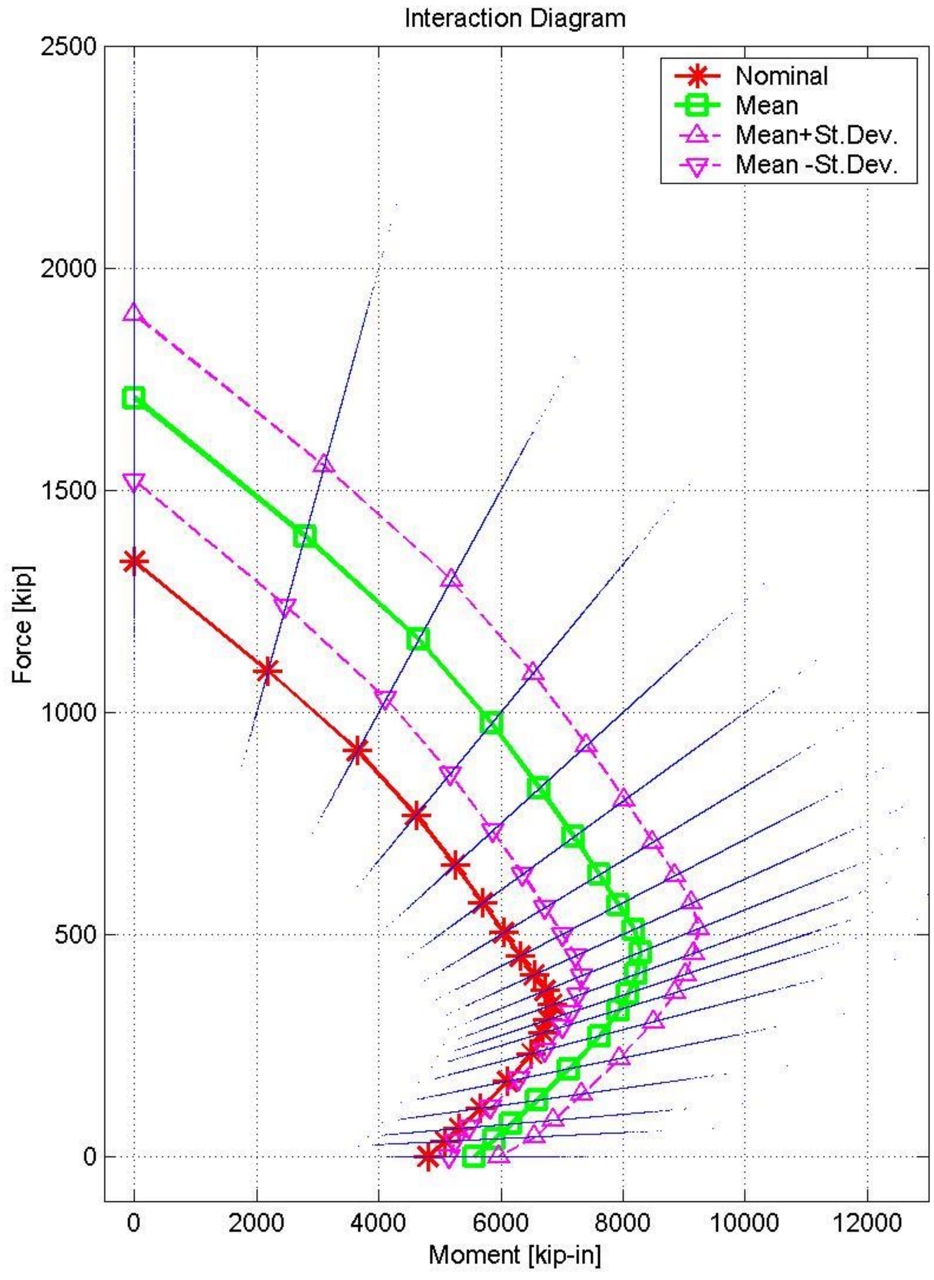
$$a_M = \frac{\sqrt{\alpha_2^2 - 4\alpha_1 \alpha_3} - \alpha_2}{2\alpha_1}$$



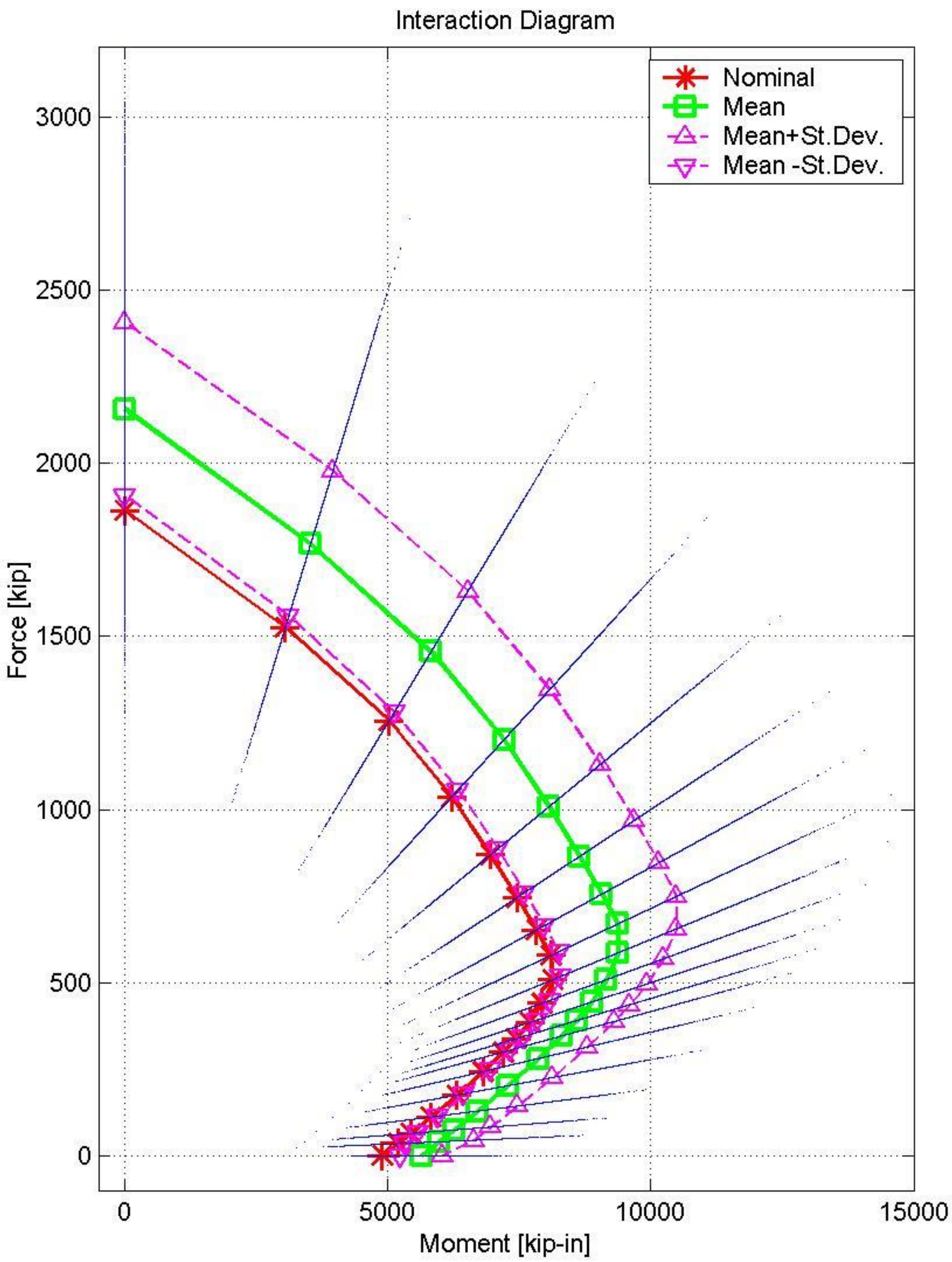
Simulated Interaction Diagrams

- concrete strength of 55 MPa
- tied columns
- cast-in-place

red dots = nominal values



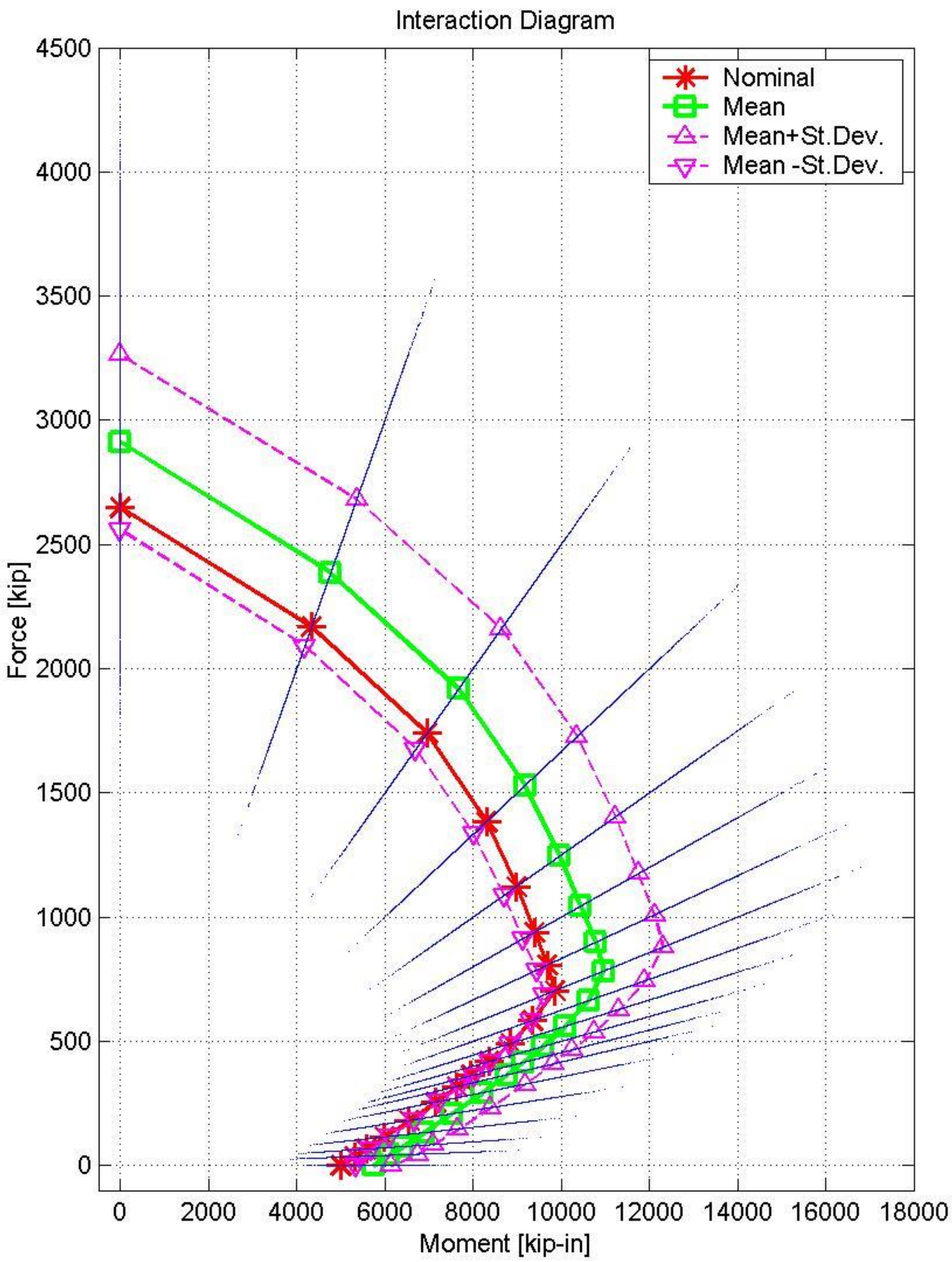
Interaction Diagrams
For Concrete 20 MPa
(tied columns, cast-in-place)



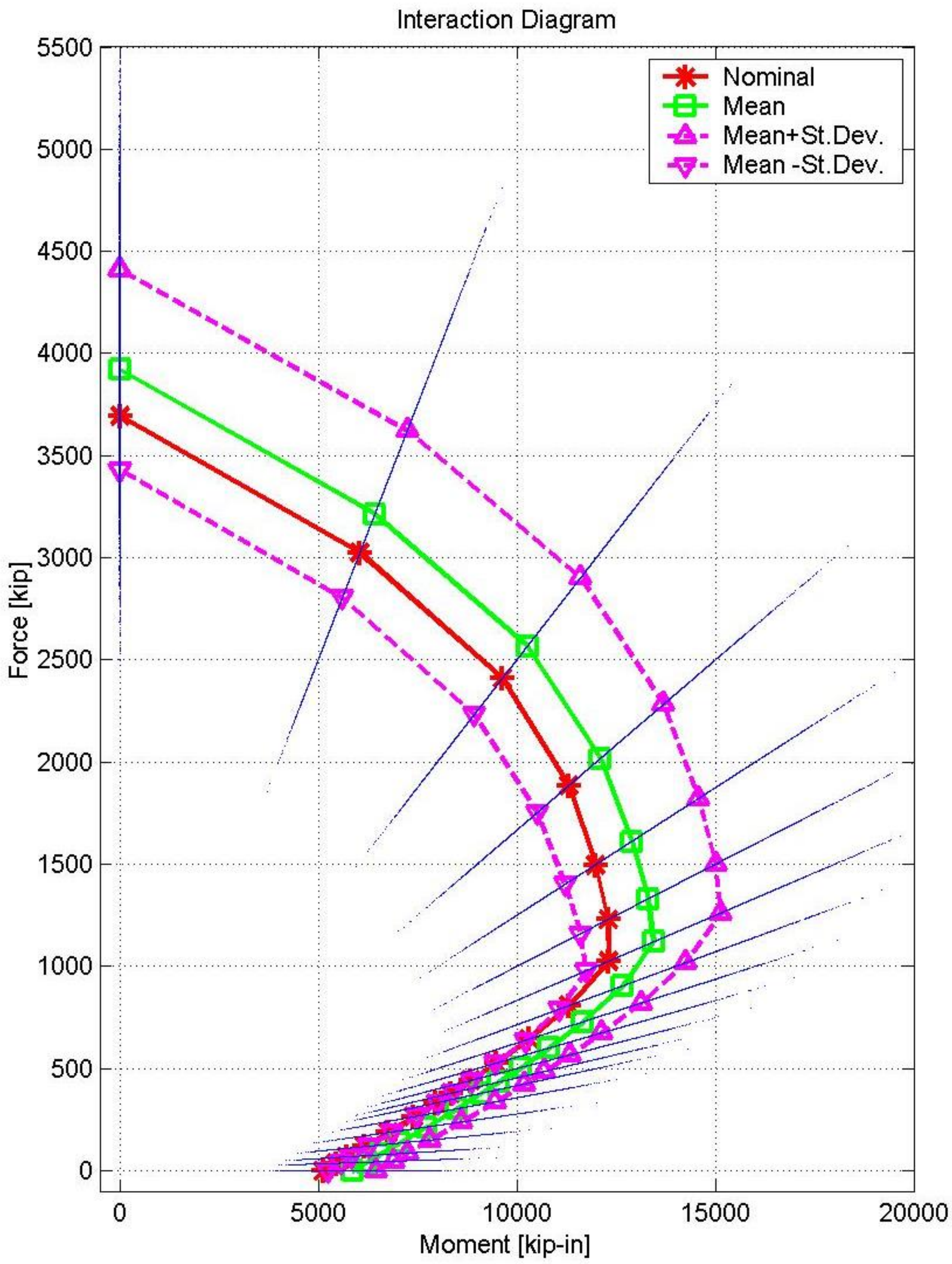
Interaction Diagrams

For Concrete 35 MPa

(tied columns, cast-in-place)

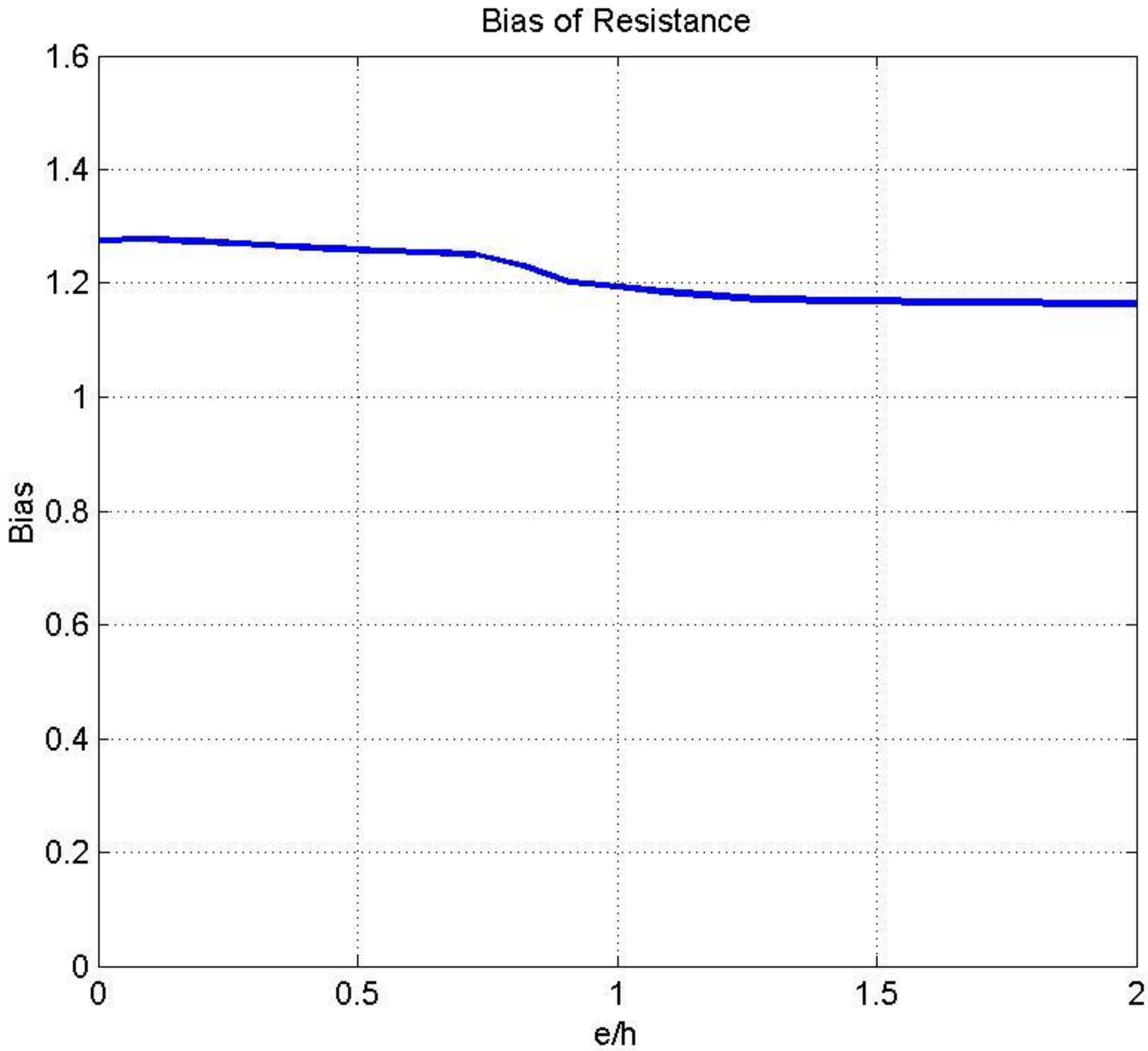


Interaction Diagrams
For Concrete 55 MPa
(tied columns, cast-in-place)



Interaction Diagrams

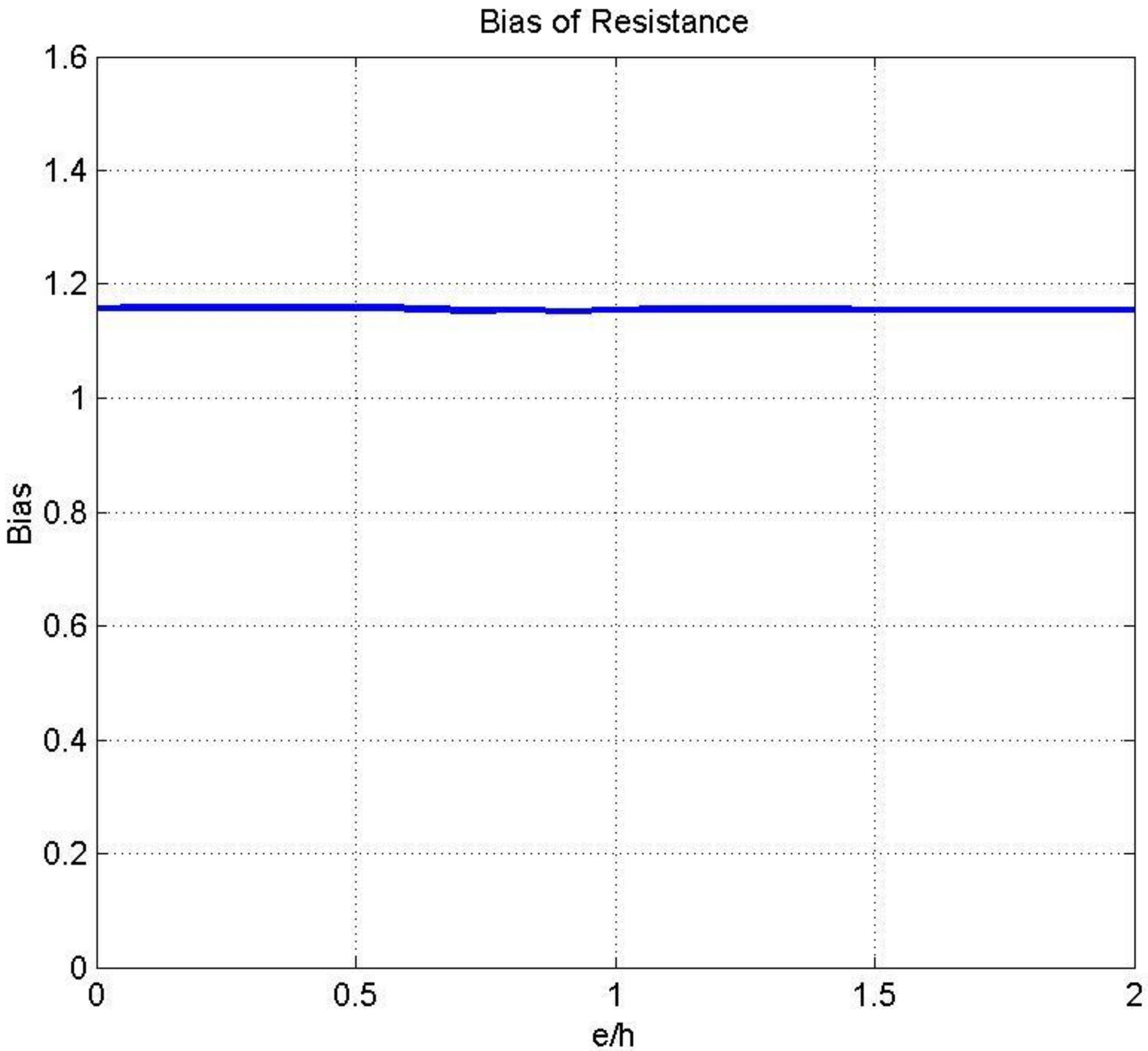
- concrete 85 MPa
- tied columns,
- cast-in-place

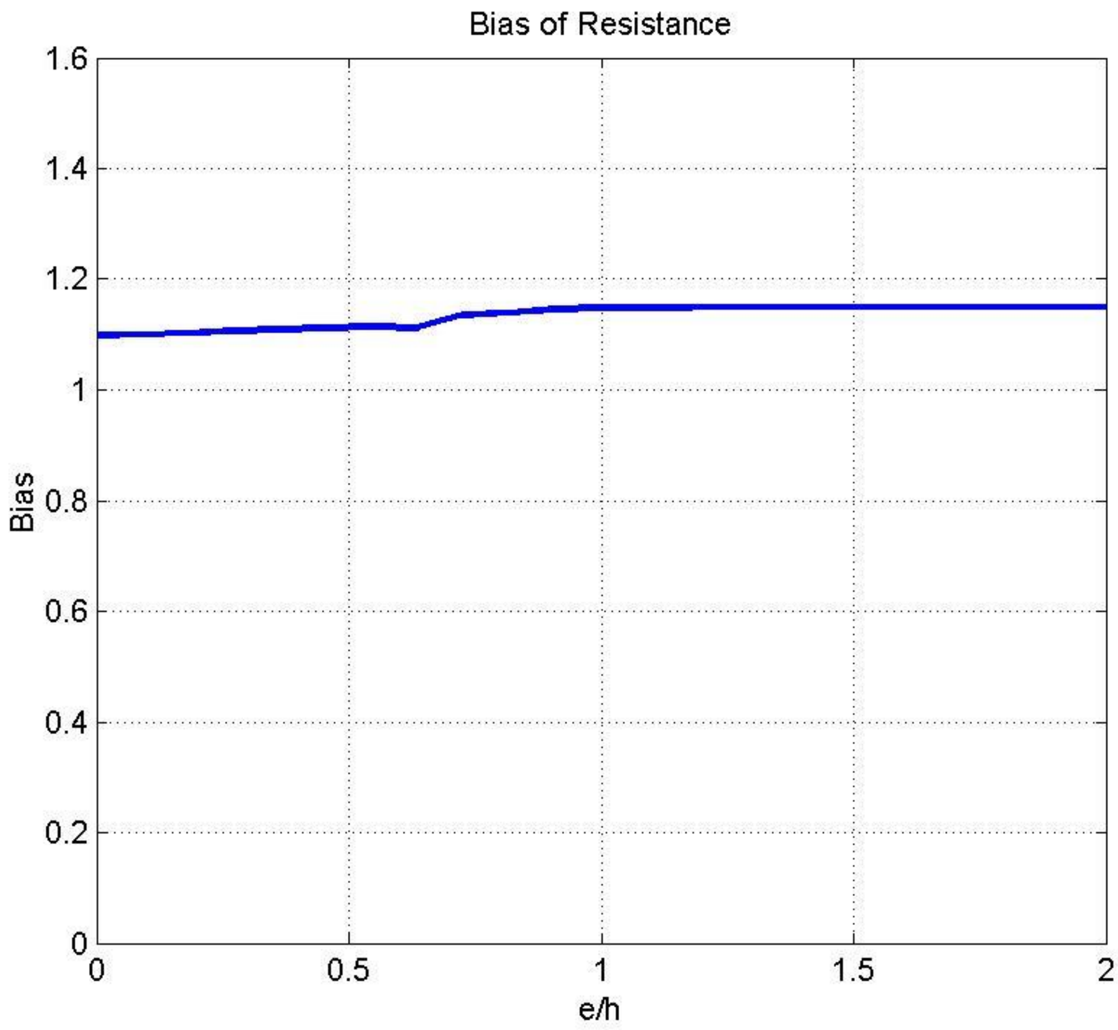


Bias for

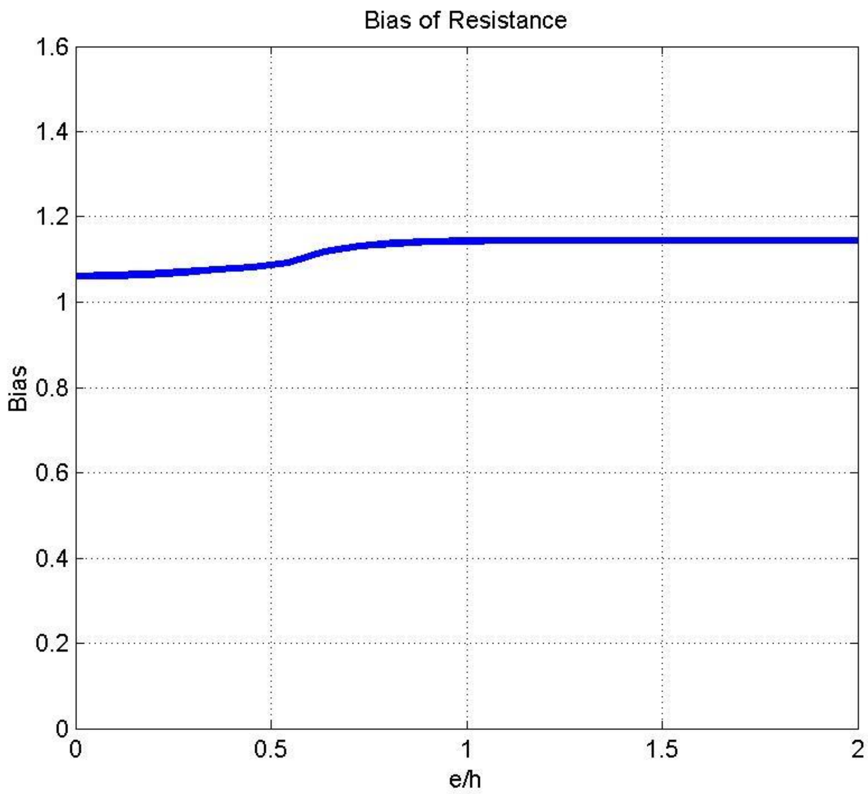
- concrete 20 MPa
- tied columns
- cast-in-place)

Bias for Concrete 35 MPa (calculated for tied columns cast-in-place)



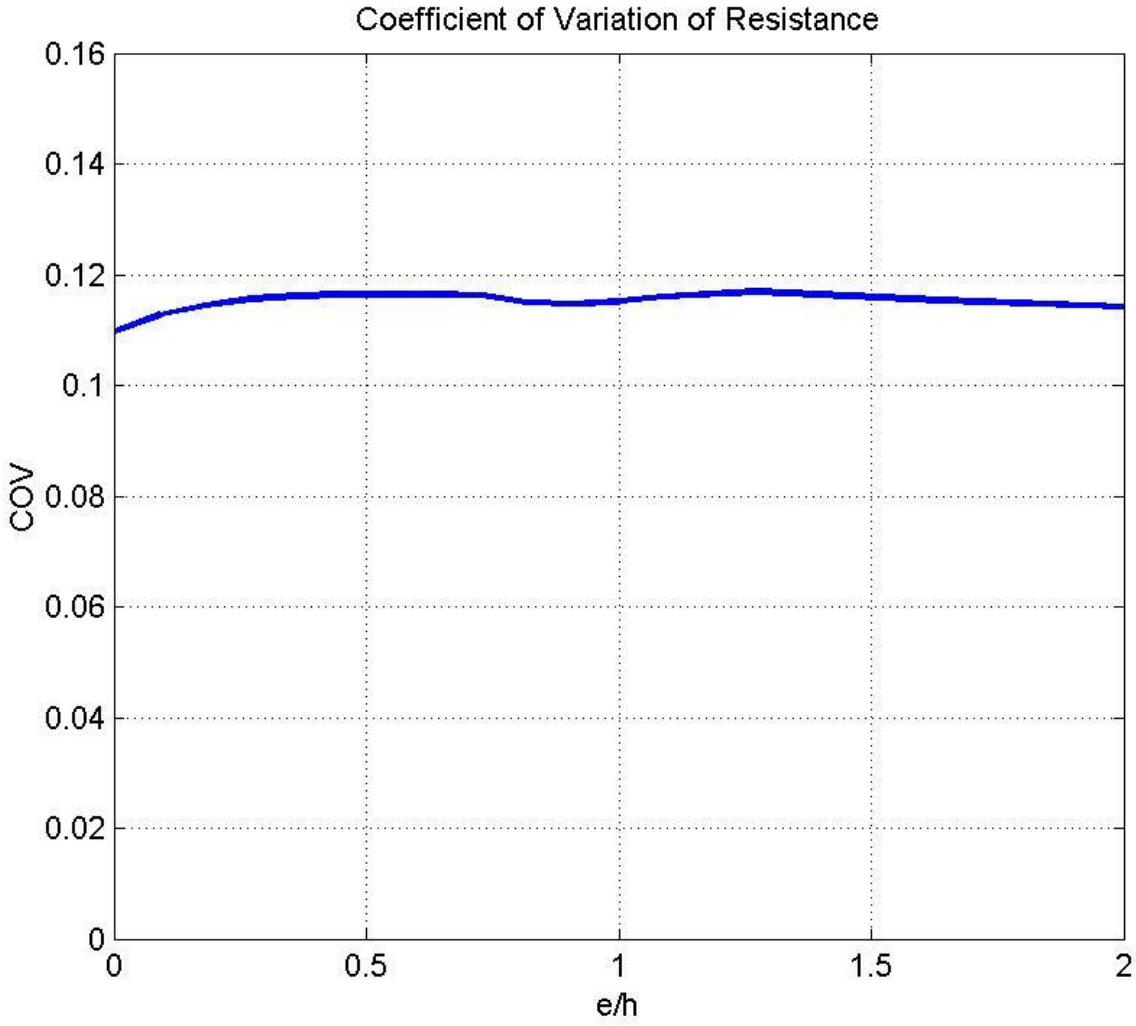


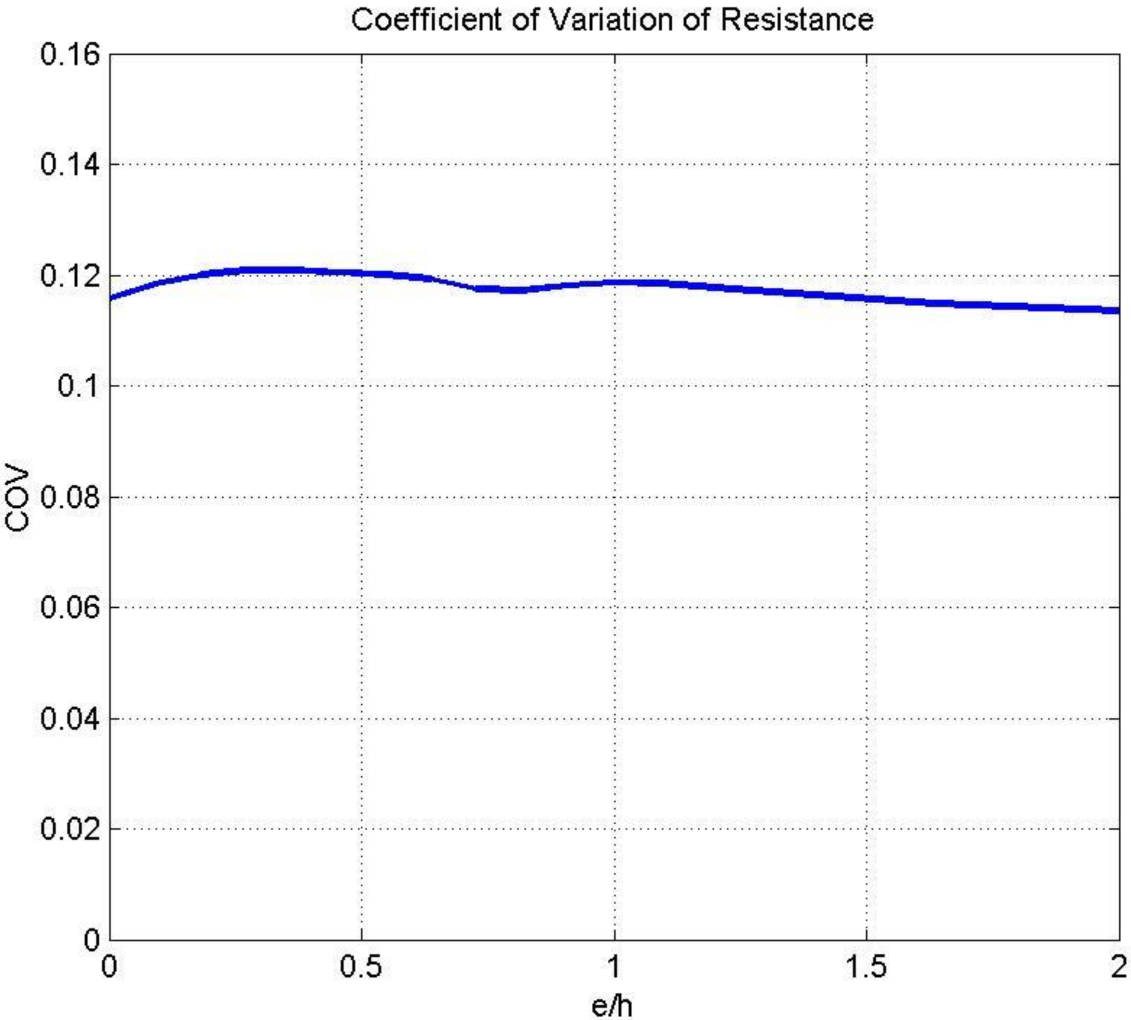
Bias for Concrete 55 MPa (calculated for tied columns cast-in-place)



Bias for Concrete 85 MPa (calculated for tied columns cast-in-place)

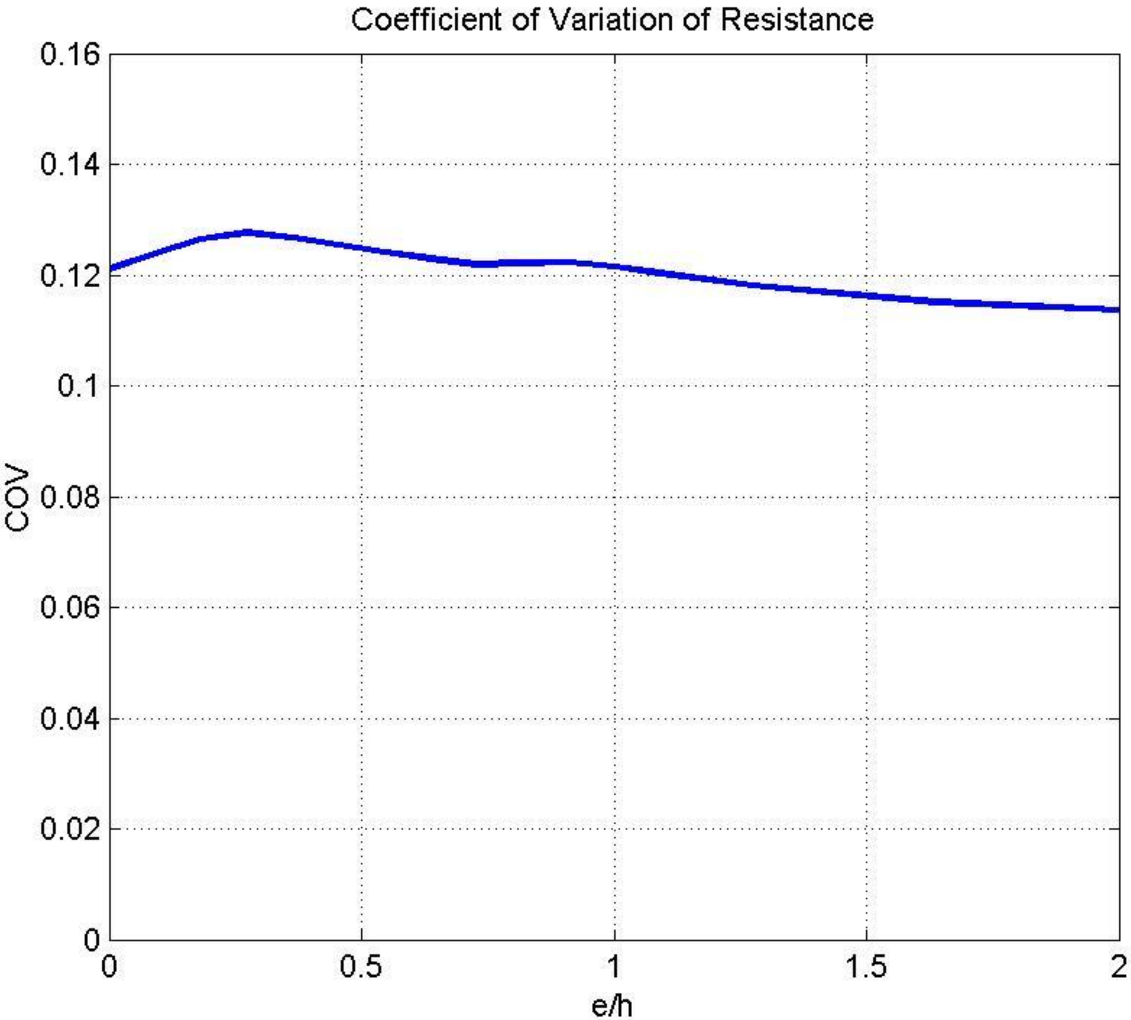
COV for Concrete 20 MPa (calculated for tied columns cast-in-place)

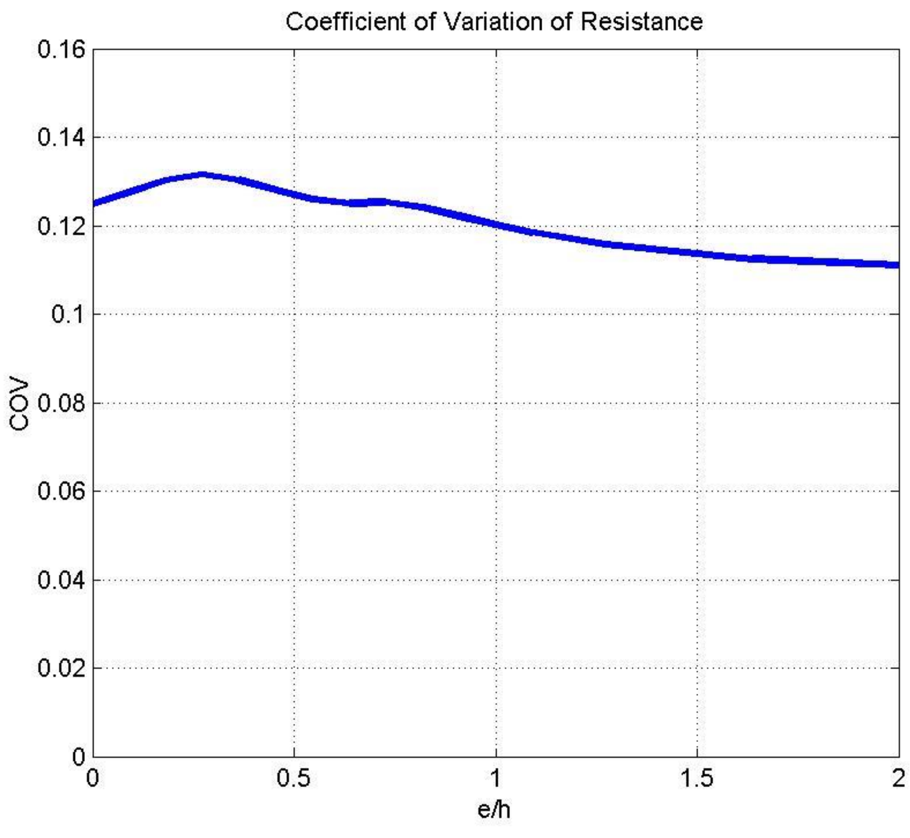




COV for Concrete 35 MPa (calculated for tied columns cast-in-place)

COV for Concrete 55 MPa (calculated for tied columns cast-in-place)





COV for Concrete 85 MPa (calculated for tied columns cast-in-place)

Statistical Parameters for Resistance (bias)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.982	1.320	1.159	1.092	1.050
	4	0.18	0.986	1.320	1.161	1.095	1.056
	8	0.36	1.001	1.302	1.162	1.103	1.069
	10	0.45	1.024	1.282	1.161	1.116	1.095
	12	0.55	1.055	1.246	1.156	1.133	1.124
	14	0.64	1.078	1.216	1.160	1.142	1.136
	16	0.73	1.095	1.202	1.161	1.147	1.141
	18	0.82	1.101	1.194	1.160	1.147	1.144
	20	0.91	1.110	1.187	1.159	1.150	1.143
	24	1.09	1.112	1.177	1.157	1.148	1.140
	40	1.82	1.116	1.169	1.156	1.147	1.141
	72	3.27	1.115	1.167	1.154	1.145	1.140
2%	0	0.00	0.997	1.292	1.160	1.096	1.055
	4	0.18	1.002	1.292	1.160	1.099	1.060
	8	0.36	1.012	1.279	1.161	1.105	1.072
	10	0.45	1.022	1.274	1.159	1.111	1.077
	12	0.55	1.021	1.269	1.151	1.115	1.110
	14	0.64	1.055	1.250	1.154	1.131	1.130
	16	0.73	1.074	1.219	1.152	1.141	1.141
	18	0.82	1.095	1.200	1.156	1.147	1.139
	20	0.91	1.102	1.187	1.157	1.146	1.143
	24	1.09	1.112	1.175	1.155	1.148	1.143
	40	1.82	1.121	1.165	1.154	1.149	1.143
	72	3.27	1.121	1.163	1.152	1.147	1.144

Highlighted values are for balanced failure

Statistical Parameters for Resistance (bias)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place bias				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	1.007	1.274	1.159	1.096	1.061
	4	0.18	1.016	1.273	1.158	1.104	1.064
	8	0.36	1.025	1.261	1.159	1.110	1.076
	10	0.45	1.028	1.262	1.159	1.110	1.084
	12	0.55	1.032	1.257	1.157	1.113	1.094
	14	0.64	1.036	1.254	1.156	1.114	1.120
	16	0.73	1.049	1.248	1.155	1.133	1.132
	18	0.82	1.074	1.228	1.154	1.138	1.136
	20	0.91	1.085	1.198	1.151	1.143	1.141
	24	1.09	1.109	1.184	1.154	1.148	1.144
	40	1.82	1.119	1.165	1.153	1.147	1.147
	72	3.27	1.125	1.158	1.154	1.150	1.143
4%	0	0.00	1.017	1.260	1.157	1.099	1.067
	4	0.18	1.022	1.256	1.160	1.104	1.068
	8	0.36	1.032	1.249	1.159	1.110	1.078
	10	0.45	1.039	1.248	1.158	1.116	1.085
	12	0.55	1.037	1.243	1.156	1.112	1.086
	14	0.64	1.041	1.241	1.155	1.115	1.106
	16	0.73	1.044	1.240	1.155	1.110	1.125
	18	0.82	1.042	1.239	1.148	1.132	1.135
	20	0.91	1.071	1.234	1.153	1.141	1.140
	24	1.09	1.095	1.194	1.149	1.148	1.147
	40	1.82	1.124	1.162	1.154	1.151	1.146
	72	3.27	1.125	1.158	1.153	1.149	1.145

Highlighted values are for balance failure

Statistical Parameters for Resistance (COV)

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
1%	0	0.00	0.1595	0.1257	0.1291	0.1322	0.1356
	4	0.18	0.1620	0.1295	0.1347	0.1381	0.1402
	8	0.36	0.1570	0.1275	0.1352	0.1395	0.1426
	10	0.45	0.1536	0.1280	0.1344	0.1430	0.1513
	12	0.55	0.1471	0.1277	0.1338	0.1427	0.1559
	14	0.64	0.1475	0.1287	0.1357	0.1402	0.1474
	16	0.73	0.1484	0.1274	0.1215	0.1361	0.1386
	18	0.82	0.1474	0.1266	0.1273	0.1301	0.1305
	20	0.91	0.1452	0.1209	0.1237	0.1263	0.1236
	24	1.09	0.1428	0.1182	0.1203	0.1204	0.1171
	40	1.82	0.1351	0.1131	0.1106	0.1100	0.1112
	72	3.27	0.1347	0.1090	0.1088	0.1077	0.1063
2%	0	0.00	0.1487	0.1172	0.1246	0.1279	0.1313
	4	0.18	0.1487	0.1195	0.1261	0.1306	0.1352
	8	0.36	0.1450	0.1205	0.1260	0.1304	0.1361
	10	0.45	0.1467	0.1213	0.1257	0.1315	0.1352
	12	0.55	0.1425	0.1217	0.1232	0.1272	0.1362
	14	0.64	0.1413	0.1200	0.1235	0.1291	0.1363
	16	0.73	0.1392	0.1176	0.1220	0.1280	0.1328
	18	0.82	0.1405	0.1187	0.1232	0.1264	0.1286
	20	0.91	0.1420	0.1184	0.1215	0.1250	0.1242
	24	1.09	0.1403	0.1176	0.1194	0.1191	0.1187
	40	1.82	0.1395	0.1141	0.1136	0.1136	0.1124
	72	3.27	0.1368	0.1103	0.1089	0.1091	0.1078

Highlighted values are for balance failure

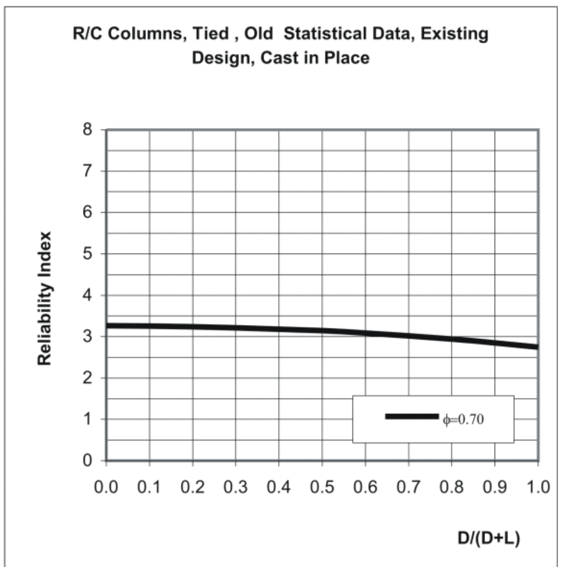
Statistical Parameters for Resistance (COV)

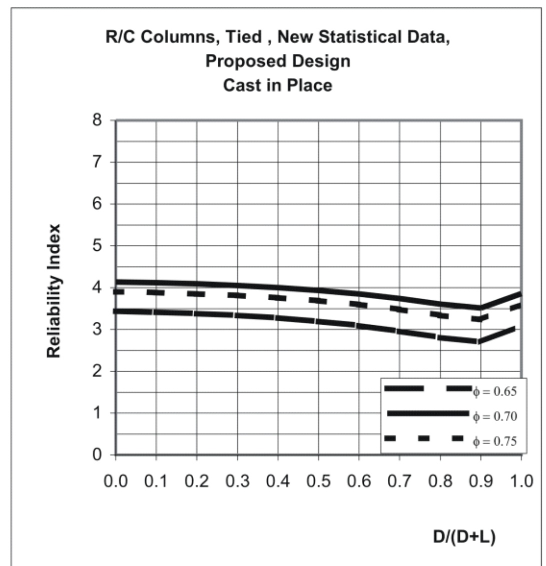
Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place COV				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
3%	0	0.00	0.1382	0.1096	0.1158	0.1211	0.1262
	4	0.18	0.1401	0.1143	0.1196	0.1255	0.1318
	8	0.36	0.1393	0.1163	0.1196	0.1256	0.1320
	10	0.45	0.1408	0.1158	0.1193	0.1271	0.1299
	12	0.55	0.1369	0.1168	0.1210	0.1228	0.1284
	14	0.64	0.1389	0.1149	0.1182	0.1240	0.1262
	16	0.73	0.1366	0.1166	0.1169	0.1219	0.1288
	18	0.82	0.1357	0.1158	0.1156	0.1213	0.1258
	20	0.91	0.1363	0.1137	0.1186	0.1214	0.1239
	24	1.09	0.1371	0.1161	0.1185	0.1176	0.1208
	40	1.82	0.1396	0.1143	0.1129	0.1137	0.1124
	72	3.27	0.1373	0.1115	0.1097	0.1098	0.1093
4%	0	0.00	0.1314	0.1048	0.1102	0.1169	0.1216
	4	0.18	0.1353	0.1092	0.1149	0.1224	0.1276
	8	0.36	0.1340	0.1116	0.1163	0.1218	0.1277
	10	0.45	0.1339	0.1133	0.1162	0.1201	0.1273
	12	0.55	0.1346	0.1135	0.1152	0.1206	0.1256
	14	0.64	0.1329	0.1117	0.1155	0.1189	0.1221
	16	0.73	0.1343	0.1123	0.1151	0.1188	0.1210
	18	0.82	0.1333	0.1124	0.1148	0.1179	0.1226
	20	0.91	0.1341	0.1125	0.1140	0.1182	0.1217
	24	1.09	0.1374	0.1125	0.1154	0.1200	0.1202
	40	1.82	0.1388	0.1149	0.1132	0.1133	0.1136
	72	3.27	0.1388	0.1114	0.1114	0.1105	0.1100

Highlighted values are for balance failure

R/C Columns, Cast-in-place, Tied D + L, e/h = 0.18 compression control

1% of reinforcement				fc = 5.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.395	2.477	2.654	2.185	1.000	0.180	0.18	3.44	4.13	3.90	3.26
0.10	0.10	0.90	0.00	1.56	1.67	2.352	2.415	2.587	2.131	1.005	0.162	0.16	3.41	4.12	3.88	3.25
0.20	0.20	0.80	0.00	1.52	1.64	2.310	2.353	2.521	2.076	1.010	0.146	0.14	3.38	4.09	3.85	3.24
0.30	0.30	0.70	0.00	1.48	1.61	2.268	2.291	2.455	2.022	1.015	0.130	0.13	3.34	4.05	3.81	3.22
0.40	0.40	0.60	0.00	1.44	1.58	2.226	2.229	2.388	1.967	1.020	0.116	0.11	3.27	4.00	3.76	3.18
0.50	0.50	0.50	0.00	1.40	1.55	2.183	2.167	2.322	1.912	1.025	0.104	0.10	3.19	3.93	3.69	3.14
0.60	0.60	0.40	0.00	1.36	1.52	2.141	2.105	2.256	1.858	1.030	0.096	0.09	3.09	3.85	3.59	3.09
0.70	0.70	0.30	0.00	1.32	1.49	2.099	2.043	2.189	1.803	1.035	0.091	0.09	2.96	3.74	3.48	3.02
0.80	0.80	0.20	0.00	1.28	1.46	2.057	1.981	2.123	1.748	1.040	0.091	0.09	2.80	3.61	3.34	2.94
0.90	0.90	0.10	0.00	1.26	1.43	2.014	1.950	2.090	1.721	1.045	0.096	0.09	2.69	3.51	3.24	2.85
1.00	1.00	0.00	0.00	1.40	1.40	1.972	2.167	2.322	1.912	1.050	0.105	0.10	3.10	3.86	3.60	2.74
										average beta			3.00	3.77	3.51	3.04



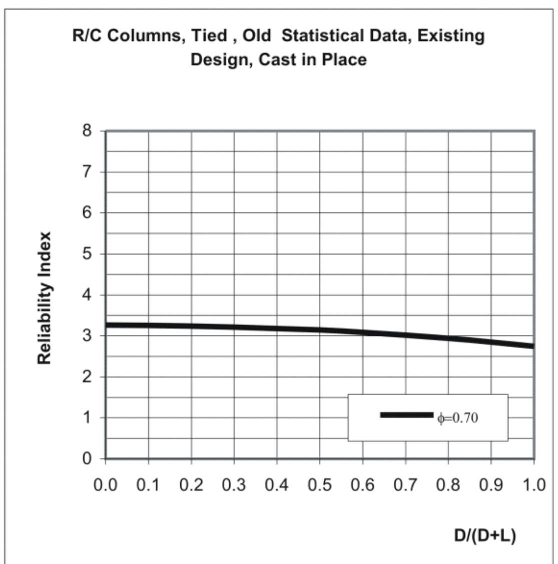


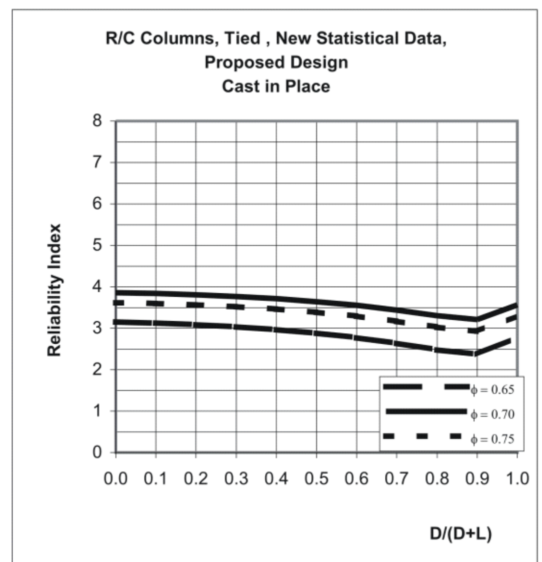
Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied D + L, e/h = 0.18 compression control

1% of reinforcement fc = 8.0 ksi

				new	old	old	mR new data/design						beta new			beta old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.395	2.336	2.503	2.061	1.000	0.180	0.18	3.15	3.86	3.62	3.26
0.10	0.10	0.90	0.00	1.56	1.67	2.352	2.278	2.440	2.010	1.005	0.162	0.16	3.12	3.84	3.60	3.25
0.20	0.20	0.80	0.00	1.52	1.64	2.310	2.219	2.378	1.958	1.010	0.146	0.14	3.09	3.81	3.56	3.24
0.30	0.30	0.70	0.00	1.48	1.61	2.268	2.161	2.315	1.907	1.015	0.130	0.13	3.04	3.77	3.52	3.22
0.40	0.40	0.60	0.00	1.44	1.58	2.226	2.102	2.253	1.855	1.020	0.116	0.11	2.97	3.71	3.46	3.18
0.50	0.50	0.50	0.00	1.40	1.55	2.183	2.044	2.190	1.804	1.025	0.104	0.10	2.88	3.64	3.39	3.14
0.60	0.60	0.40	0.00	1.36	1.52	2.141	1.986	2.127	1.752	1.030	0.096	0.09	2.78	3.55	3.29	3.09
0.70	0.70	0.30	0.00	1.32	1.49	2.099	1.927	2.065	1.700	1.035	0.091	0.09	2.64	3.44	3.17	3.02
0.80	0.80	0.20	0.00	1.28	1.46	2.057	1.869	2.002	1.649	1.040	0.091	0.09	2.48	3.30	3.03	2.94
0.90	0.90	0.10	0.00	1.26	1.43	2.014	1.840	1.971	1.623	1.045	0.096	0.09	2.37	3.21	2.93	2.85
1.00	1.00	0.00	0.00	1.40	1.40	1.972	2.044	2.190	1.804	1.050	0.105	0.10	2.79	3.56	3.30	2.74
										average beta			2.69	3.48	3.21	3.04

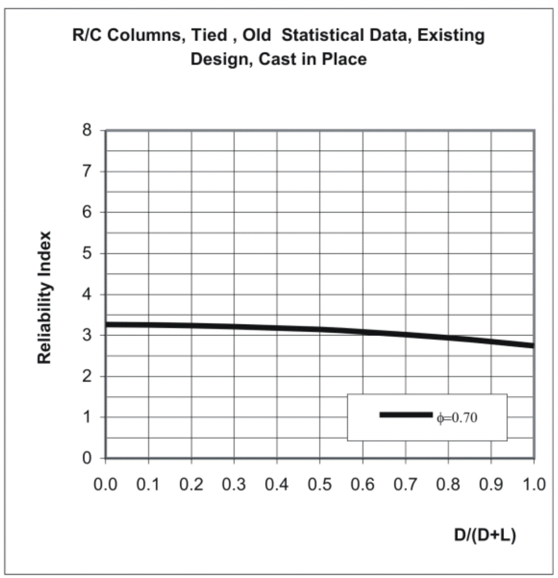


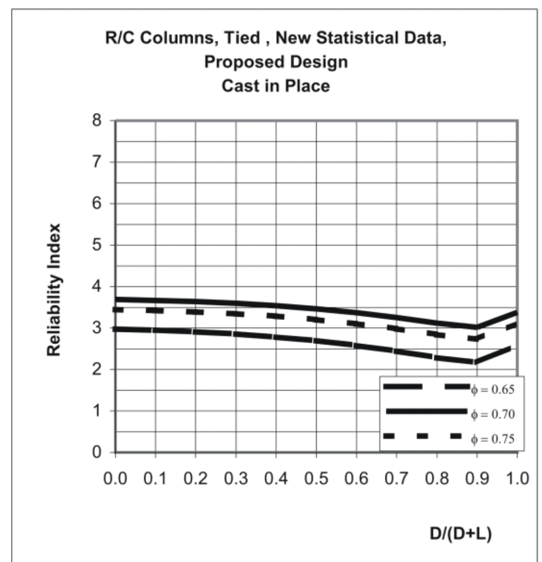


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied D + L, e/h = 0.18 compression control

1% of reinforcement				fc = 12.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.395	2.253	2.414	1.988	1.000	0.180	0.18	2.98	3.69	3.45	3.26
0.10	0.10	0.90	0.00	1.56	1.67	2.352	2.196	2.353	1.938	1.005	0.162	0.16	2.95	3.67	3.42	3.25
0.20	0.20	0.80	0.00	1.52	1.64	2.310	2.140	2.293	1.888	1.010	0.146	0.14	2.91	3.64	3.39	3.24
0.30	0.30	0.70	0.00	1.48	1.61	2.268	2.084	2.233	1.839	1.015	0.130	0.13	2.85	3.59	3.34	3.22
0.40	0.40	0.60	0.00	1.44	1.58	2.226	2.028	2.172	1.789	1.020	0.116	0.11	2.78	3.54	3.28	3.18
0.50	0.50	0.50	0.00	1.40	1.55	2.183	1.971	2.112	1.739	1.025	0.104	0.10	2.69	3.46	3.20	3.14
0.60	0.60	0.40	0.00	1.36	1.52	2.141	1.915	2.052	1.690	1.030	0.096	0.09	2.58	3.37	3.10	3.09
0.70	0.70	0.30	0.00	1.32	1.49	2.099	1.859	1.991	1.640	1.035	0.091	0.09	2.45	3.26	2.98	3.02
0.80	0.80	0.20	0.00	1.28	1.46	2.057	1.802	1.931	1.590	1.040	0.091	0.09	2.28	3.12	2.84	2.94
0.90	0.90	0.10	0.00	1.26	1.43	2.014	1.774	1.901	1.565	1.045	0.096	0.09	2.17	3.02	2.73	2.85
1.00	1.00	0.00	0.00	1.40	1.40	1.972	1.971	2.112	1.739	1.050	0.105	0.10	2.60	3.38	3.12	2.74
										average beta			2.49	3.29	3.02	3.04



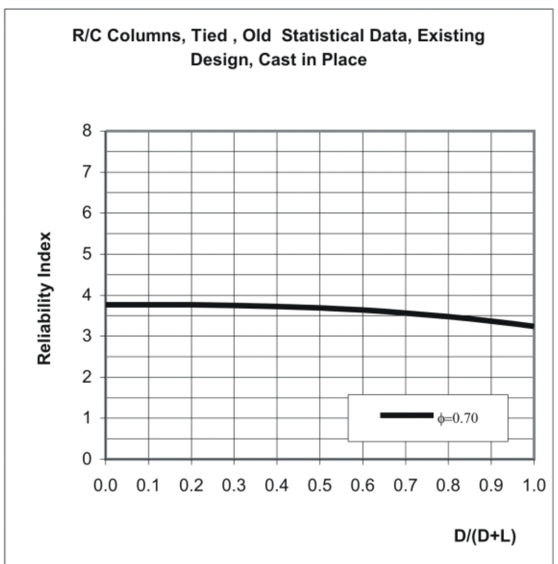


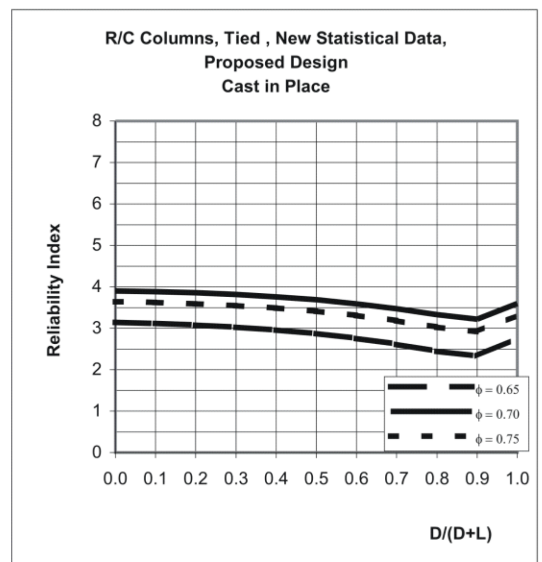
Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied D + L, e/h = 0.18 compression control

3% of reinforcement fc = 12.0 ksi

				new	old	old	mR new data/design						beta new			beta old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.467	2.270	2.432	2.003	1.000	0.180	0.18	3.14	3.90	3.64	3.77
0.10	0.10	0.90	0.00	1.56	1.67	2.424	2.213	2.371	1.953	1.005	0.162	0.16	3.11	3.88	3.62	3.77
0.20	0.20	0.80	0.00	1.52	1.64	2.380	2.156	2.310	1.903	1.010	0.146	0.14	3.08	3.85	3.59	3.77
0.30	0.30	0.70	0.00	1.48	1.61	2.337	2.100	2.250	1.853	1.015	0.130	0.13	3.03	3.81	3.55	3.75
0.40	0.40	0.60	0.00	1.44	1.58	2.293	2.043	2.189	1.803	1.020	0.116	0.11	2.96	3.76	3.49	3.73
0.50	0.50	0.50	0.00	1.40	1.55	2.250	1.986	2.128	1.752	1.025	0.104	0.10	2.87	3.69	3.41	3.69
0.60	0.60	0.40	0.00	1.36	1.52	2.206	1.929	2.067	1.702	1.030	0.096	0.09	2.76	3.59	3.31	3.64
0.70	0.70	0.30	0.00	1.32	1.49	2.163	1.873	2.006	1.652	1.035	0.091	0.09	2.61	3.47	3.18	3.56
0.80	0.80	0.20	0.00	1.28	1.46	2.119	1.816	1.946	1.602	1.040	0.091	0.09	2.44	3.33	3.03	3.47
0.90	0.90	0.10	0.00	1.26	1.43	2.076	1.788	1.915	1.577	1.045	0.096	0.09	2.32	3.22	2.92	3.36
1.00	1.00	0.00	0.00	1.40	1.40	2.032	1.986	2.128	1.752	1.050	0.105	0.10	2.77	3.60	3.32	3.24
										average beta			2.66	3.51	3.22	3.58



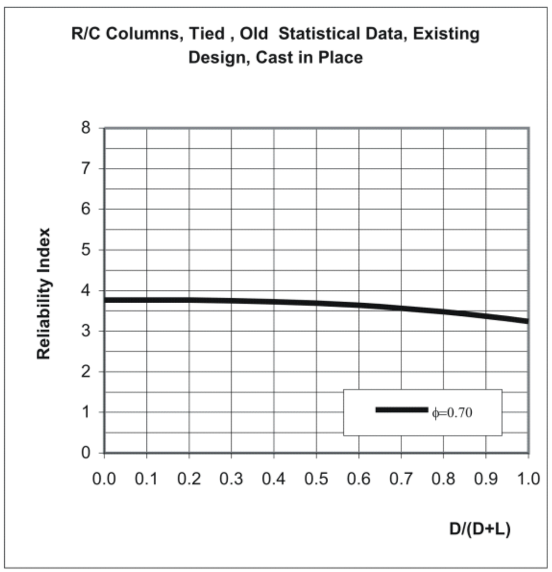


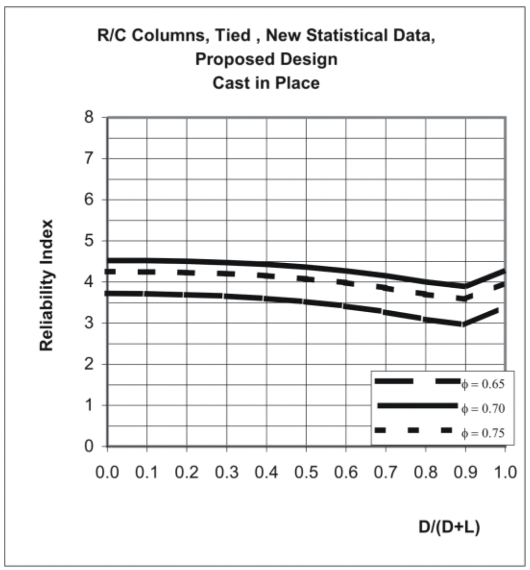
Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied **D + L,** **e/h = 0.18 compression control**

3% of reinforcement fc = 5.0 ksi

				new	old	old	mR new data/design						beta new			beta old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.467	2.470	2.647	2.180	1.000	0.180	0.18	3.72	4.52	4.25	3.77
0.10	0.10	0.90	0.00	1.56	1.67	2.424	2.409	2.581	2.125	1.005	0.162	0.16	3.71	4.52	4.24	3.77
0.20	0.20	0.80	0.00	1.52	1.64	2.380	2.347	2.515	2.071	1.010	0.146	0.14	3.69	4.50	4.23	3.77
0.30	0.30	0.70	0.00	1.48	1.61	2.337	2.285	2.448	2.016	1.015	0.130	0.13	3.66	4.47	4.20	3.75
0.40	0.40	0.60	0.00	1.44	1.58	2.293	2.223	2.382	1.962	1.020	0.116	0.11	3.60	4.43	4.15	3.73
0.50	0.50	0.50	0.00	1.40	1.55	2.250	2.162	2.316	1.907	1.025	0.104	0.10	3.52	4.36	4.08	3.69
0.60	0.60	0.40	0.00	1.36	1.52	2.206	2.100	2.250	1.853	1.030	0.096	0.09	3.41	4.27	3.98	3.64
0.70	0.70	0.30	0.00	1.32	1.49	2.163	2.038	2.184	1.798	1.035	0.091	0.09	3.27	4.15	3.85	3.56
0.80	0.80	0.20	0.00	1.28	1.46	2.119	1.976	2.117	1.744	1.040	0.091	0.09	3.09	4.00	3.69	3.47
0.90	0.90	0.10	0.00	1.26	1.43	2.076	1.945	2.084	1.717	1.045	0.096	0.09	2.96	3.89	3.58	3.36
1.00	1.00	0.00	0.00	1.40	1.40	2.032	2.162	2.316	1.907	1.050	0.105	0.10	3.41	4.27	3.98	3.24
										average beta			3.31	4.18	3.89	3.58

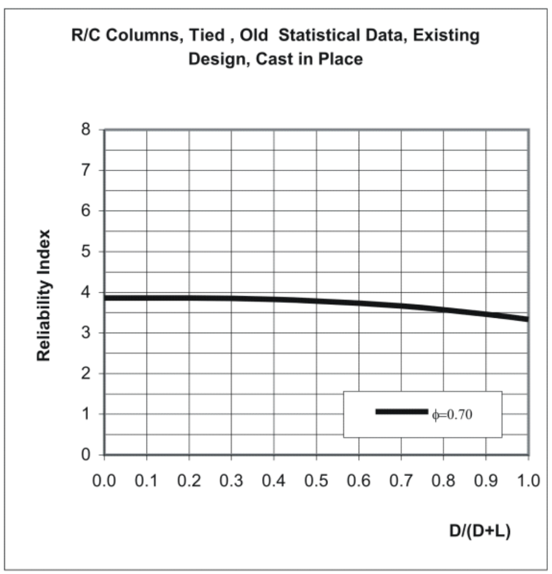


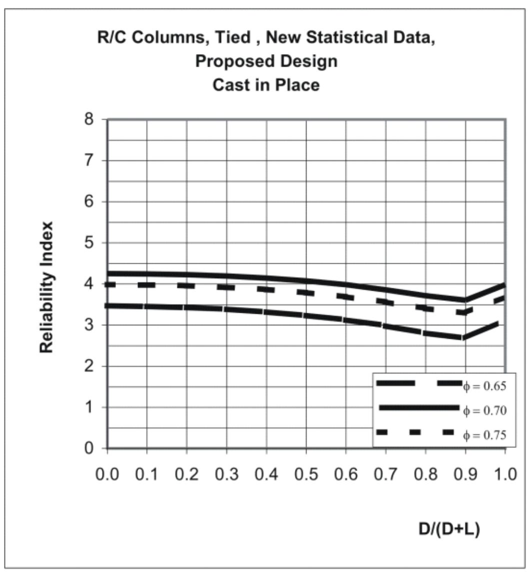


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied **D + L,** **e/h = 0.64 balance control**

3% of reinforcement				fc = 8.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.516	2.377	2.546	2.097	1.000	0.180	0.18	3.47	4.25	3.99	3.86
0.10	0.10	0.90	0.00	1.56	1.67	2.472	2.317	2.483	2.045	1.005	0.162	0.16	3.45	4.25	3.98	3.86
0.20	0.20	0.80	0.00	1.52	1.64	2.427	2.258	2.419	1.992	1.010	0.146	0.14	3.43	4.23	3.95	3.86
0.30	0.30	0.70	0.00	1.48	1.61	2.383	2.198	2.355	1.940	1.015	0.130	0.13	3.38	4.19	3.92	3.85
0.40	0.40	0.60	0.00	1.44	1.58	2.338	2.139	2.292	1.887	1.020	0.116	0.11	3.32	4.14	3.87	3.82
0.50	0.50	0.50	0.00	1.40	1.55	2.294	2.079	2.228	1.835	1.025	0.104	0.10	3.24	4.07	3.79	3.79
0.60	0.60	0.40	0.00	1.36	1.52	2.250	2.020	2.164	1.782	1.030	0.096	0.09	3.12	3.98	3.69	3.73
0.70	0.70	0.30	0.00	1.32	1.49	2.205	1.961	2.101	1.730	1.035	0.091	0.09	2.98	3.86	3.56	3.66
0.80	0.80	0.20	0.00	1.28	1.46	2.161	1.901	2.037	1.678	1.040	0.091	0.09	2.81	3.71	3.41	3.57
0.90	0.90	0.10	0.00	1.26	1.43	2.116	1.872	2.005	1.651	1.045	0.096	0.09	2.68	3.60	3.29	3.46
1.00	1.00	0.00	0.00	1.40	1.40	2.072	2.079	2.228	1.835	1.050	0.105	0.10	3.13	3.99	3.70	3.34
										average beta			3.02	3.90	3.60	3.67



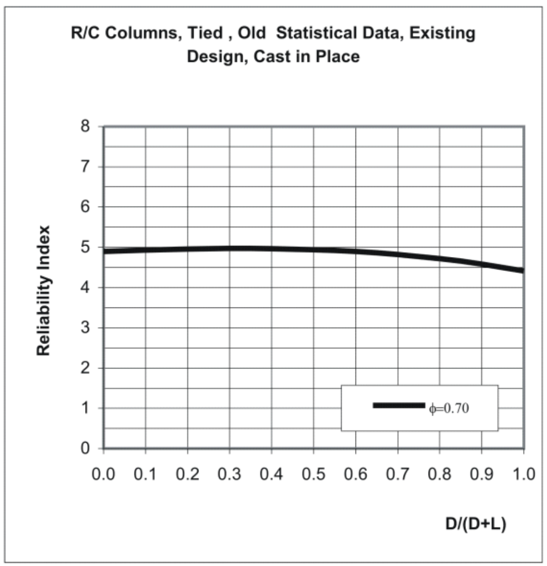


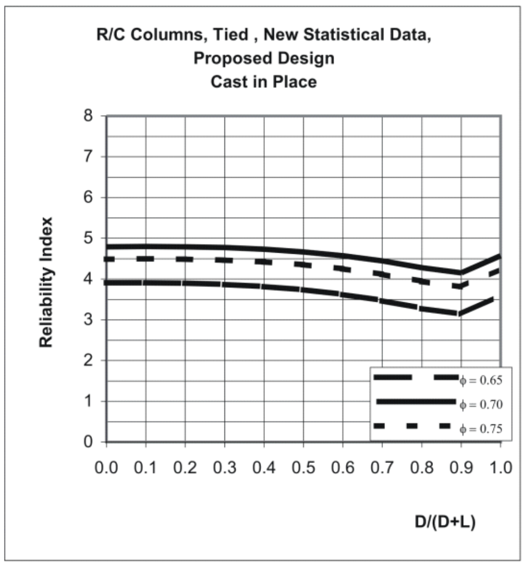
Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied **D + L,** **e/h = 3.27 tension control**

3% of reinforcement fc = 8.0 ksi

				new	old	old	mR new data/design						beta new			beta old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.732	2.453	2.629	2.165	1.000	0.180	0.18	3.91	4.79	4.49	4.90
0.10	0.10	0.90	0.00	1.56	1.67	2.684	2.392	2.563	2.111	1.005	0.162	0.16	3.91	4.80	4.49	4.93
0.20	0.20	0.80	0.00	1.52	1.64	2.636	2.331	2.497	2.056	1.010	0.146	0.14	3.90	4.79	4.49	4.96
0.30	0.30	0.70	0.00	1.48	1.61	2.588	2.269	2.431	2.002	1.015	0.130	0.13	3.87	4.77	4.46	4.97
0.40	0.40	0.60	0.00	1.44	1.58	2.539	2.208	2.366	1.948	1.020	0.116	0.11	3.82	4.73	4.42	4.97
0.50	0.50	0.50	0.00	1.40	1.55	2.491	2.147	2.300	1.894	1.025	0.104	0.10	3.74	4.67	4.35	4.94
0.60	0.60	0.40	0.00	1.36	1.52	2.443	2.085	2.234	1.840	1.030	0.096	0.09	3.62	4.57	4.25	4.89
0.70	0.70	0.30	0.00	1.32	1.49	2.395	2.024	2.169	1.786	1.035	0.091	0.09	3.47	4.45	4.12	4.82
0.80	0.80	0.20	0.00	1.28	1.46	2.346	1.963	2.103	1.732	1.040	0.091	0.09	3.28	4.28	3.94	4.71
0.90	0.90	0.10	0.00	1.26	1.43	2.298	1.932	2.070	1.705	1.045	0.096	0.09	3.13	4.15	3.81	4.58
1.00	1.00	0.00	0.00	1.40	1.40	2.250	2.147	2.300	1.894	1.050	0.105	0.10	3.62	4.57	4.25	4.41
										average beta			3.51	4.48	4.15	4.82

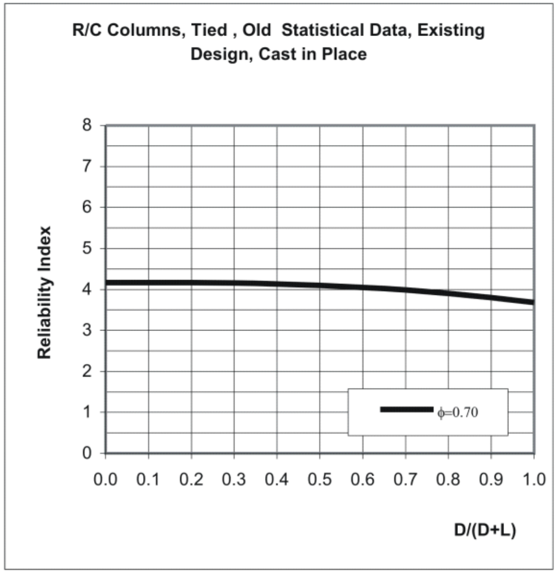


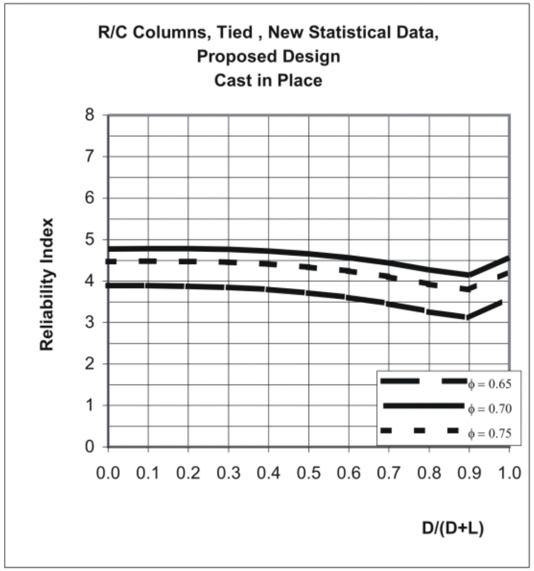


Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

R/C Columns, Cast-in-place, Tied **D + L,** **e/h = 3.27 tension control**

3% of reinforcement				fc = 12.0 ksi									beta			beta
				new	old	old	mR new data/design						new			old
D/D+L	D	L	S	Q	Q	mR	0.75	0.70	0.85	mQ	sQ	VQ	0.85	0.70	0.75	0.70
0.00	0.00	1.00	0.00	1.60	1.70	2.732	2.438	2.613	2.152	1.000	0.180	0.18	3.89	4.78	4.47	4.16
0.10	0.10	0.90	0.00	1.56	1.67	2.684	2.377	2.547	2.098	1.005	0.162	0.16	3.89	4.79	4.48	4.17
0.20	0.20	0.80	0.00	1.52	1.64	2.636	2.316	2.482	2.044	1.010	0.146	0.14	3.88	4.78	4.47	4.17
0.30	0.30	0.70	0.00	1.48	1.61	2.588	2.256	2.417	1.990	1.015	0.130	0.13	3.85	4.76	4.45	4.16
0.40	0.40	0.60	0.00	1.44	1.58	2.539	2.195	2.351	1.936	1.020	0.116	0.11	3.80	4.72	4.41	4.14
0.50	0.50	0.50	0.00	1.40	1.55	2.491	2.134	2.286	1.883	1.025	0.104	0.10	3.72	4.66	4.34	4.10
0.60	0.60	0.40	0.00	1.36	1.52	2.443	2.073	2.221	1.829	1.030	0.096	0.09	3.60	4.56	4.24	4.05
0.70	0.70	0.30	0.00	1.32	1.49	2.395	2.012	2.155	1.775	1.035	0.091	0.09	3.45	4.43	4.10	3.98
0.80	0.80	0.20	0.00	1.28	1.46	2.346	1.951	2.090	1.721	1.040	0.091	0.09	3.26	4.27	3.93	3.90
0.90	0.90	0.10	0.00	1.26	1.43	2.298	1.920	2.057	1.694	1.045	0.096	0.09	3.11	4.14	3.79	3.80
1.00	1.00	0.00	0.00	1.40	1.40	2.250	2.134	2.286	1.883	1.050	0.105	0.10	3.60	4.56	4.24	3.68
										average beta			3.49	4.46	4.13	4.00





Reliability Indices Calculated for R/C Column Made of Ordinary Concrete for D+L Load Combination

Reliability Indices

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place beta				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
1%	0	0.00					
	4	0.18	3.04	4.37	3.77	3.48	3.29
	8	0.36					3.29
	10	0.45	3.31	4.32	3.78	3.44	
	12	0.55					3.20
	14	0.64	3.61	4.11	3.75	3.58	
	16	0.73					
	18	0.82					
	20	0.91	3.75	4.25		3.96	
	24	1.09			4.16		
	40	1.82					
	72	3.27	4.04	4.57	4.53	4.53	4.56

Highlighted values are for balance failure

Reliability Indices

Design case			Tied				
% of reinf.	e (in)	e/h	Cast-in-place beta				
			old	f_c'= 3.0 ksi	f_c'= 5.0 ksi	f_c'= 8.0 ksi	f_c'=12.0 ksi
3%	0	0.00					
	4	0.18	3.58	4.75	4.18	3.82	3.51
	8	0.36					
	10	0.45					
	12	0.55	3.71	4.61			3.71
	14	0.64	3.67		4.22	3.90	
	16	0.73					
	18	0.82	3.88			4.06	3.93
	20	0.91	3.90	4.52	4.19		
	24	1.09					
	40	1.82					
	72	3.27	4.00	4.45	4.49	4.48	4.46

Conclusions

- **Eccentricly loaded columns require a special approach**

- **The uncertainty is not only in load but also in the eccentricity**

- **There is a need to modify the resistance factors for moment and axial load**