LATERAL TORSIONAL BUCKLING OF BEAMS, RESISTANCES AND CRITICAL MOMENTS

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Beam in bending:

- Ideal beam – without imperfections:

  characteristic is critical moment M_{cr}
- Real beam – with imperfections

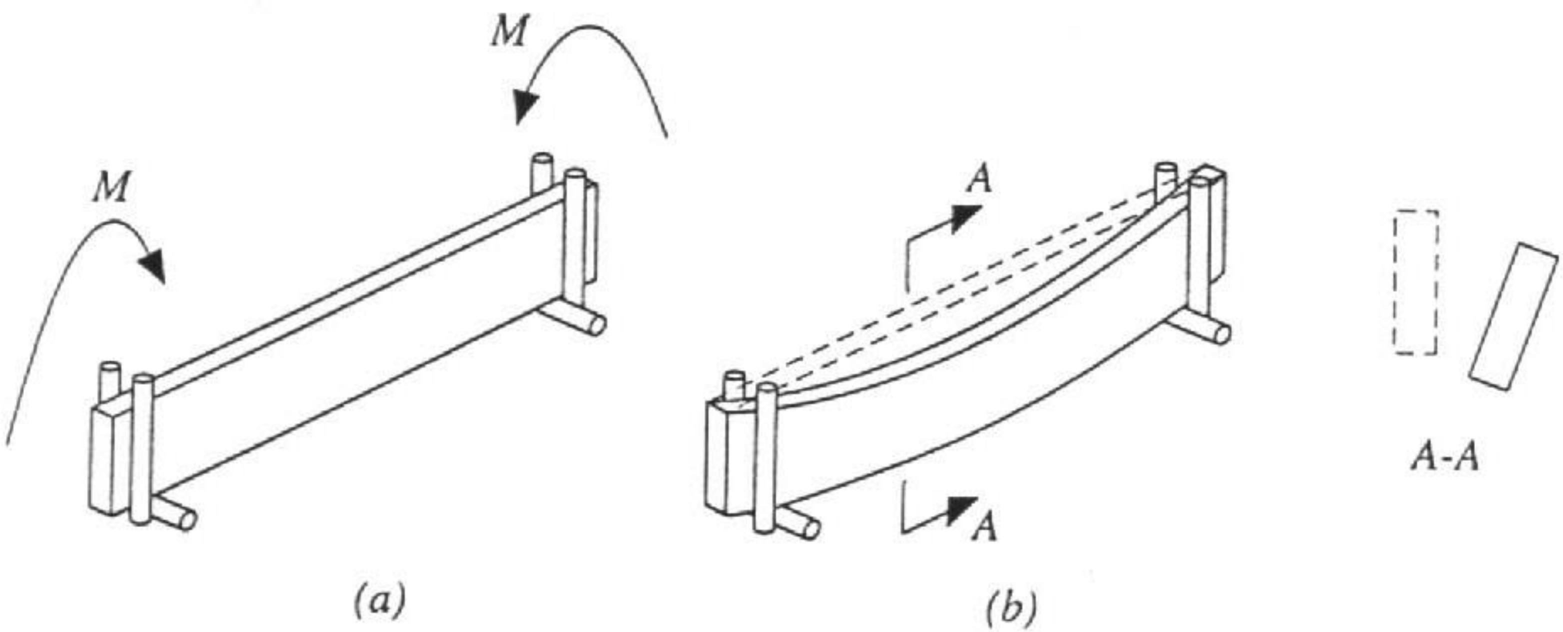
  characteristic is resistance of beam $M_{b,Rd}$

A. Ideal beam – without imperfections: Critical moment M_{cr}

Elastic critical moment M_{cr}

„fork“ boundary conditions, uniform bending moment M,

uniform nonwarping cross-section: warping stiffness $EI_w = 0\ kNm^4$



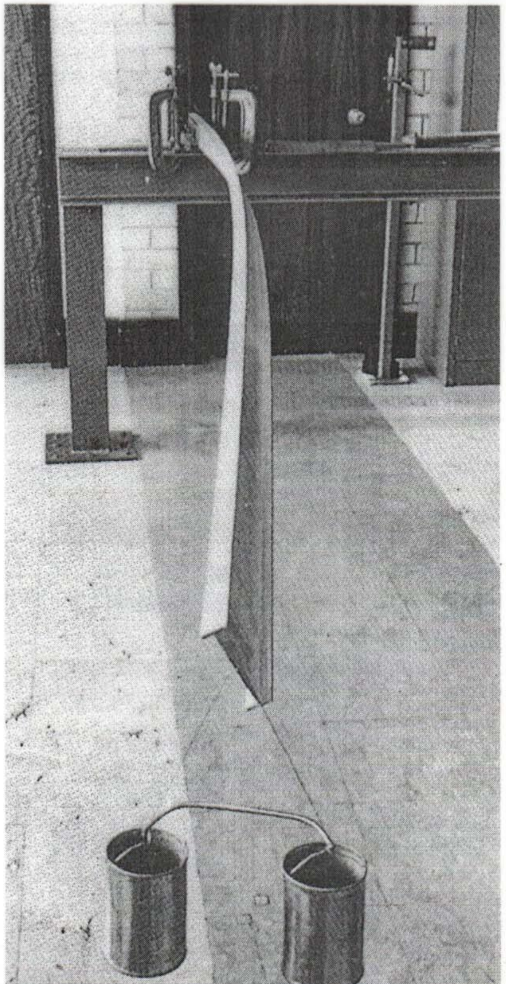
$$M_{cr} = \frac{\pi \sqrt{EI_z GI_t}}{L}$$

EI_z bending stiffness

GI_t torsional stiffness

L span

Lateral Torsional Buckling



Positive influence of warping stiffness on M_{cr}

- Rectangular cross-section

  $I_w \approx 0\ m^6$

  $I_w = b^3h^3/36$

- Double symmetric cross-section

$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L} \qquad M_{cr} = \frac{\pi\sqrt{EI_z(GI_t + \pi^2 EI_w / L^2)}}{L}$$

Positive influence of vertical deflection on M_{cr}

$$M_{cr} = \xi \frac{\pi\sqrt{EI_z(GI_t + \pi^2 EI_w / L^2)}}{L}, \qquad \xi \geq \mathbf{1}$$

$$\xi = \frac{1}{\sqrt{\left(1 - \frac{I_z}{I_y}\right)\left[1 - \frac{I_z}{I_y}\frac{GI_t}{EI_z}\left(1 + \frac{\pi^2 EI_w}{L^2 GI_t}\right)\right]}}$$

Broude 1953 in Russia,
Trahair cca 20 years later in Australia

Examples:

1) IPE 360, $L = 4.5\,\mathrm{m}$

$I_z/I_y = 0.064$

$K = 1.029$

$\sqrt{GI_t/EI_z} = 0.118$

$\xi = 1.035$

2) HD 360x162,

$L = 4.5\,\mathrm{m}$:

$I_z/I_y = 0.360$

$K = 1.526$

$\sqrt{GI_t/EI_z} = 0.078$

$\xi = 1.255$

$\frac{I_z}{I_y}$	$K = \frac{\pi}{L}\sqrt{\frac{EI_\omega}{GI_t}}$	$\sqrt{GI_t/EI_z}$			
		0	0.125	0.25	0.5
0	$K \geq 0$	1	1	1	1
0.1	0	1.054	1.055	1.057	1.068
	1		1.056	1.061	1.081
	3		1.062	1.089	1.217
0.3	0	1.195	1.198	1.207	1.243
	1		1.201	1.218	1.296
	3		1.224	1.326	2.390
0.6	0	1.581	1.589	1.612	1.715
	1		1.596	1.644	1.890
	3		1.661	2.000	-

M_{cr} for any boundary conditions and any type of loading

- 1958 Mrázik, obtained coefficients for 11 cases:

$$M_{cr} = \beta_0 \frac{\pi^2 EI_z}{(k_z L)^2} \left\{ \left(z_g + \beta_1 z_j\right) + \sqrt{\left(z_g + \beta_1 z_j\right)^2 + \beta_2 \left[\frac{I_w}{I_z} \left(\frac{k_z}{k_w} \right)^2 + \frac{GI_t}{EI_z} \left(\frac{k_z L}{\pi} \right)^2 \right]} \right\}$$

- 1960 Clark, Hill, colected coefficients for 21 cases:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(k_z L)^2} \left\{ \left(C_2 z_g + C_3 z_j\right) + \sqrt{\left(C_2 z_g + C_3 z_j\right)^2 + \frac{I_w}{I_z} \left[1 + \frac{GI_t}{EI_w} \left(\frac{k_z L}{\pi} \right)^2 \right]} \right\}$$

M_{cr} for any boundary conditions and any type of loading

- 1974 Djalaly, obtained coefficients for 37 cases:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ (C_2 z_g + C_3 z_j) + \sqrt{(C_2 z_g + C_3 z_j)^2 + \frac{I_w}{I_z} \left[1 + \frac{G I_t}{E I_w} \left(\frac{k_z L}{\pi} \right)^2 \right]} \right\}$$

- ENV 1993-1-1:1992 a ENV 1999-1-1:1998

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left[\pm (C_2 z_g \pm C_3 z_j) + \sqrt{(C_2 z_g \pm C_3 z_j)^2 + \frac{I_w}{I_z} \left(\frac{k_z}{k_w} \right)^2 + \frac{G I_t}{E I_z} \left(\frac{k_z L}{\pi} \right)^2} \right]$$

Relations between cofficients of different formulae

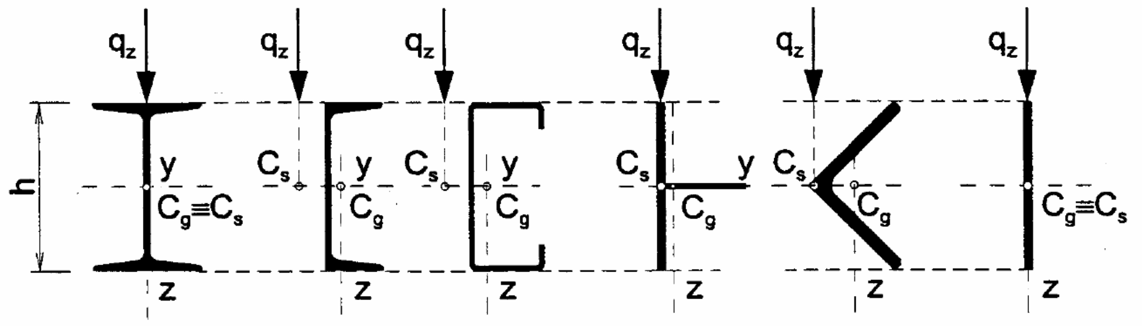
$$\beta_0 = C_1 C_2 \ , \quad \beta_1 = C_3 / C_2 \ , \quad \beta_2 = 1 / C_2^2 \ ,$$

$$C_1 = \beta_0 \sqrt{\beta_2} \ , \ C_2 = 1 / \sqrt{\beta_2} \ , \ C_3 = \beta_1 / \sqrt{\beta_2}$$

Formulae from ČSN and STN 73 1401

- cl. H.2: loading prependicular to axis of symmetry

$$\lambda = \gamma \frac{2L_z}{h}\sqrt{\frac{I_y}{I_z}}$$

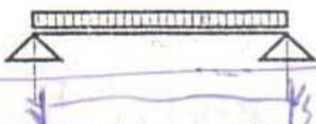
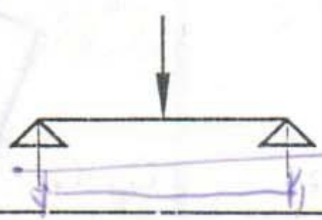


$$M_{cr} = \kappa_1 \frac{\pi^2 E I_z}{(k_z L)^2}\left\{ z_g + \sqrt{z_g^2 + \kappa_2\left[\frac{I_w}{I_z}\left(\frac{k_z}{k_w}\right)^2 + \frac{G I_t}{E I_z}\left(\frac{k_z L}{\pi}\right)^2\right]}\right\}$$

- It is simplification of Mráziks formula

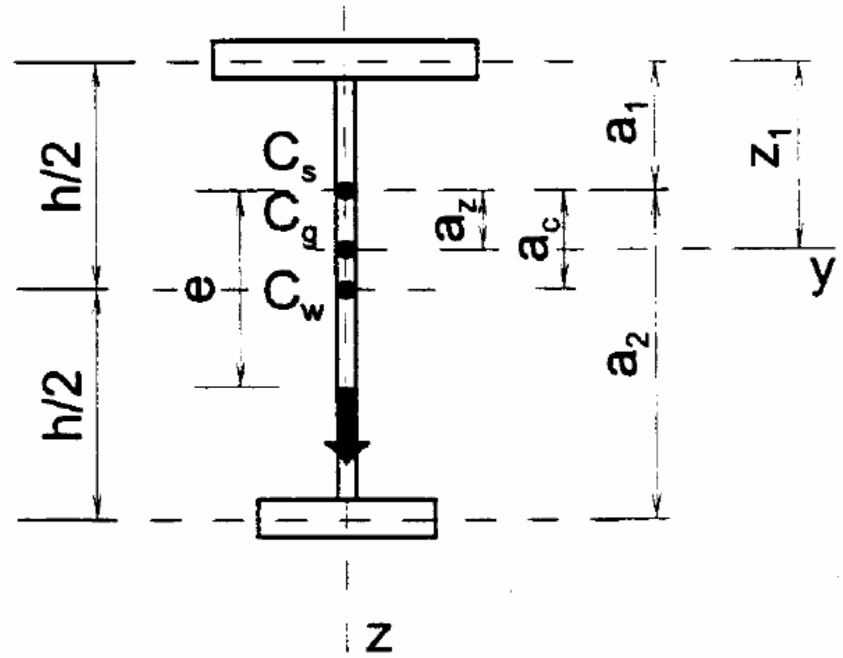
$$\kappa_1 = \beta_0 ,\ \ \kappa_2 = \beta_1$$

Tabuľka H.1 - Súčinitele κ_1, κ_2

Schéma zaťaženia	Obojstrané Uloženie			$k_z = L_z / L$	$k_\omega = L_\omega / L$	κ_1	κ_2
	ohyb v rovine		krútenie				
	zx	*yx*					
	K	K	K	1,0	1,0	1,00	1,00
	K	K	V	1,0	0,5	1,33	0,75
	K	V	V	0,5	0,5	1,00	1,00
	K	K	K	1,0	1,0	0,53	4,68
	K	K	V	1,0	0,5	0,52	6,33
	K	V	V	0,5	0,5	0,29	11,29
	V	V	V	0,5	0,5	1,61	1,36
	K	K	K	1,0	1,0	0,76	3,26
	K	K	V	1,0	0,5	0,87	2,83
	K	V	V	0,5	0,5	0,50	4,99
	V	V	V	0,5	0,5	1,23	1,00

Formulae from ČSN and STN 73 1401

cl. H.6: loading in the direction of symmetry axis



e, a_c are used with sign + or -

Formulae from ČSN and STN 73 1401

cl. H.6: loading in direction of symmetry axis

$$M_{cr} = \frac{1}{\kappa_M^2}\frac{\pi^2 EI_z}{(k_z L)^2}\left[\kappa\left(z_g + a_c\right) + \sqrt{\kappa^2\left(z_g + a_c\right)^2 + \frac{I_w}{I_z}\left(\frac{k_z}{k_w}\right)^2 + \frac{GI_t}{EI_z}\left(\frac{k_z L}{\pi}\right)^2}\right]$$

It is again simplification of Mrazics formula

$$\beta_0 \approx \frac{\kappa}{\kappa_M^2}, \qquad \beta_1 z_j \approx a_c, \qquad \beta_2 \approx \frac{1}{\kappa^2}$$

Nosník

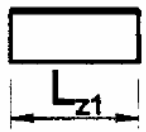
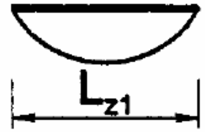
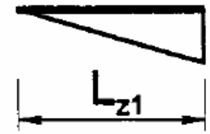
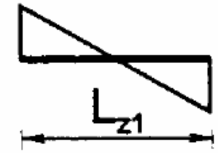
Tvar momentovej plochy					
κ_M	1,0	0,94	0,86	0,75	0,63

Tabelle 8 — Druckkraftbeiwerte k_c

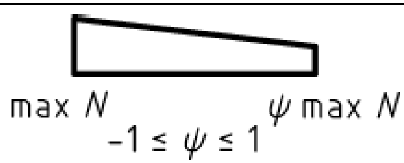
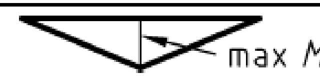
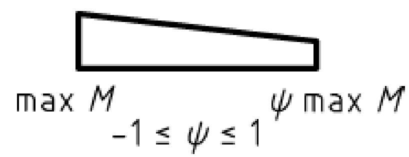
	Normalkraftverlauf	k_c
1		1,00
2		0,94
3		0,86
4		$\frac{1}{1{,}33 - 0{,}33\,\psi}$

Tabelle 10 — Momentenbeiwerte ζ

Zeile	Momentenverlauf	ζ
1		1,00
2		1,12
3		1,35
4		$1{,}77 - 0{,}77\,\psi$

$$k_c = \frac{1}{\sqrt{\xi}}$$

Relations between cofficients of different formulae

$$C_1 \approx \frac{1}{\kappa_M^2}\,, \qquad C_2 \approx \kappa, \qquad C_3 z_j \approx \kappa a_c\,,$$

$$\kappa_M \approx \frac{1}{\sqrt{C_1}}, \qquad \kappa \approx C_2\,, \qquad \kappa a_c \approx C_3 z_j\,.$$

M_{cr} according to DIN 18 800

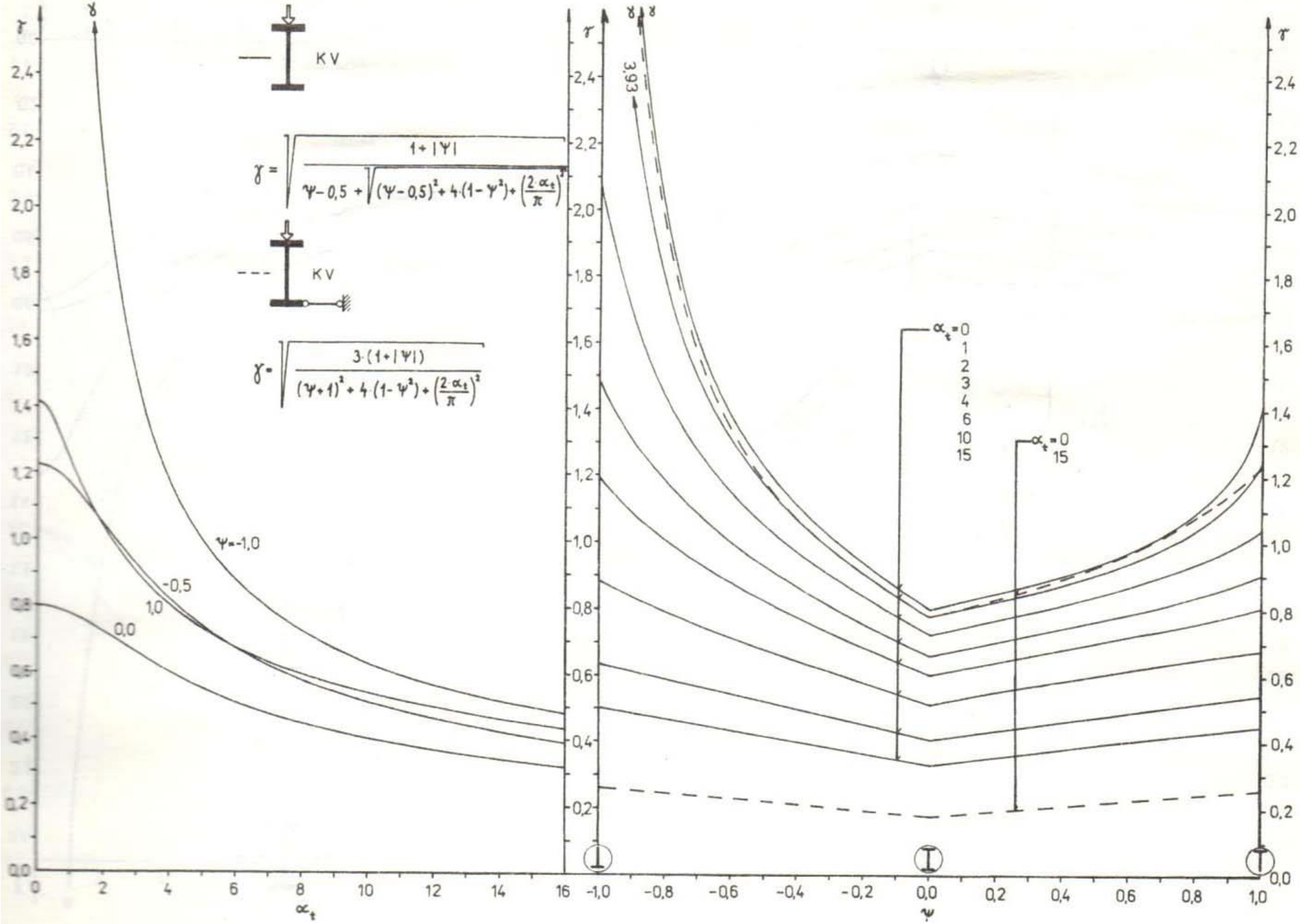
$$M_{\mathrm{Ki,y}} = \zeta \cdot N_{\mathrm{Ki,z}} \left(\sqrt{c^2 + 0{,}25 z_{\mathrm{p}}^2} + 0{,}5 z_{\mathrm{p}} \right)$$

$$\lambda = \gamma \frac{\kappa_M \, L_{z1}}{i_{z1}}$$

$$\lambda = \gamma \frac{2 L_z}{h} \sqrt{\frac{I_y}{I_z}}$$

$$\lambda = \pi \sqrt{E \, W_{el,y} \; / \; M_{cr}}$$

$$M_{cr} = \pi^2 \, \frac{E W_{y,el}}{\lambda^2}$$



KV
$\gamma = \sqrt{\dfrac{1+|\psi|}{\psi-0{,}5+\sqrt{(\psi-0{,}5)^2+4\cdot(1-\psi^2)+\left(\frac{2\cdot\alpha_t}{\pi}\right)^2}}}$
KV
$\gamma = \sqrt{\dfrac{3\cdot(1+|\psi|)}{(\psi+1)^2+4\cdot(1-\psi^2)+\left(\frac{2\cdot\alpha_t}{\pi}\right)^2}}$
$\psi = -1{,}0$
-0,5
1,0
0,0
α_t
ψ
γ
3,93
$\alpha_t = 0$
1
2
3
4
6
10
15
$\alpha_t = 0$
15

Not correct things in European prestandards ENV 1993, ENV 1999

- Some coefficients C_i are incorrect (e.g. instead of C_1=0.411 it is used C_1=1.73,etc.)
- No information is given about used torsion boundary conditions (k_w=1) in the cases of end moments
- No information is given about used lateral bending boundary conditions (fixed is left beam end)

Not correct things in European prestandards ENV 1993, ENV 1999

- No definition of M_{cr} location (location is in middle of span)
- Influence of torsion stiffness and warping stiffness are not taken into account in calculation of C_1 coefficients
- For beams with nonsymmetric cross-sections under nonsymmetric bending end moments are C_i coefficients incorrect

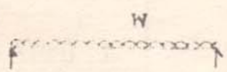
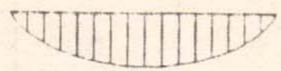
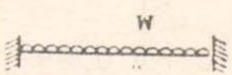
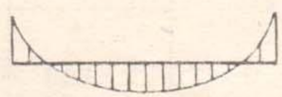
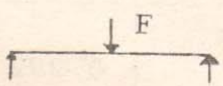
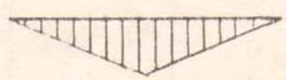
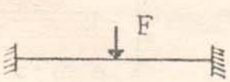
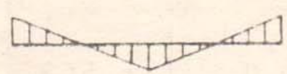
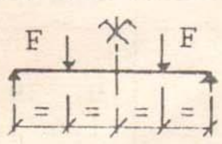
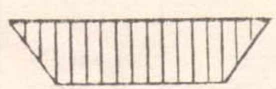
ENV 1993-1-1, ENV 1999-1-1

Tabulka F.1.1 Hodnoty součinitelů C_1, C_2 a C_3 v závislosti na velikosti součinitele k. Zatížení koncovými momenty

Zatížení a podmínky podepření	Průběh ohybového momentu	Hodnoty k	Hodnoty součinitelů		
			C_1	C_2	C_3
	$\psi = +1$	1,0 0,7 0,5	1,000 1,000 1,000	–	1,000 1,113 1,144
	$\psi = +3/4$	1,0 0,7 0,5	1,141 1,270 1,305	–	0,998 1,565 2,283
	$\psi = +1/2$	1,0 0,7 0,5	1,323 1,473 1,514	–	0,992 1,556 2,271
M ψ M	$\psi = -1$	1,0 0,7 0,5	2,752 3,063 3,149	–	0,000 0,000 0,000

ENV 1993-1-1, ENV 1999-1-1

Tabulka F.1.2 Hodnoty součinitelů C_1, C_2 a C_3 v závislosti na velikosti součinitele k. Případy příčného zatížení

Zatížení a podmínky podepření	Průběh ohybového momentu	Hodnoty k	Hodnoty součinitelů		
			C_1	C_2	C_3
		1,0 0,5	1,132 0,972	0,459 0,304	0,525 0,980
		1,0 0,5	1,285 0,712	1,562 0,652	0,753 1,070
		1,0 0,5	1,365 1,070	0,553 0,432	1,730 3,050
		1,0 0,5	1,565 0,938	1,267 0,715	2,640 4,800
		1,0 0,5	1,046 1,010	0,430 0,410	1,120 1,890

Tab.13E Beam with concentrated central load F and boundary conditions $k_y = 1$, $k_z = 1$, $k_\omega = 1$

E - **E**xact values of $\overline{M}_{cr}$ calculated by program IBDSQ

R - **R**atio of approximate values $\overline{M}_{cr}$ calculated by 3-coefficients ENV´s formula using ENV's $C_1 = 1.365$, $C_2 = 0.553$, $C_3 = 1.730$ and exact values $\overline{M}_{cr}$.

$K_{\omega t}$	$\bar{z}_g$	E / R	$\bar{z}_j$ = 4	2	1	0.5	0	-0.5	-1	-2	-4
0	-4	E	0.645	0.395	0.309	0.274	0.243	0.217	0.194	0.157	0.108
		R	20.138	9.844	2.779	1.650	1.211	0.998	0.879	0.761	0.688
	-2	E	1.588	0.934	0.662	0.547	0.450	0.370	0.305	0.215	0.124
		R	10.071	7.175	3.720	1.964	1.168	0.883	0.765	0.688	0.681
	-1	E	2.410	1.571	1.141	0.930	0.730	0.554	0.414	0.248	0.129
		R	7.257	5.195	3.256	1.996	1.103	0.782	0.690	0.676	0.706
	0	E	3.384	2.464	1.947	1.662	1.348	0.971	0.515	0.258	0.129
		R	5.612	3.912	2.614	1.796	1.013	0.643	0.712	0.750	0.761
	1	E	4.429	3.471	2.911	2.582	2.143	1.020	0.515	0.258	0.129
		R	4.627	3.204	2.239	1.667	1.080	0.984	0.975	0.886	0.827
	2	E	5.497	4.492	3.859	3.428	2.642	1.020	0.515	0.258	0.129
		R	4.002	2.808	2.067	1.665	1.342	1.699	1.472	1.079	0.904
		E	7.606	6.421	5.498	4.692	3.055	1.020	0.515	0.258	0.129

$0.9 \le R \le 1.1$ | $0.8 \le R < 0.9$ & $1.1 < R \le 1.2$ | $R < 0.8$ & $R > 1.2$

$$\overline{M}_{cr} = \frac{M_{cr}}{M_{cr,0}}, \quad M_{cr,0} = \frac{\pi}{k_z L}\sqrt{EI_z GI_t}, \quad K_{\omega t} = \frac{\pi}{k_\omega L}\sqrt{\frac{EI_\omega}{GI_t}}, \quad \bar{z}_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}}, \quad \bar{z}_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}}$$

Tab.13E Beam with concentrated central load F and boundary conditions $k_y = 1$, $k_z = 1$, $k_\omega = 1$

E - **E**xact values of $\overline{M}_{cr}$ calculated by program IBDSQ

R - **R**atio of approximate values $\overline{M}_{cr}$ calculated by 3-coefficients ENV´s formula using ENV's $C_1 = 1.365$, $C_2 = 0.553$, $C_3 = 1.730$ and exact values $\overline{M}_{cr}$.

$K_{\omega t}$	$\bar{z}_g$	E/R	$\bar{z}_j$ = 4	2	1	0.5	0	-0.5	-1	-2	-4
0	-4	E	0.645	0.395	0.309	0.274	0.243	0.217	0.194	0.157	0.108
		R	20.138	9.844	2.779	1.650	1.211	0.998	*0.879*	0.761	0.688
	-2	E	1.588	0.934	0.662	0.547	0.450	0.370	0.305	0.215	0.124
		R	10.071	7.175	3.720	1.964	*1.168*	*0.883*	0.765	0.688	0.681
	-1	E	2.410	1.571	1.141	0.930	0.730	0.554	0.414	0.248	0.129
		R	7.257	5.195	3.256	1.996	*1.103*	0.782	0.690	0.676	0.706
	0	E	3.384	2.464	1.947	1.662	1.348	0.971	0.515	0.258	0.129
		R	5.612	3.912	2.614	1.796	1.013	0.643	0.712	0.750	0.761
	1	E	4.429	3.471	2.911	2.582	2.143	1.020	0.515	0.258	0.129
		R	4.627	3.204	2.239	1.667	1.080	0.984	0.975	*0.886*	*0.827*
	2	E	5.497	4.492	3.859	3.428	2.642	1.020	0.515	0.258	0.129
		R	4.002	2.808	2.067	1.665	1.342	1.699	1.472	1.079	0.904
	4	E	7.606	6.421	5.498	4.692	3.055	1.020	0.515	0.258	0.129
		R	3.287	2.430	1.989	1.837	2.073	4.048	4.224	1.861	*1.112*
0.25	-4	E	0.730	0.442	0.352	0.316	0.285	0.259	0.236	0.200	0.150
		R	17.807	8.850	2.546	1.509	1.093	*0.886*	0.765	0.634	0.526
	-2	E	1.802	1.014	0.723	0.610	0.516	0.440	0.380	0.292	0.195
		R	8.874	6.627	3.452	1.831	1.074	0.785	0.652	0.537	0.462
0.5	-4	E	0.866	0.523	0.421	0.381	0.347	0.318	0.293	0.253	0.197
		R	15.043	7.632	2.386	1.445	1.047	*0.844*	0.723	0.589	0.474
	-2	E	2.057	1.151	0.833	0.712	0.614	0.535	0.471	0.376	0.264
		R	7.785	5.881	3.121	1.730	1.037	0.752	0.615	0.489	0.401
	-1	E	3.010	1.846	1.331	1.112	0.926	0.776	0.657	0.491	0.316
		R	5.820	4.453	2.872	1.808	1.023	0.682	0.538	0.425	0.359
	0	E	4.091	2.803	2.149	1.828	1.520	1.241	1.006	0.684	0.390
		R	4.648	3.456	2.407	1.702	1.004	0.604	0.447	0.352	0.314
	1	E	5.229	3.890	3.185	2.820	2.444	2.055	1.622	0.871	0.445
		R	3.923	2.870	2.068	1.561	1.005	0.564	0.376	0.325	0.299
	2	E	6.382	5.009	4.272	3.885	3.484	2.663	1.622	0.871	0.445
		R	3.450	2.526	1.880	1.489	1.049	0.710	0.552	0.395	0.327
	4	E	8.655	7.167	6.288	5.766	5.123	2.663	1.622	0.871	0.445
		R	2.891	2.181	1.745	1.504	1.250	1.588	1.430	0.670	0.402

$K_{\omega t}$	$\bar{z}_g$	E/R	$\bar{z}_j$ = 4	2	1	0.5	0	-0.5	-1	-2	-4
1	-4	E	1.101	0.740	0.621	0.572	0.530	0.492	0.459	0.403	0.323
		R	20.523	13.022	5.647	2.301	1.021	0.639	0.477	0.335	0.237
	-2	E	2.028	1.308	1.043	0.935	0.841	0.761	0.691	0.580	0.431
		R	12.049	8.757	4.935	2.553	1.009	0.544	0.383	0.262	0.189
	-1	E	2.695	1.792	1.420	1.261	1.121	0.999	0.894	0.728	0.515
		R	9.412	6.896	4.229	2.451	0.999	0.489	0.330	0.222	0.164
	0	E	3.458	2.421	1.948	1.732	1.534	1.356	1.199	0.947	0.630
		R	7.600	5.481	3.533	2.233	0.987	0.437	0.278	0.182	0.138
	1	E	4.286	3.162	2.613	2.349	2.096	1.858	1.640	1.271	0.795
		R	6.347	4.486	2.973	1.996	0.976	0.399	0.233	0.146	0.114
	2	E	5.155	3.973	3.376	3.079	2.785	2.498	2.222	1.725	1.032
		R	5.458	3.800	2.566	1.799	0.969	0.384	0.200	0.116	0.091
	4	E	6.945	5.695	5.045	4.708	4.367	3.855	2.999	1.968	1.105
		R	4.315	2.972	2.075	1.548	0.971	0.451	0.218	0.121	0.092
2	-4	E	1.980	1.502	1.319	1.240	1.167	1.102	1.041	0.936	0.774
		R	11.487	6.642	3.192	1.767	1.007	0.674	0.510	0.357	0.246
	-2	E	2.942	2.201	1.900	1.768	1.646	1.536	1.435	1.260	0.998
		R	8.354	5.332	3.002	1.816	0.997	0.615	0.442	0.297	0.203
	-1	E	3.561	2.689	2.317	2.150	1.995	1.852	1.722	1.496	1.156
		R	7.159	4.696	2.807	1.787	0.990	0.585	0.407	0.266	0.181
	0	E	4.258	3.271	2.830	2.626	2.435	2.256	2.091	1.799	1.359
		R	6.203	4.133	2.590	1.728	0.983	0.559	0.374	0.235	0.160
4	-1	E	5.565	4.745	4.375	4.200	4.032	3.871	3.717	3.431	2.935
		R	4.677	2.867	1.839	1.367	0.986	0.720	0.550	0.371	0.238
	0	E	6.180	5.285	4.876	4.682	4.494	4.313	4.140	3.814	3.247
		R	4.357	2.733	1.798	1.355	0.982	0.711	0.536	0.353	0.223
	1	E	6.835	5.875	5.430	5.215	5.010	4.807	4.614	4.248	3.604
		R	4.069	2.602	1.753	1.339	0.978	0.705	0.525	0.337	0.208
	2	E	7.535	6.515	6.035	5.800	5.575	5.355	5.140	4.733	4.008
		R	3.811	2.479	1.705	1.321	0.975	0.702	0.515	0.322	0.193
	4	E	9.035	7.915	7.370	7.105	6.845	6.590	6.340	5.855	4.969
		R	3.379	2.260	1.612	1.282	0.969	0.702	0.506	0.298	0.168

$0.9 \le R \le 1.1$ | *$0.8 \le R < 0.9$ & $1.1 < R \le 1.2$* | $R < 0.8$ & $R > 1.2$

$$\overline{M}_{cr} = \frac{M_{cr}}{M_{cr,0}}, \quad M_{cr,0} = \frac{\pi}{k_z L}\sqrt{EI_z GI_t}, \quad K_{\omega t} = \frac{\pi}{k_\omega L}\sqrt{\frac{EI_\omega}{GI_t}}, \quad \bar{z}_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}}, \quad \bar{z}_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}}$$

$0.9 \le R \le 1.1$ | *$0.8 \le R < 0.9$ & $1.1 < R \le 1.2$* | $R < 0.8$ & $R > 1.2$

$$\overline{M}_{cr} = \frac{M_{cr}}{M_{cr,0}}, \quad M_{cr,0} = \frac{\pi}{k_z L}\sqrt{EI_z GI_t}, \quad K_{\omega t} = \frac{\pi}{k_\omega L}\sqrt{\frac{EI_\omega}{GI_t}}, \quad \bar{z}_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}}, \quad \bar{z}_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}}$$

prEN 1993-3: October 2001

(6) The sign convention for determining z_j, see Figure B.3, is:

- z is negative for the compression flange
- z_j is positive when the flange with the larger value of I_z is in compression at the point of largest moment.

Draft note: According to Prof. Ivan Balaz this sentence is not correct and should be deleted – e.g. see fixed beams. z_j is clearly defined by $z_j = z_s - 0{,}5 \int_A (y^2 + z^2) \frac{z}{I_y} dA$

(7) The sign convention for determining z_g is:

- for gravity loads z_g is negative for loads applied above the shear centre
- in the general case z_g is negative for loads acting towards the shear centre from their point of application.

Draft note: Prof. Ivan Balaz suggests that sign convention should be changed. Reference is also made to DIN 18800 T2 and the format of Eq.(19).

prENV 1993-3: October 2001

Draft note: Modifications proposed by Prof. Ivan Balaz

(8) For an I-section with unequal flanges:

$$I_w = \left(1 - \psi_f^2\right) I_z \left(\frac{h_s}{2}\right)^2 \qquad \text{(B.7)}$$

where $\psi_f = \dfrac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$

I_{fc} is the second moment of area of the compression flange about the minor axis of the cross-section

I_{ft} is the second moment of area of the tension flange about the minor axis of the cross-section

h_s is the distance between the shear centres of the flanges.

EN 1993-1-1 – no formulae for calculation of M_{cr}
EN 1999-1-1 and NA to STN EN 1993-1-1 contain our proposals how to calculate M_{cr}

EN 1999-1-1 and NA to STN EN 1993-1-1

$$M_{\mathrm{cr}} = \frac{\pi^2 E I_{\mathrm{z}}}{L^2} \sqrt{\frac{L^2 G I_{\mathrm{t}}}{\pi^2 E I_{\mathrm{z}}} + \frac{I_{\mathrm{w}}}{I_{\mathrm{z}}}} = \frac{\pi \sqrt{E I_{\mathrm{z}} G I_{\mathrm{t}}}}{L} \sqrt{1 + \frac{\pi^2 E I_{\mathrm{w}}}{L^2 G I_{\mathrm{t}}}}$$

$$M_{\mathrm{cr}} = \mu_{\mathrm{cr}} \frac{\pi \sqrt{E I_{\mathrm{z}} G I_{\mathrm{t}}}}{L}$$

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{\mathrm{wt}}^2 + (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})^2} - (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})\right]$$

$$\kappa_{\mathrm{wt}} = \frac{\pi}{k_{\mathrm{w}} L}\sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{t}}}} \qquad \zeta_{\mathrm{g}} = \frac{\pi z_{\mathrm{g}}}{k_{\mathrm{z}} L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}} \qquad \zeta_{\mathrm{j}} = \frac{\pi z_{\mathrm{j}}}{k_{\mathrm{z}} L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}}$$

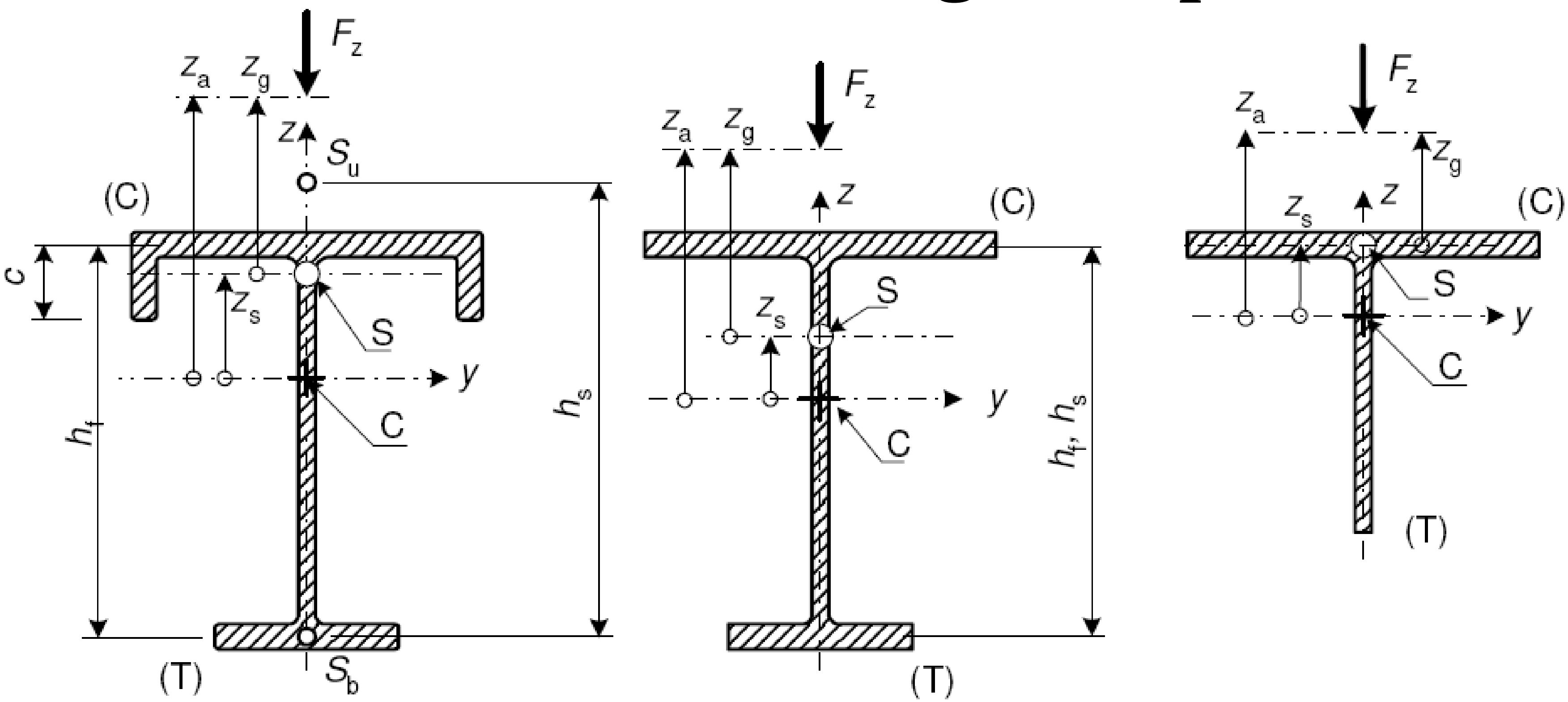
$$I_{\mathrm{w}} = (1-\psi_{\mathrm{f}}^2) I_{\mathrm{z}} (h_{\mathrm{s}}/2)^2 \qquad \psi_{\mathrm{f}} = \frac{I_{\mathrm{fc}} - I_{\mathrm{ft}}}{I_{\mathrm{fc}} + I_{\mathrm{ft}}} \qquad z_{\mathrm{g}} = z_{\mathrm{a}} - z_{\mathrm{s}}$$

$$z_{\mathrm{j}} = z_{\mathrm{s}} - \frac{0{,}5}{I_{\mathrm{y}}}\int_A (y^2 + z^2)\, z\, dA \qquad z_{\mathrm{j}} = 0{,}45\psi_{\mathrm{f}} h_{\mathrm{s}}\left(1+\frac{c}{2h_{\mathrm{f}}}\right)$$

For symmetric cross-section to axis y-y is $z_j = 0$

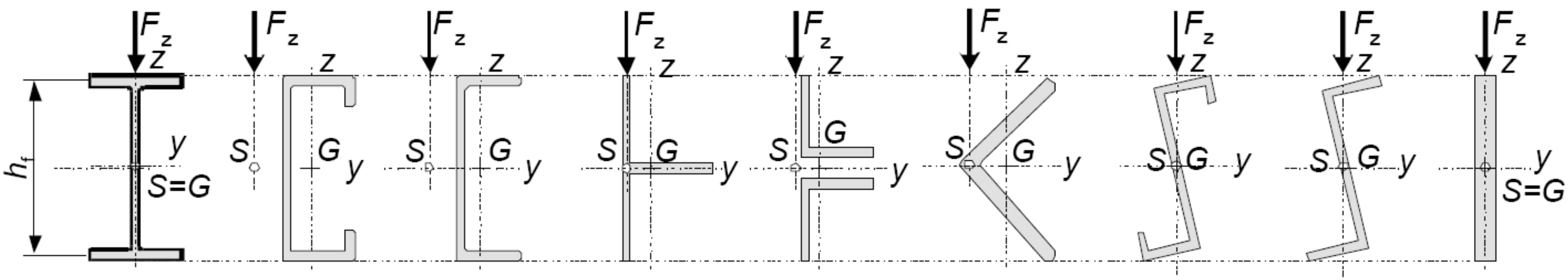
C1, C2, C3 are cofficients depending on type of loading and boundary conditions

In STN 73 1401 positive axis z goes down, in STN EN goes up !!!



a) Loading in the direction of symmetry axis

Loading perpendicular to axis of symmetry:
b) single symmetric,
c) double- or point-symmetric



$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}} \left[\sqrt{1 + \kappa_{\mathrm{wt}}^2 + (C_2 \zeta_{\mathrm{g}} - C_3 \zeta_{\mathrm{j}})^2} - (C_2 \zeta_{\mathrm{g}} - C_3 \zeta_{\mathrm{j}}) \right]$$

(1) For beams with uniform cross-sections symmetrical about major axis, centrally symmetric and doubly symmetric cross-sections loaded perpendicular to the major axis in the plane going through the shear centre,

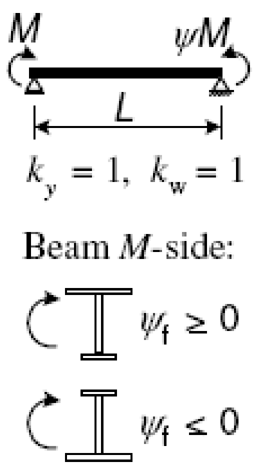
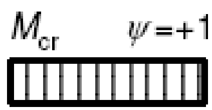
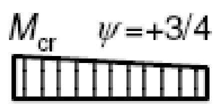
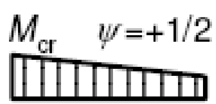
$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}} \left[\sqrt{1 + \kappa_{wt}^2 + (C_2 \zeta_g)^2} - C_2 \zeta_g \right]$$

(2) For end-moment loading $C_2 = 0$ and for transverse loads applied at the shear centre $z_{\mathrm{g}} = 0$. For these cases:

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}} \sqrt{1 + \kappa_{wt}^2}$$

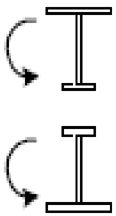
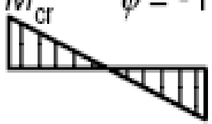
$$\kappa_{\mathrm{wt}} = 0: \; \mu_{\mathrm{cr}} = C_1 / k_{\mathrm{z}}$$

Table I.1 - Values of factors C_1 and C_3 corresponding to various end moment ratios ψ, values of buckling length factor k_z and cross-section parameters ψ_f and κ_{wt}.
End moment loading of the simply supported beam with buckling length factors $k_y = 1$ for major axis bending and $k_w = 1$ for torsion

Loading and support conditions. Cross-section monosymmetry factor ψ_f	Bending moment diagram. End moment ratio ψ. M-side, ψM-side	k_z [2)]	Values of factors: C_1 [1)] $C_{1,0}$	C_1 [1)] $C_{1,1}$	C_3: $\psi_f = -1$	C_3: $-0,9 \le \psi_f \le 0$	C_3: $0 \le \psi_f \le 0,9$	C_3: $\psi_f = 1$
		1,0	1,000	1,000	1,000			
		0,7L	1,016	1,100	1,025		1,000	
		0,7R	1,016	1,100	1,025		1,000	
		0,5	1,000	1,127	1,019			
		1,0	1,139	1,141	1,000			
		0,7L	1,210	1,313	1,050		1,000	
		0,7R	1,109	1,201	1,000			
		0,5	1,139	1,285	1,017			
		1,0	1,312	1,320	1,150	1,000		
		0,7L	1,480	1,616	1,160		1,000	
		0,7R	1,213	1,317	1,000			
		0,5	1,310	1,482	1,150	1,000		

1) $C_1 = C_{1,0} + (C_{1,1} - C_{1,0})\kappa_{wt} \le C_{1,1}$, ($C_1 = C_{1,0}$ for $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ for $\kappa_{wt} \ge 1$)

2) $0,7L$ = left end fixed, $0,7R$ = right end fixed

Loading and support conditions. Cross-section monosymmetry factor ψ_f	Bending moment diagram. End moment ratio ψ. M-side, ψM-side	k_z 2)	C_1 1)		C_3			
			$C_{1,0}$	$C_{1,1}$	$\psi_f = -1$	$-0{,}9 \le \psi_f \le 0$	$0 \le \psi_f \le 0{,}9$	$\psi_f = 1$
 $\psi_f \le 0$; $\psi_f \ge 0$	 M_{cr} $\psi = -1/2$	1,0	2,331	2,591	1,85	1,000	$1{,}3 - 1{,}2\psi_f$	-0,70
		0,7L	3,078	3,399	2,70	1,450	$1 - 1{,}2\psi_f$	-1,15
		0,7R	1,711	1,897	1,45	0,780	$0{,}9 - 0{,}75\psi_f$	-0,53
		0,5	2,230	2,579	2,00	0,950	$0{,}75 - \psi_f$	-0,85
	 M_{cr} $\psi = -3/4$	1,0	2,547	2,852	2,00	1,000	$0{,}55 - \psi_f$	-1,45
$\psi_f = \dfrac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$		0,7L	2,592	2,770	2,00	0,850	$0{,}23 - 0{,}9\psi_f$	-1,55
		0,7R	1,829	2,027	1,55	0,700	$0{,}68 - \psi_f$	-1,07
		0,5	2,352	2,606	2,00	0,850	$0{,}35 - \psi_f$	-1,45
	 M_{cr} $\psi = -1$	1,0	2,555	2,733	2,00	$-\psi_f$		-2,00
		0,7L	1,921	2,103	1,55	0,380	-0,580	-1,55
		0,7R	1,921	2,103	1,55	0,580	-0,380	-1,55
		0,5	2,223	2,390	1,88	$0{,}125 - 0{,}7\psi_f$	$-0{,}125 - 0{,}7\psi_f$	-1,88

1) $C_1 = C_{1,0} + (C_{1,1} - C_{1,0})\kappa_{wt} \le C_{1,1}$, ($C_1 = C_{1,0}$ for $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ for $\kappa_{wt} \ge 1$)

2) $0{,}7L$ = left end fixed, $0{,}7R$ = right end fixed

For any ratio of end moments when k_z=1

$$C_1 = (0{,}310 + 0{,}428\psi + 0{,}262\psi^2)^{-0.5}$$

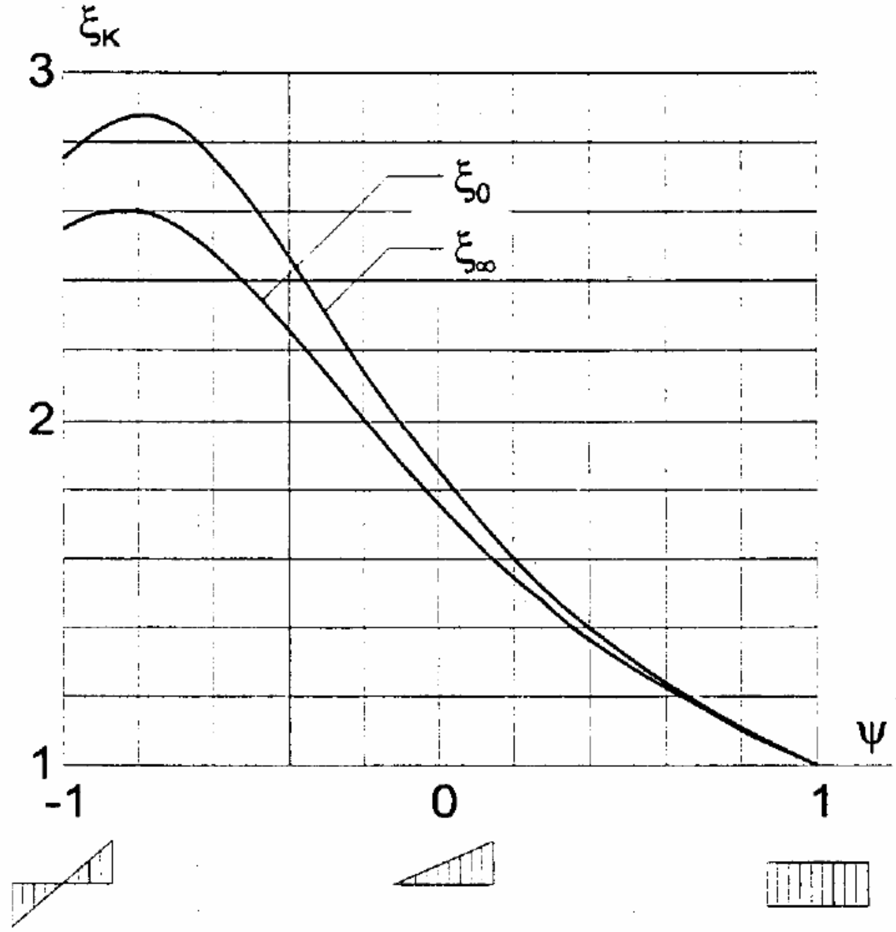
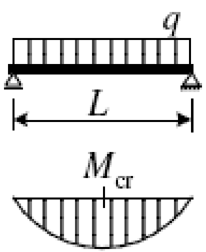
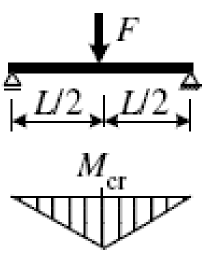


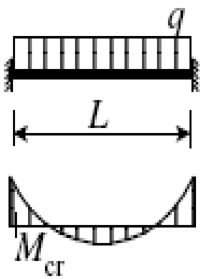
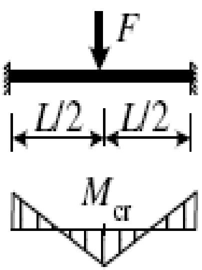
Table I.2 - Values of factors C_1, C_2 and C_3 corresponding to various transverse loading cases, values of buckling length factors k_y, k_z, k_w, cross-section monosymmetry factor ψ_f and torsion parameter κ_{wt}.

Loading and support conditions	Buckling length factors			Values of factors							
				C_1 1)		C_2			C_3		
	k_y	k_z	k_w	$C_{1,0}$	$C_{1,1}$	⊥ $\psi_f = -1$	I I I $-0{,}9 \le \psi_f \le 0{,}9$	T $\psi_f = 1$	⊥ $\psi_f = -1$	I I I $-0{,}9 \le \psi_f \le 0{,}9$	T $\psi_f = 1$
	1	1	1	1,127	1,132	0,33	0,459	0,50	0,93	0,525	0,38
	1	1	0,5	1,128	1,231	0,33	0,391	0,50	0,93	0,806	0,38
	1	0,5	1	0,947	0,997	0,25	0,407	0,40	0,84	0,478	0,44
	1	0,5	0,5	0,947	0,970	0,25	0,310	0,40	0,84	0,674	0,44
	1	1	1	1,348	1,363	0,52	0,553	0,42	1,00	0,411	0,31
	1	1	0,5	1,349	1,452	0,52	0,580	0,42	1,00	0,666	0,31
	1	0,5	1	1,030	1,087	0,40	0,449	0,42	0,80	0,338	0,31

1) $C_1 = C_{1,0} + (C_{1,1} - C_{1,0})\kappa_{wt} \le C_{1,1}$, ($C_1 = C_{1,0}$ for $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ for $\kappa_{wt} \ge 1$).

2) Parameter ψ_f refers to the middle of the span.

3) Values of critical moments M_{cr} refer to the cross section, where M_{max} is located

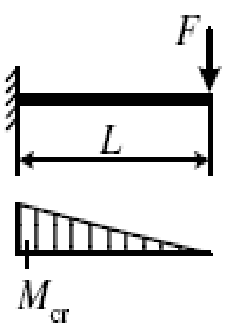
						$\psi_f = -1$	$-0{,}5 \le \psi_f \le 0{,}5$	$\psi_f = 1$	$\psi_f = -1$	$-0{,}5 \le \psi_f \le 0{,}5$	$\psi_f = 1$
	0,5	1	1	2,576	2,608	1,00	1,562	0,15	1,00	-0,859	-1,99
	0,5	0,5	1	1,490	1,515	0,56	0,900	0,08	0,61	-0,516	-1,20
	0,5	0,5	0,5	1,494	1,746	0,56	0,825	0,08	0,61	0,002712	-1,20
	0,5	1	1	1,683	1,726	1,20	1,388	0,07	1,15	-0,716	-1,35
	0,5	0,5	1	0,936	0,955	0,69	0,763	0,03	0,64	-0,406	-0,76
	0,5	0,5	0,5	0,937	1,057	0,69	0,843	0,03	0,64	-0,0679	-0,76

1) $C_1 = C_{1,0} + \left(C_{1,1} - C_{1,0}\right)\kappa_{wt} \le C_{1,1}$, $\left(C_1 = C_{1,0}\right.$ for $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ for $\left.\kappa_{wt} \ge 1\right)$.

2) Parameter ψ_f refers to the middle of the span.

3) Values of critical moments M_{cr} refer to the cross section, where M_{max} is located

Table I.3 - Relative non-dimensional critical moment μ_{cr} for cantilever $(k_y = k_z = k_w = 2)$ loaded by concentrated end load F.

Loading and support conditions	$\frac{\pi}{L}\sqrt{\frac{EI_w}{GI_t}} = k_w \kappa_{wt} = \kappa_{wt0}$	$\frac{\pi z_g}{L}\sqrt{\frac{EI_z}{GI_t}} = k_z \zeta_g = \zeta_g$	(T) (C) (C) (T) $\frac{\pi z_j}{L}\sqrt{\frac{EI_z}{GI_t}} = k_z \zeta_j = \zeta_{j0}$ (C) (T) (T) (C)						
			-4	-2	-1	0	1	2	4
	0	4	0,107	0,156	0,194	0,245	0,316	0,416	0,759
		2	0,123	0,211	0,302	0,463	0,759	1,312	4,024
		0	0,128	0,254	0,478	1,280	3,178	5,590	10,730
		-2	0,129	0,258	0,508	1,619	3,894	6,500	11,860
		-4	0,129	0,258	0,511	1,686	4,055	6,740	12,240
	0,5	4	0,151	0,202	0,240	0,293	0,367	0,475	0,899
		2	0,195	0,297	0,393	0,560	0,876	1,528	5,360
		0	0,261	0,495	0,844	1,815	3,766	6,170	11,295
		-2	0,329	0,674	1,174	2,423	4,642	7,235	12,595
		-4	0,364	0,723	1,235	2,529	4,843	7,540	13,100
	1	4	0,198	0,257	0,301	0,360	0,445	0,573	1,123
		2	0,268	0,391	0,502	0,691	1,052	1,838	6,345
		0	0,401	0,750	1,243	2,431	4,456	6,840	11,920
		-2	0,629	1,326	2,115	3,529	5,635	8,115	13,365
		-4	0,777	1,474	2,264	3,719	5,915	8,505	13,960

a) For $z_j = 0$, $z_g = 0$ and $\kappa_{wt0} \leq 8$: $\mu_{cr} = 1{,}27 + 1{,}14\,\kappa_{wt0} + 0{,}017\,\kappa_{wt0}^2$.

b) For $z_j = 0$, $-4 \leq \zeta_g \leq 4$ and $\kappa_{wt} \leq 4$, μ_{cr} may be calculated also from formulae (I.7) and (I.8), where the following approximate values of the factors C_1, C_2 should be used for the cantilever under tip load F:

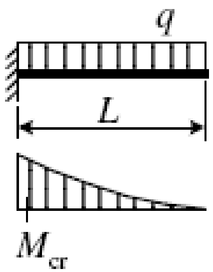
$$C_1 = 2{,}56 + 4{,}675\,\kappa_{wt} - 2{,}62\,\kappa_{wt}^2 + 0{,}5\kappa_{wt}^3, \qquad \text{if} \quad \kappa_{wt} \leq 2$$

$$C_1 = 5{,}55 \qquad \text{if} \quad \kappa_{wt} > 2$$

$$C_2 = 1{,}255 + 1{,}566\kappa_{wt} - 0{,}931\,\kappa_{wt}^2 + 0{,}245\kappa_{wt}^3 - 0{,}024\kappa_{wt}^4, \qquad \text{if} \quad \zeta_g \geq 0$$

$$C_2 = 0{,}192 + 0{,}585\kappa_{wt} - 0{,}054\kappa_{wt}^2 - (0{,}032 + 0{,}102\,\kappa_{wt} - 0{,}013\kappa_{wt}^2)\,\zeta_g, \qquad \text{if} \quad \zeta_g < 0$$

Table I.4 - Relative non-dimensional critical moment μ_{cr} for cantilever $(k_y = k_z = k_w = 2)$ loaded by uniformly distributed load q

Loading and support conditions	$\frac{\pi}{L}\sqrt{\frac{EI_w}{GI_t}} = k_w \kappa_{wt} = \kappa_{wt0}$	$\frac{\pi z_g}{L}\sqrt{\frac{EI_z}{GI_t}} = k_z \zeta_g = \zeta_{g0}$	(T) (C) (C) (T) $\frac{\pi z_j}{L}\sqrt{\frac{EI_z}{GI_t}} = k_z \zeta_j = \zeta_{j0}$ (C) (T) (T) (C)						
			-4	-2	-1	0	1	2	4
	0	4	0,113	0,173	0,225	0,304	0,431	0,643	1,718
		2	0,126	0,225	0,340	0,583	1,165	2,718	13,270
		0	0,132	0,263	0,516	2,054	6,945	12,925	25,320
		-2	0,134	0,268	0,537	3,463	10,490	17,260	30,365
		-4	0,134	0,270	0,541	4,273	12,715	20,135	34,005
	0,5	4	0,213	0,290	0,352	0,443	0,586	0,823	2,046
		2	0,273	0,421	0,570	0,854	1,505	3,229	14,365
		0	0,371	0,718	1,287	3,332	8,210	14,125	26,440
		-2	0,518	1,217	2,418	6,010	12,165	18,685	31,610
		-4	0,654	1,494	2,950	7,460	14,570	21,675	35,320
	1	4	0,336	0,441	0,522	0,636	0,806	1,080	2,483
		2	0,449	0,663	0,865	1,224	1,977	3,873	15,575
		0	0,664	1,263	2,172	4,762	9,715	15,530	27,735
		-2	1,109	2,731	4,810	8,695	14,250	20,425	33,075
		-4	1,623	3,558	6,025	10,635	16,880	23,555	36,875

a) For $z_j = 0$, $z_g = 0$ and $\kappa_{wt0} \leq 8$: $\mu_{cr} = 2{,}04 + 2{,}68\,\kappa_{wt0} + 0{,}021\,\kappa_{wt0}^2$.

b) For $z_j = 0$, $-4 \leq \zeta_g \leq 4$ and $\kappa_{wt} \leq 4$, μ_{cr} may be calculated also from formula (I.7) and (I.8), where the following approximate values of the factors C_1, C_2 should be used for the cantilever under uniform load q:

$$C_1 = 4{,}11 + 11{,}2\,\kappa_{wt} - 5{,}65\,\kappa_{wt}^2 + 0{,}975\,\kappa_{wt}^3, \qquad \text{if } \kappa_{wt} \leq 2$$

$$C_1 = 12 \qquad \text{if } \kappa_{wt} > 2$$

$$C_2 = 1{,}661 + 1{,}068\,\kappa_{wt} - 0{,}609\,\kappa_{wt}^2 + 0{,}153\,\kappa_{wt}^3 - 0{,}014\,\kappa_{wt}^4, \qquad \text{if } \zeta_g \geq 0$$

$$C_2 = 0{,}535 + 0{,}426\,\kappa_{wt} - 0{,}029\,\kappa_{wt}^2 - (0{,}061 + 0{,}074\,\kappa_{wt} - 0{,}0085\,\kappa_{wt}^2)\,\zeta_g, \qquad \text{if } \zeta_g < 0$$

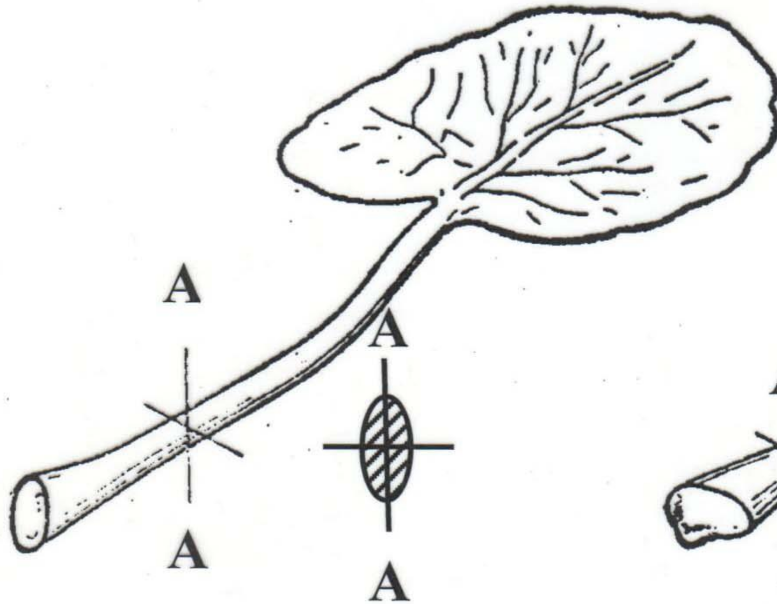
Calculation of M_{cr} with the help of computer programs (no limitation)

- DRILL (emeritus Prof. Friemann, TU Darmstadt)
- BT II (Prof. Ostrerrieder)
- IBDSQ (Dickel, Prof. Rothert, …)
- RSTAB (Dlubal)
- CalcMcr (Baláž)
- LTBeam (CTICM Paris)
- ALPHAcr (Lennert, TU Graz)
- many others

B. Real beam – with imperfections: Resistance of beam $M_{b,Rd}$

When it is not necessary to verify LTB of beam ?

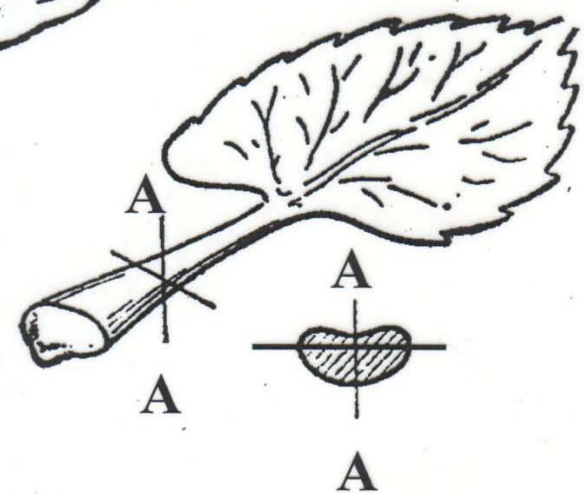
- beam flange in compression is lateraly supported
- beam has closed profile (big torsional stiffness)
- beam is loaded in direction of minor stiffness



Osika

OSIKA

Die Espe



Breza

BRZOZA

Die Birke

Structure magazine, February 2008

Avoiding Structural Failures During Construction

Part 2

By David B. Peraza, P.E. and Dennis M. McCann, Ph.D., P.E.

Figure 4: The Marcy Pedestrian Bridge following collapse during placement of the concrete deck.

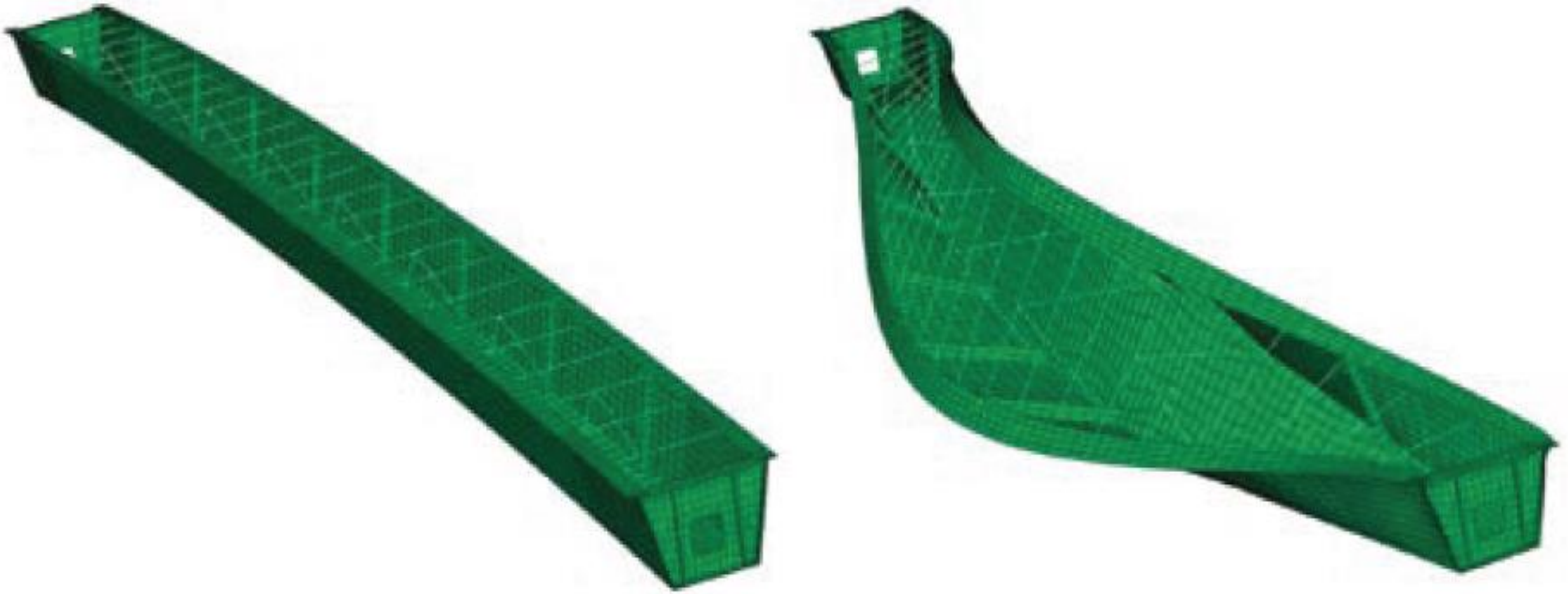


Figure 5: Finite element model of the Marcy Pedestrian Bridge showing the undeformed shape and the global lateral-torsional buckling mode.

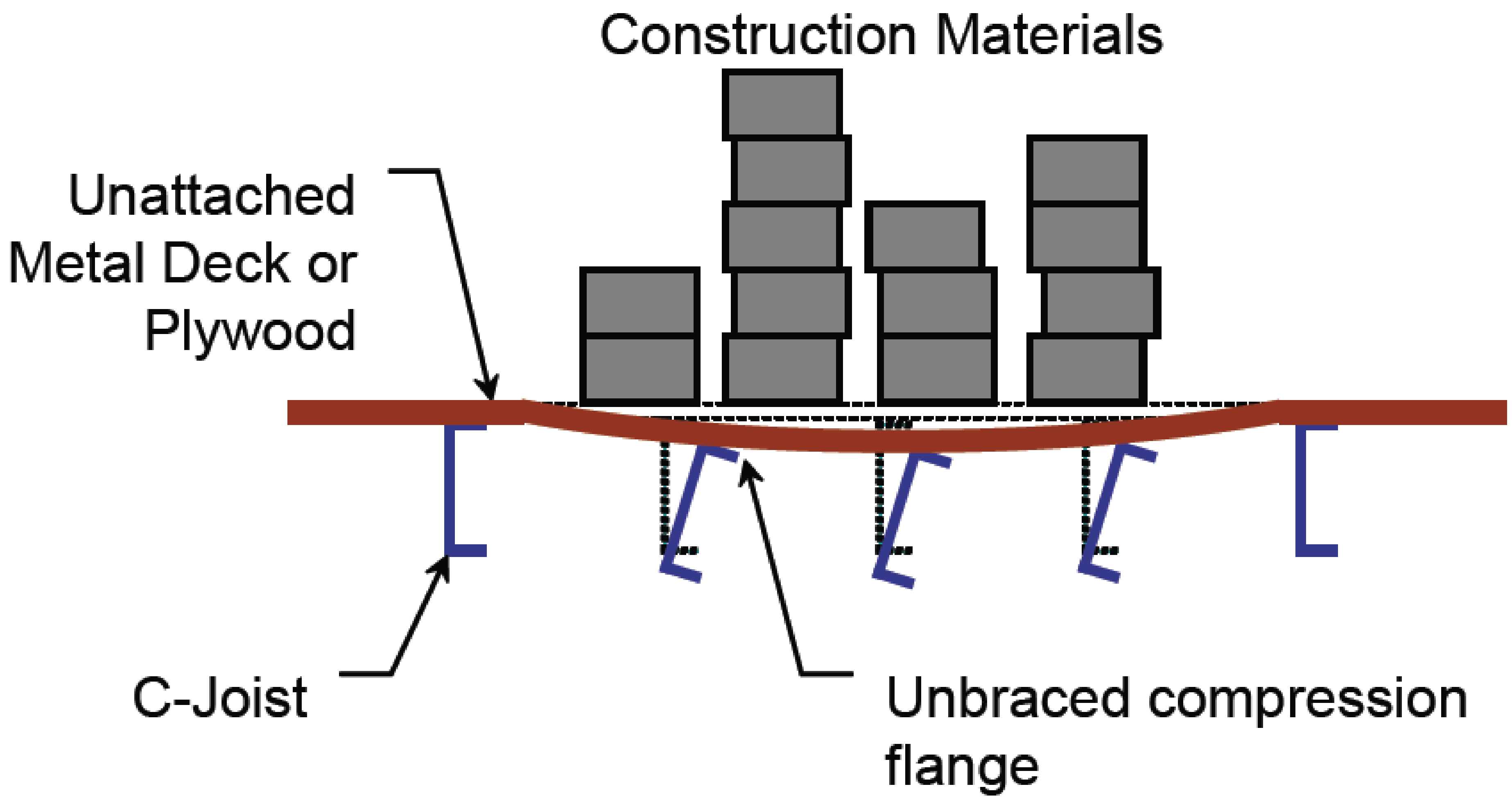


Figure 3: Light gage metal joists are vulnerable to lateral torsional buckling if their compression flange is not properly braced. Contractors may not be aware of the importance of attaching decking before loading a floor.

EUROPEAN STANDARD

NORME EUROPÉENNE

EUROPÄISCHE NORM

EN 1993-1-1

May 2005

ICS 91.010.30; 91.080.10

Supersedes ENV 1993-1-1:1992

English version

Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings

Eurocode 3: Calcul des structures en acier - Partie 1-1: Règles générales et règles pour les bâtiments

Eurocode 3: Bemessung und Konstruktion von Stahlbauten - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau

6.3.2.1 Buckling resistance

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$

where W_y is the appropriate section modulus as follows:

- $W_y = W_{pl,y}$ for Class 1 or 2 cross-sections
- $W_y = W_{el,y}$ for Class 3 cross-sections
- $W_y = W_{eff,y}$ for Class 4 cross-sections

χ_{LT} is the reduction factor for lateral-torsional buckling.

6.3.2.2 Lateral torsional buckling curves – General case

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but } \chi_{LT} \le 1{,}0$$

where $\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0{,}2\right) + \bar{\lambda}_{LT}^2\right]$

α_{LT} is an imperfection factor

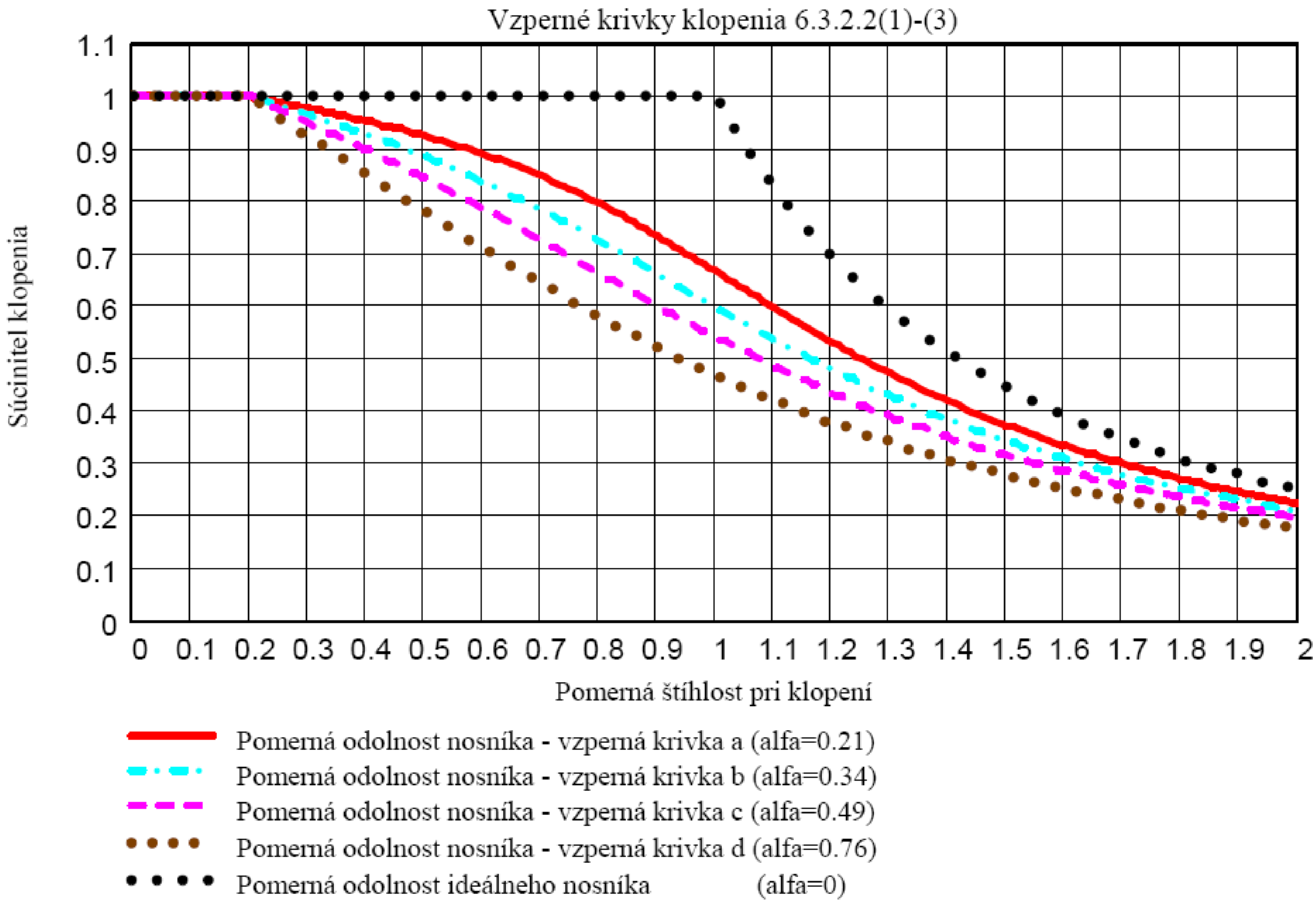
$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$

M_{cr} is the elastic critical moment for lateral-torsional buckling

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**a** **b**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**
Other cross-sections	-	**d**

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

Súčiniteľ klopenia χ_{LT} ako funkcia pomernej štíhlosti pri klopení λ_{LT} a miery imperfekcie pri klopení α_{LT} podľa 1993-1-1, 6.3.2.2(1)-(3) (6.56):



$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$ *is the relative slenderness*

$$W_y = \beta_w W_{pl}$$

$$\overline{\lambda}_{LT} = \sqrt{\frac{M_{pl}}{M_{cr}}} \sqrt{\beta_w}$$

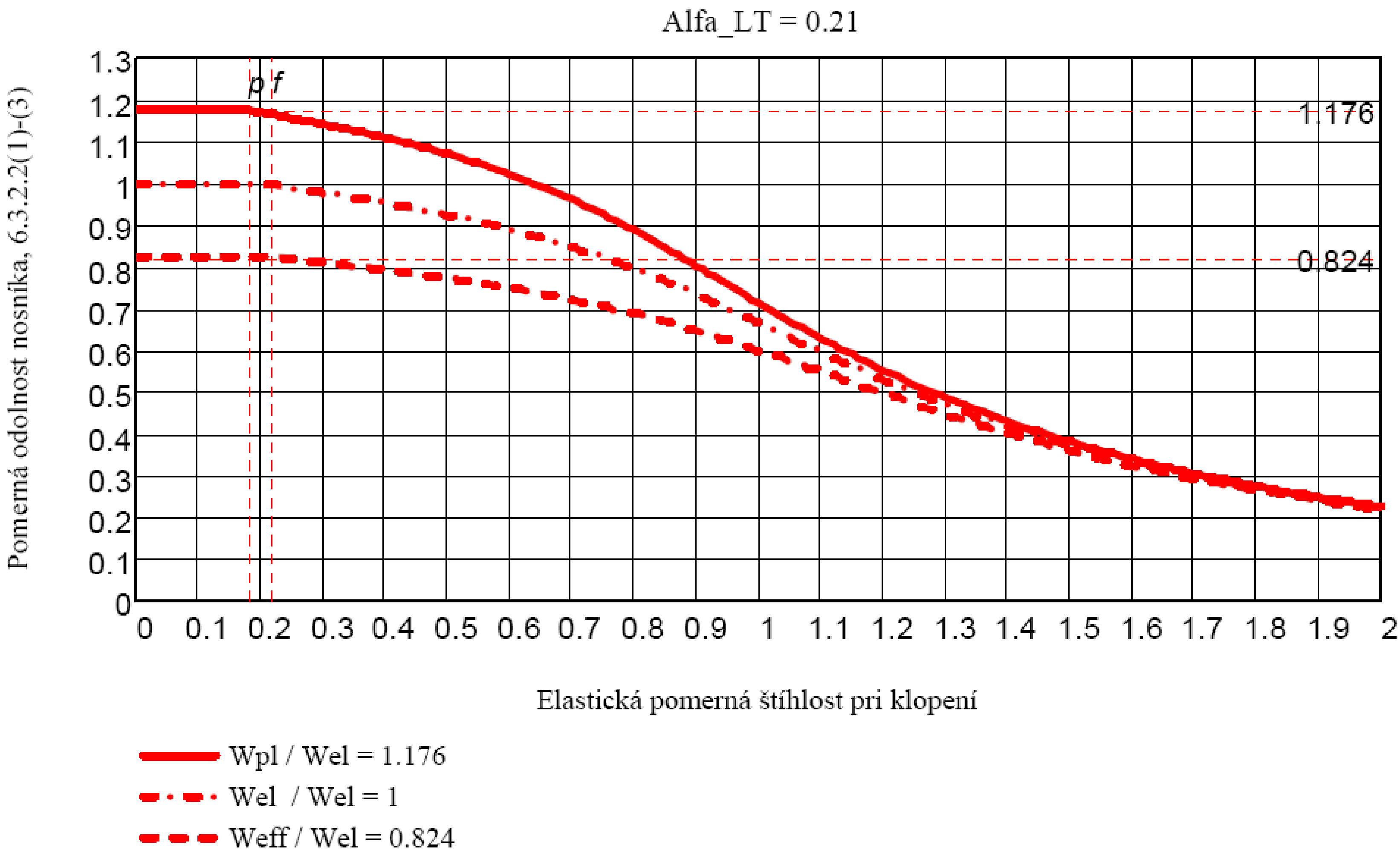
$$\frac{\gamma_{M1} M_{b,Rd}}{M_{pl}} = \frac{\beta_w}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \left(\sqrt{\frac{M_{pl}}{M_{cr}}}\sqrt{\beta_w}\right)^2}}, \quad \text{if} \quad \sqrt{\frac{M_{pl}}{M_{cr}}} \geq \frac{\bar{\lambda}_{0,LT}}{\sqrt{\beta_w}} ;$$

$$\frac{\gamma_{M1} M_{b,Rd}}{M_{pl}} = \beta_w$$

$$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\sqrt{\frac{M_{pl}}{M_{cr}}}\sqrt{\beta_w} - \bar{\lambda}_{0,LT}\right) + \left(\sqrt{\frac{M_{pl}}{M_{cr}}}\sqrt{\beta_w}\right)^2\right]$$

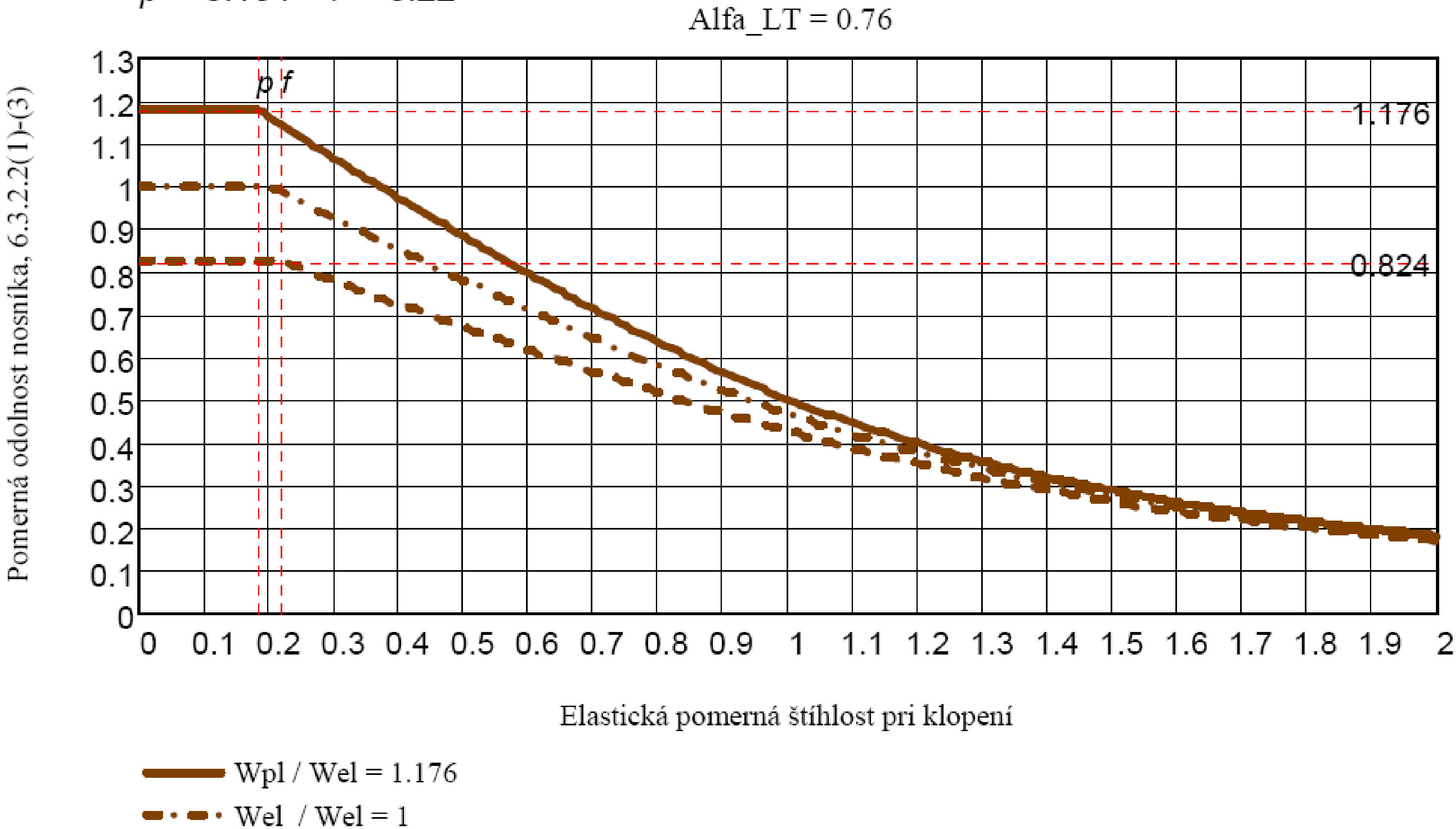
Pomerná odolnosť nosníka $\chi_{LTel} = M_{bRd} / (M_{el} / \gamma_{M1})$ ako funkcia elastickej pomernej štíhlosti pri klopení $_{LTel}$, pomerného prierezového modulu α_W a miery imperfekcie pri klopení α_{LT} podľa EN 1993-1-1, 6.3.2.2(1)-(3) (6.56):

štíhlosti: $p = 0.184$ $\quad f = 0.22$



Pomerná odolnosť nosníka $\chi_{LTel} = M_{bRd} / (M_{el} / \gamma_{M1})$ ako funkcia elastickej pomernej štíhlosti pri klopení λ_{LTel}, pomerného prierezového modulu α_W a miery imperfekcie pri klopení α_{LT} podľa EN 1993-1-1, 6.3.2.2(1)-(3) (6.56):

štíhlosti: $p = 0.184$ $f = 0.22$

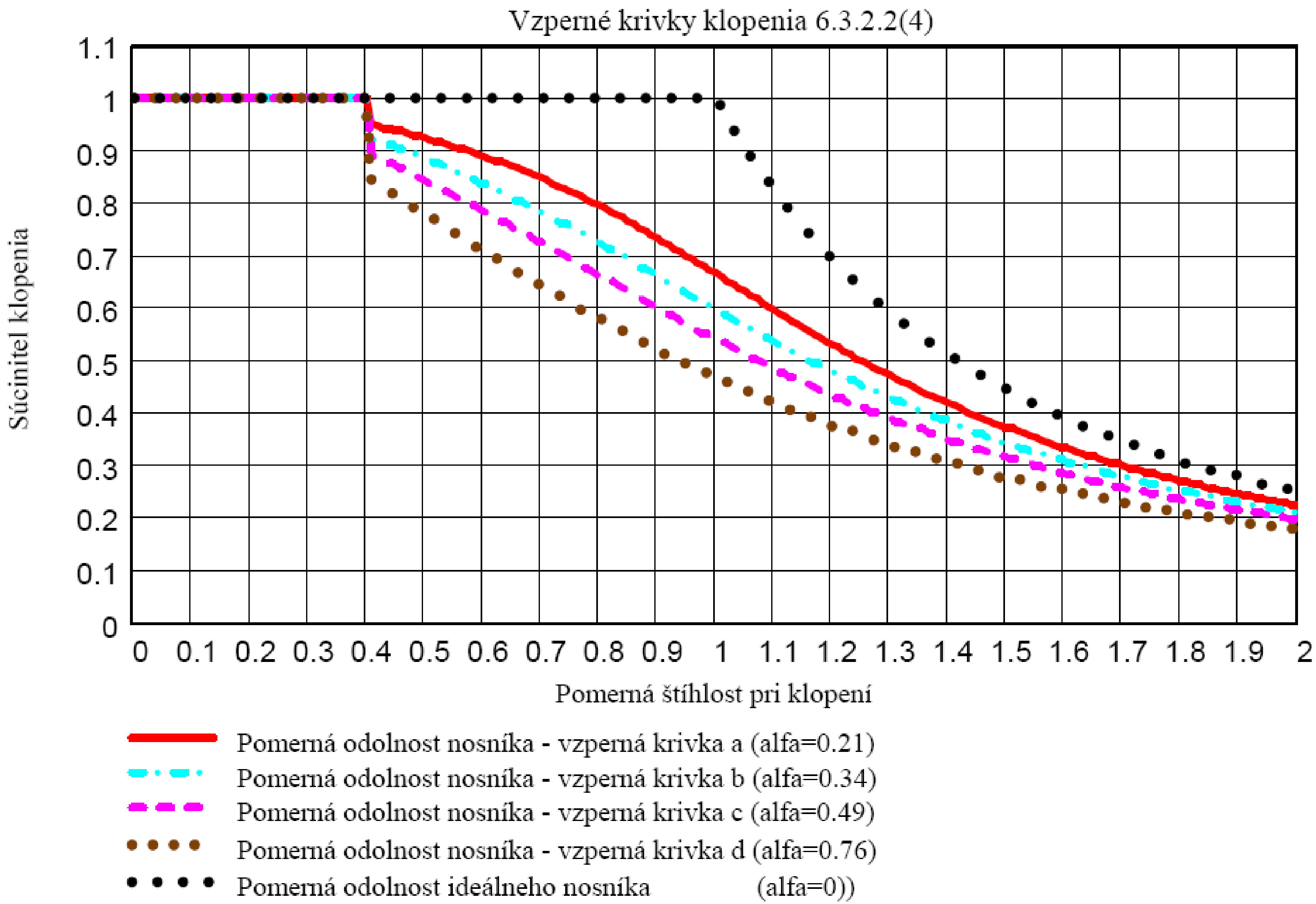


(4) For slendernesses $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0}$ (see 6.3.2.3) or for $\frac{M_{Ed}}{M_{cr}} \leq \bar{\lambda}_{LT,0}{}^2$

lateral torsional buckling

effects may be ignored and only cross sectional checks apply.

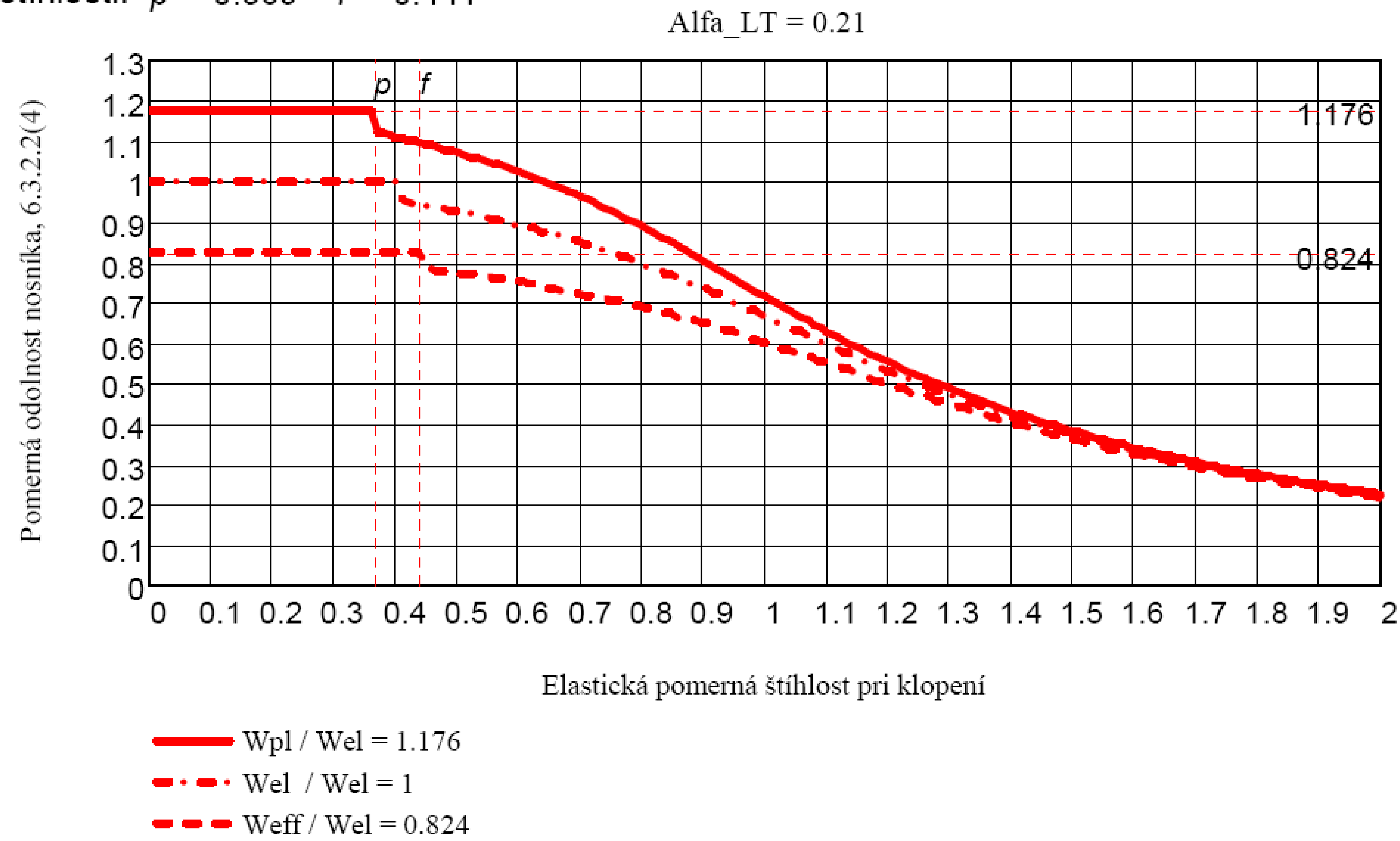
Súčiniteľ klopenia χ_{LT} ako funkcia pomernej štíhlosti pri klopení λ_{LT} a miery imperfekcie pri klopení α_{LT} podľa 1993-1-1, 6.3.2.2(1)-(3) (6.56) s využitím 6.3.2.2(4):



Pomerná odolnosť nosníka $\chi_{LTelm} = M_{bRd} / (M_{el} / \gamma_{M1})$ ako funkcia elastickej pomernej štíhlosti pri klopení λ_{LTel}, pomerného prierezového modulu α_W a miery imperfekcie pri klopení α_{LT} podľa EN 1993-1-1, 6.3.2.2(4) (6.56):

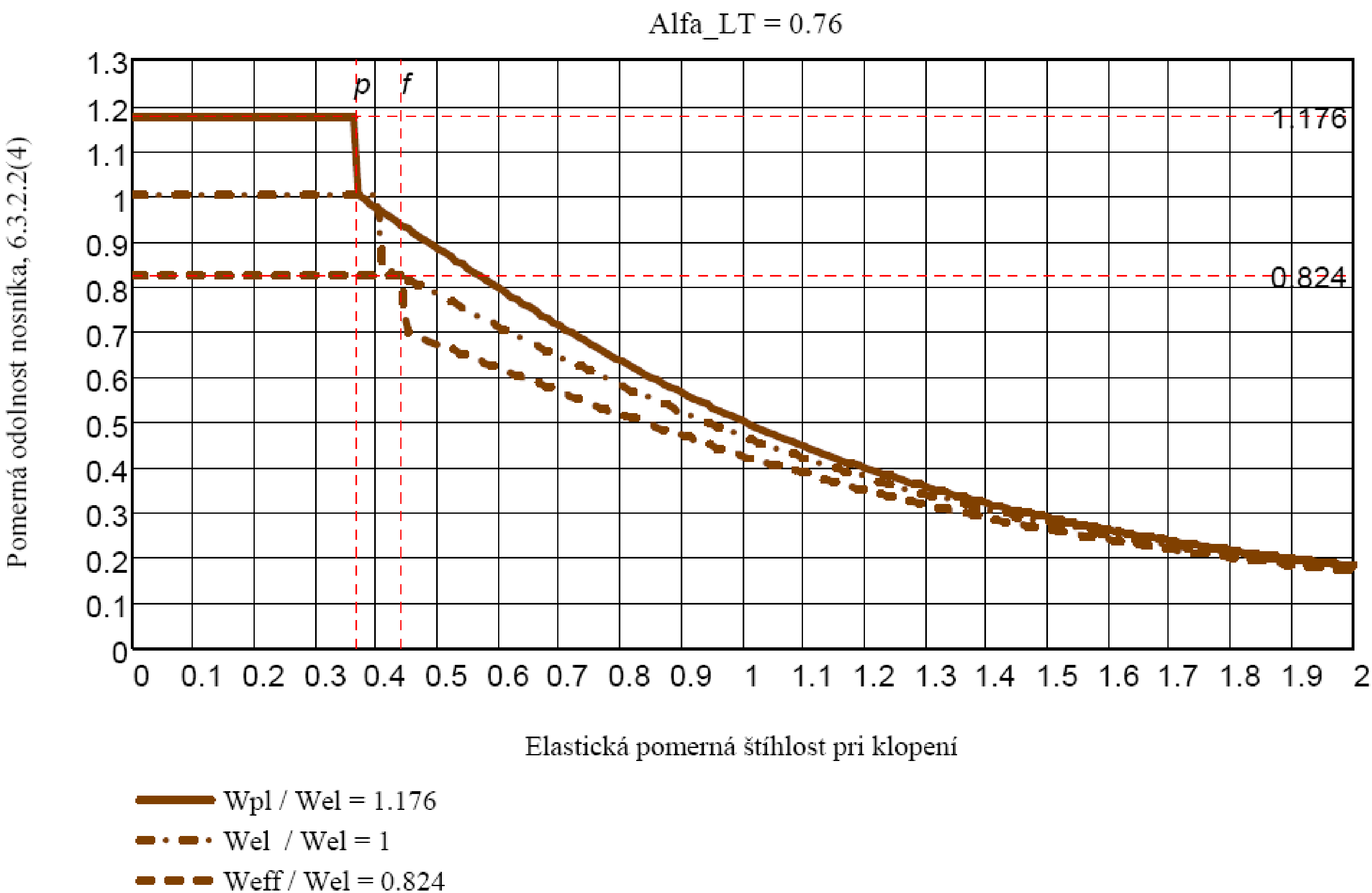
Miera imperfekcie: $\alpha := 0.21$

štíhlosti: $p = 0.369$ $f = 0.441$



Miera imperfekcie: $\alpha := 0.76$

štíhlosti: $p = 0.369$ $f = 0.441$



6.3.2.3 Lateral torsional buckling curves for rolled sections or equivalent welded sections

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \begin{cases} \chi_{LT} \leq 1{,}0 \\ \chi_{LT} \leq \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases}$$

$$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}^2\right]$$

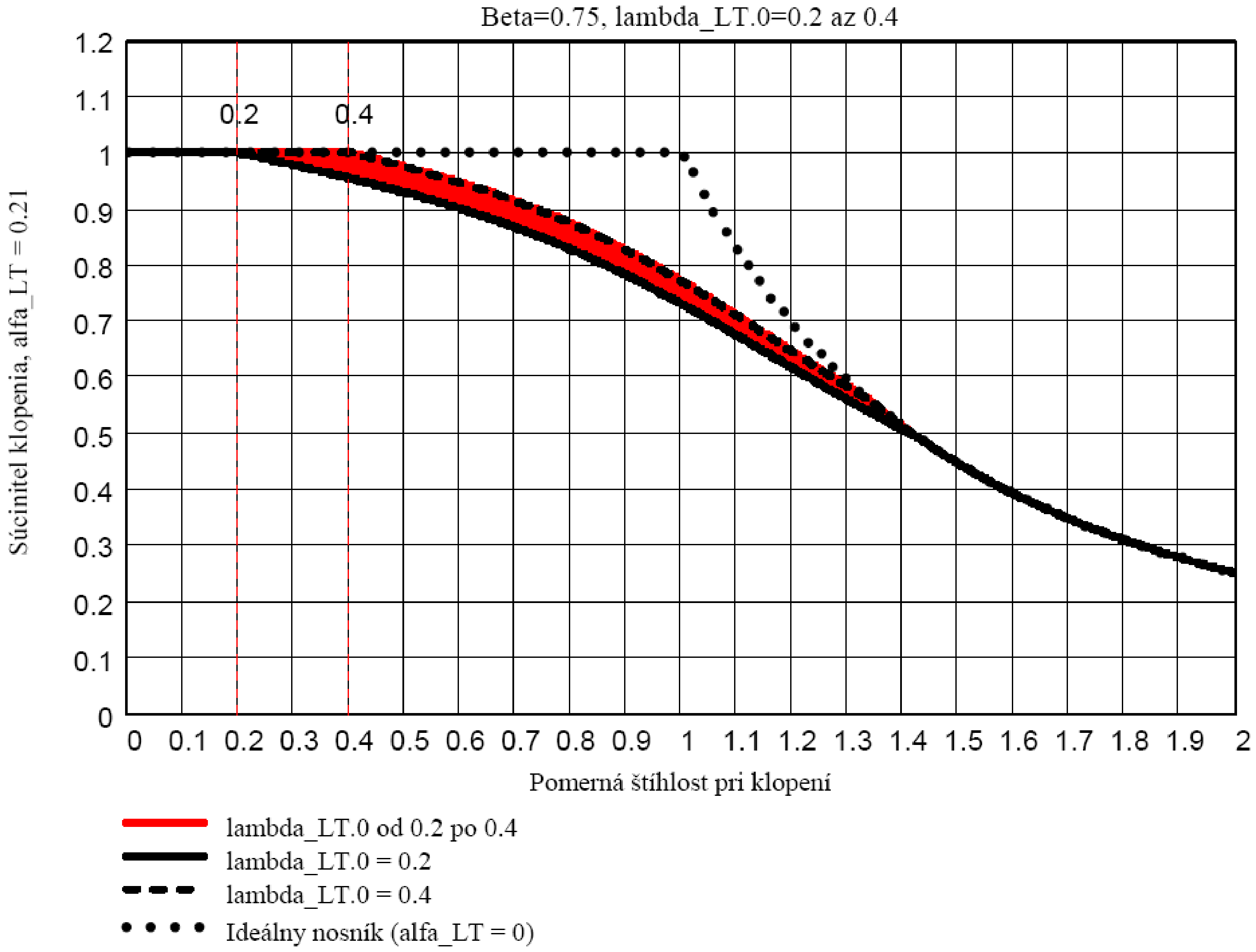
$\bar{\lambda}_{LT,0} = 0{,}4$ (maximum value)

$\beta = 0{,}75$ (minimum value)

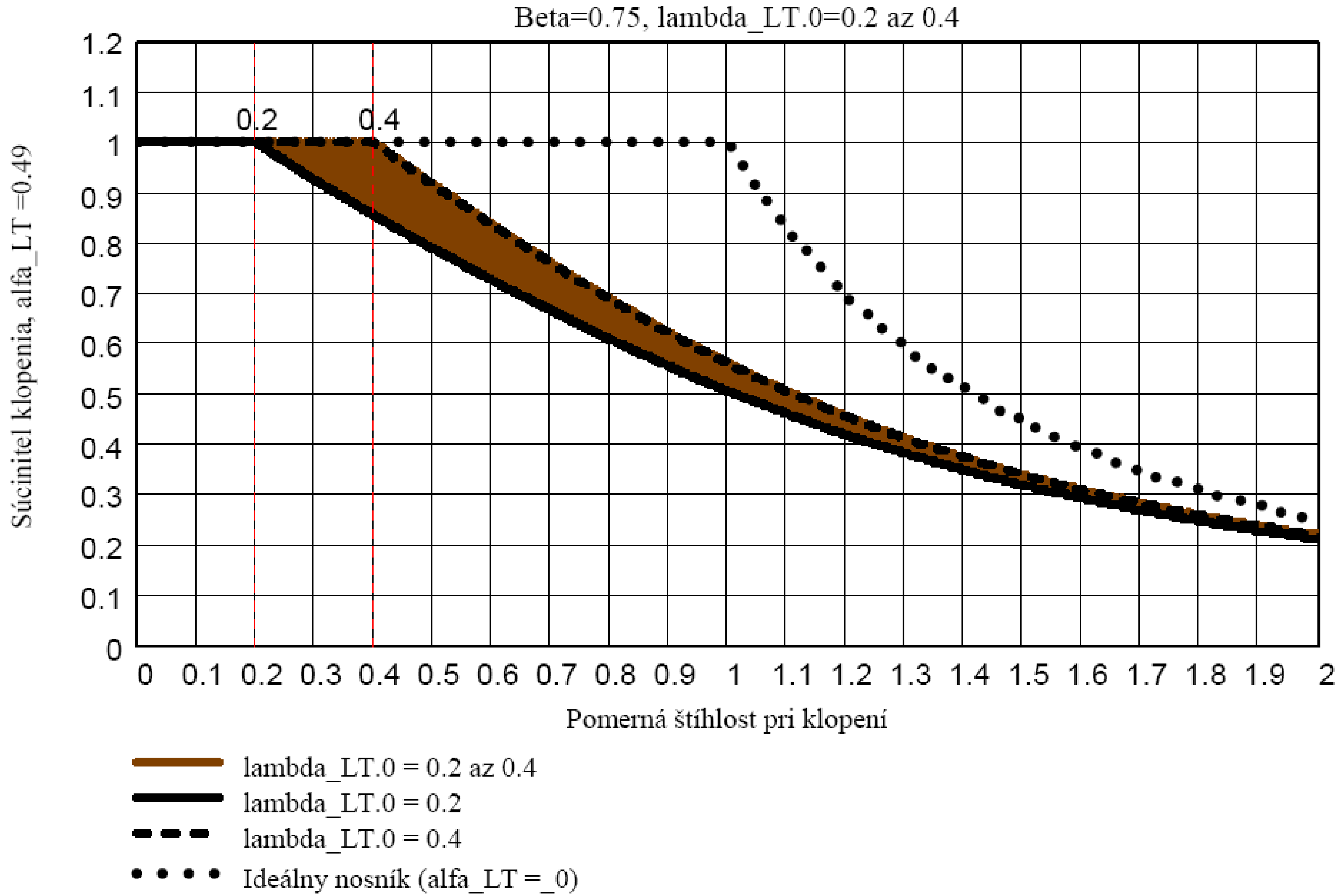
Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**b** **c**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**

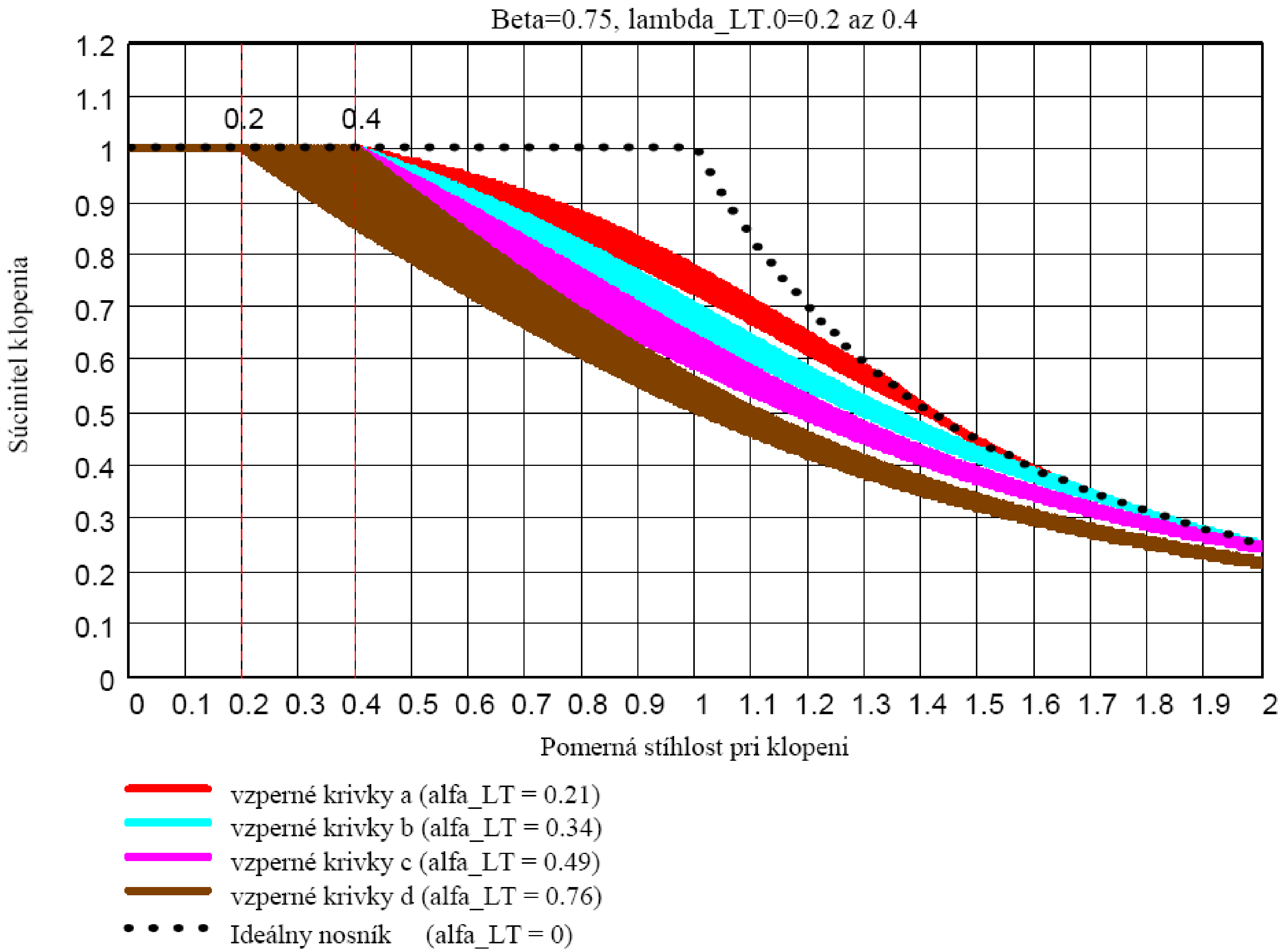
Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	0,49	0,76

Súčiniteľ klopenia χ_{LT} ako funkcia miery imperfekcie pri klopení α_{LT} = 0.21, parametra β = 0.75, pomernej štíhlosti pri klopení $\lambda_{\_LT}$ = 0 až 2 a parametra $\lambda_{\_LT.0}$ = 0.2 až 0.4 podľa EN 1993-1-1, 6.3.2.3(1), (6.57):



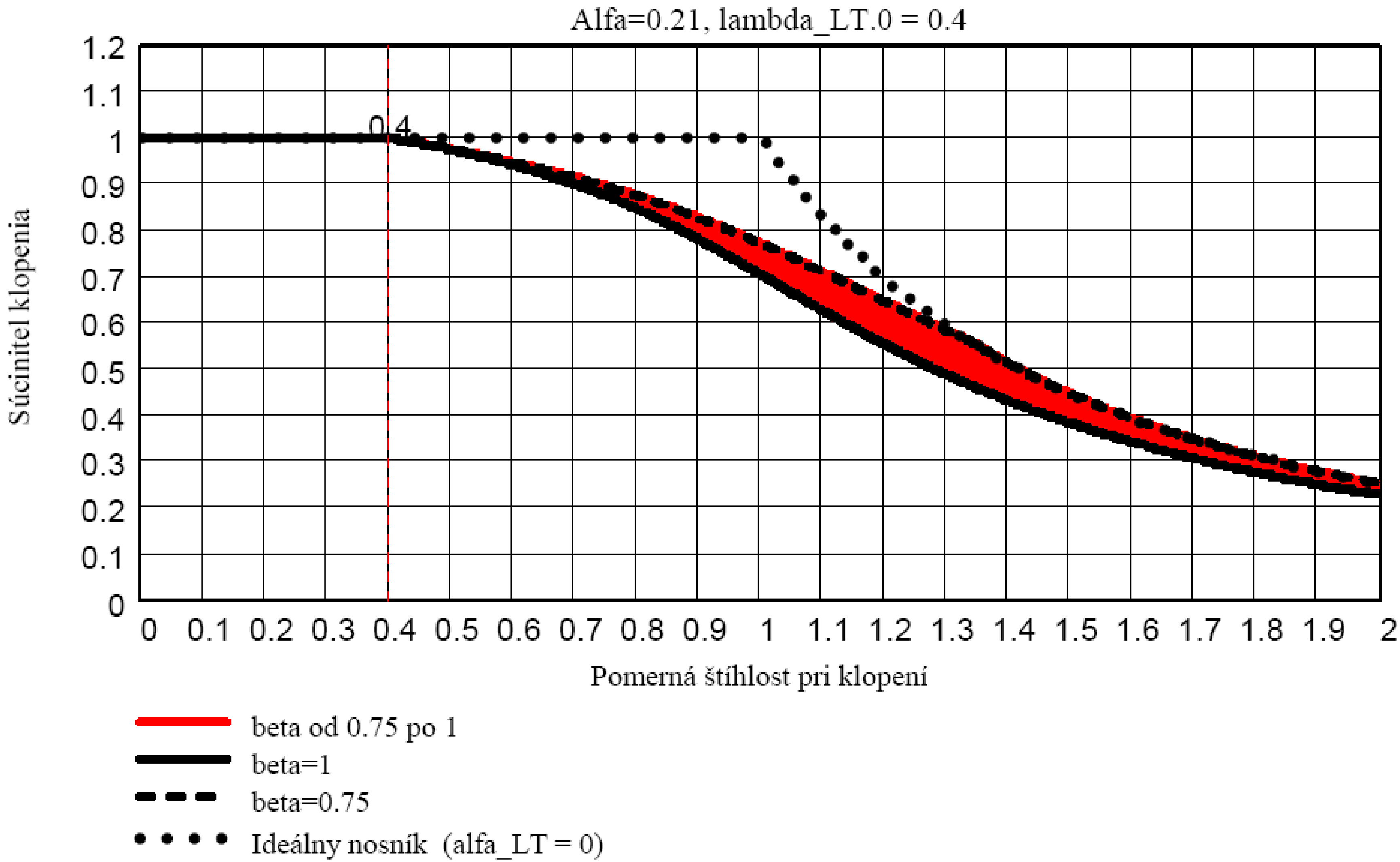
Súčiniteľ klopenia χ_{LT} ako funkcia miery imperfekcie pri klopení α_{LT} = 0.76, parametra β = 0.75, pomernej štíhlosti pri klopení λ_{LT} = 0 až 2 a parametra $\lambda\_{LT.0}$ = 0.2 až 0.4 podľa EN 1993-1-1, 6.3.2.3(1), (6.57):



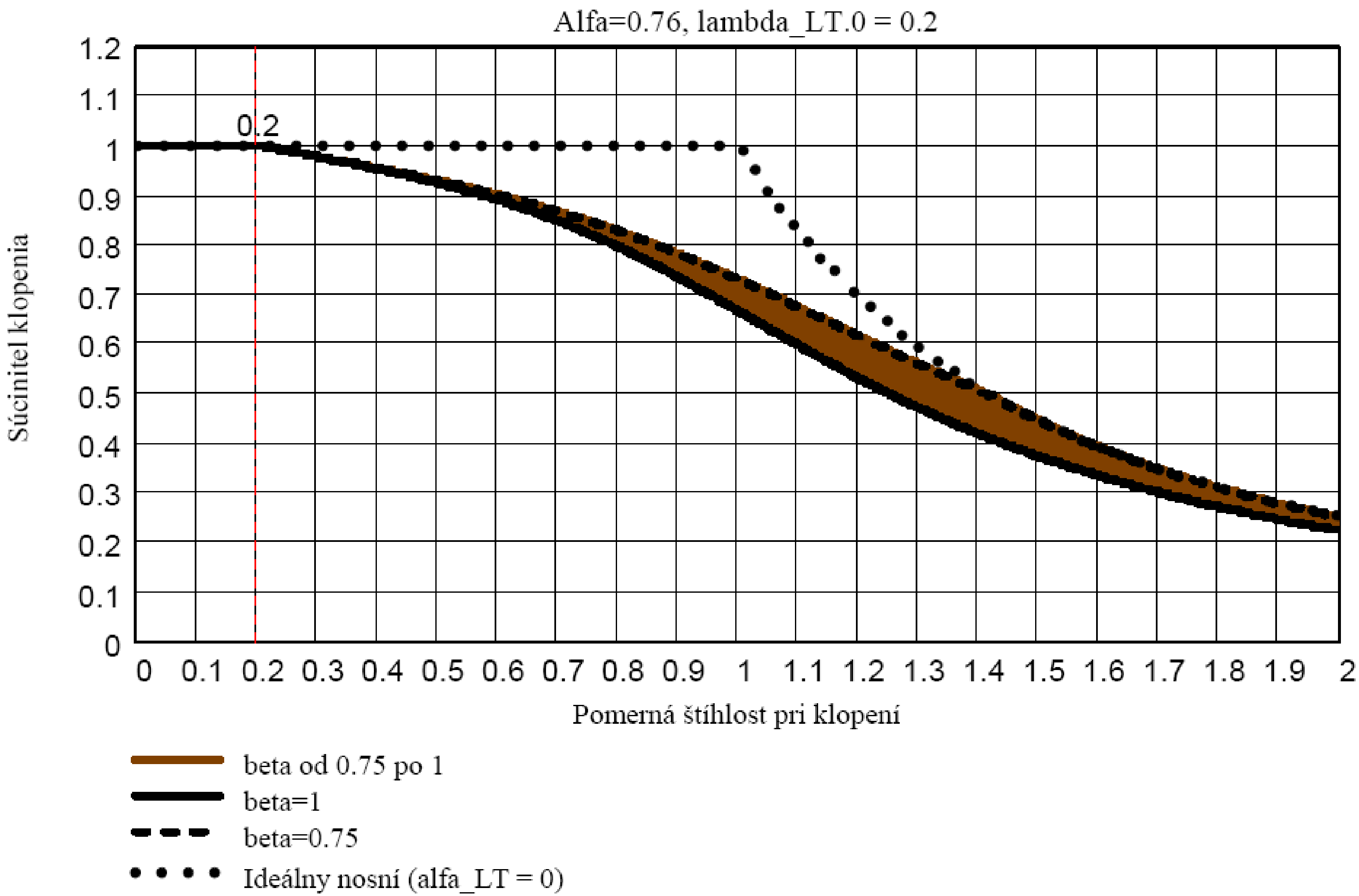


Beta=0.75, lambda_LT.0=0.2 az 0.4
Súcinitel klopenia
Pomerná stíhlost pri klopeni
0.2
0.4
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
vzperné krivky a (alfa_LT = 0.21)
vzperné krivky b (alfa_LT = 0.34)
vzperné krivky c (alfa_LT = 0.49)
vzperné krivky d (alfa_LT = 0.76)
Ideálny nosník (alfa_LT = 0)

Súčiniteľ klopenia χ_{LT} ako funkcia miery imperfekcie pri klopení α_{LT} = 0.21, parametra λ_LT.0 = 0.4 , pomernej štíhlosti pri klopení λ_LT = 0 až 2 a parametra β = 0.75 až 1.0 podľa EN 1993-1-1, 6.3.2.3(1), (6.57):



Súčiniteľ klopenia χ_{LT} ako funkcia miery imperfekcie pri klopení α_{LT} = 0.76, parametra λ_LT.0 = 0.2 , pomernej štíhlosti pri klopení λ_LT = 0 až 2 a parametra β = 0.75 až 1.0 podľa EN 1993-1-1, 6.3.2.3(1), (6.57):



$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \quad \text{ale} \quad \chi_{LT,mod} \leq 1$$

$$f = 1 - 0{,}5(1 - k_c)[1 - 2{,}0(\bar{\lambda}_{LT} - 0{,}8)^2] \quad ale \quad f \leq 1{,}0$$

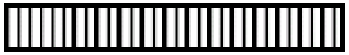
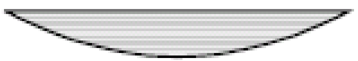
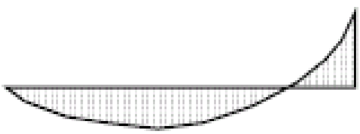
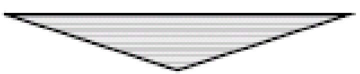
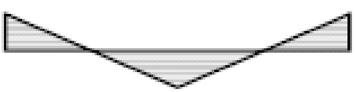
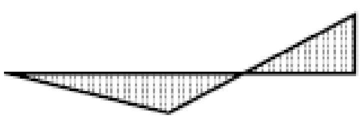
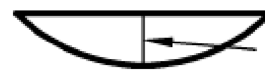
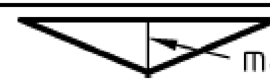
Priebeh momentov	k_c
 $\psi = 1$	1,0
 $-1 \leq \psi \leq 1$	$\dfrac{1}{1{,}33 - 0{,}33\psi}$
	0,94
	0,90
	0,91
	0,86
	0,77
	0,82

Tabelle 8 — Druckkraftbeiwerte k_c

	Normalkraftverlauf	k_c
1	 max N	1,00
2	 max N	0,94
3	 max N	0,86
4	 max N ψ max N $-1 \leq \psi \leq 1$	$\dfrac{1}{1{,}33 - 0{,}33\,\psi}$

Tabelle 10 — Momentenbeiwerte ζ

Zeile	Momentenverlauf	ζ
1	 max M	1,00
2	 max M	1,12
3	 max M	1,35
4	 max M ψ max M $-1 \leq \psi \leq 1$	$1{,}77 - 0{,}77\,\psi$

$$k_c = \frac{1}{\sqrt{\xi}}$$

Interval in which increasing of reduction coefficient χ_{LT} by factor f is effective

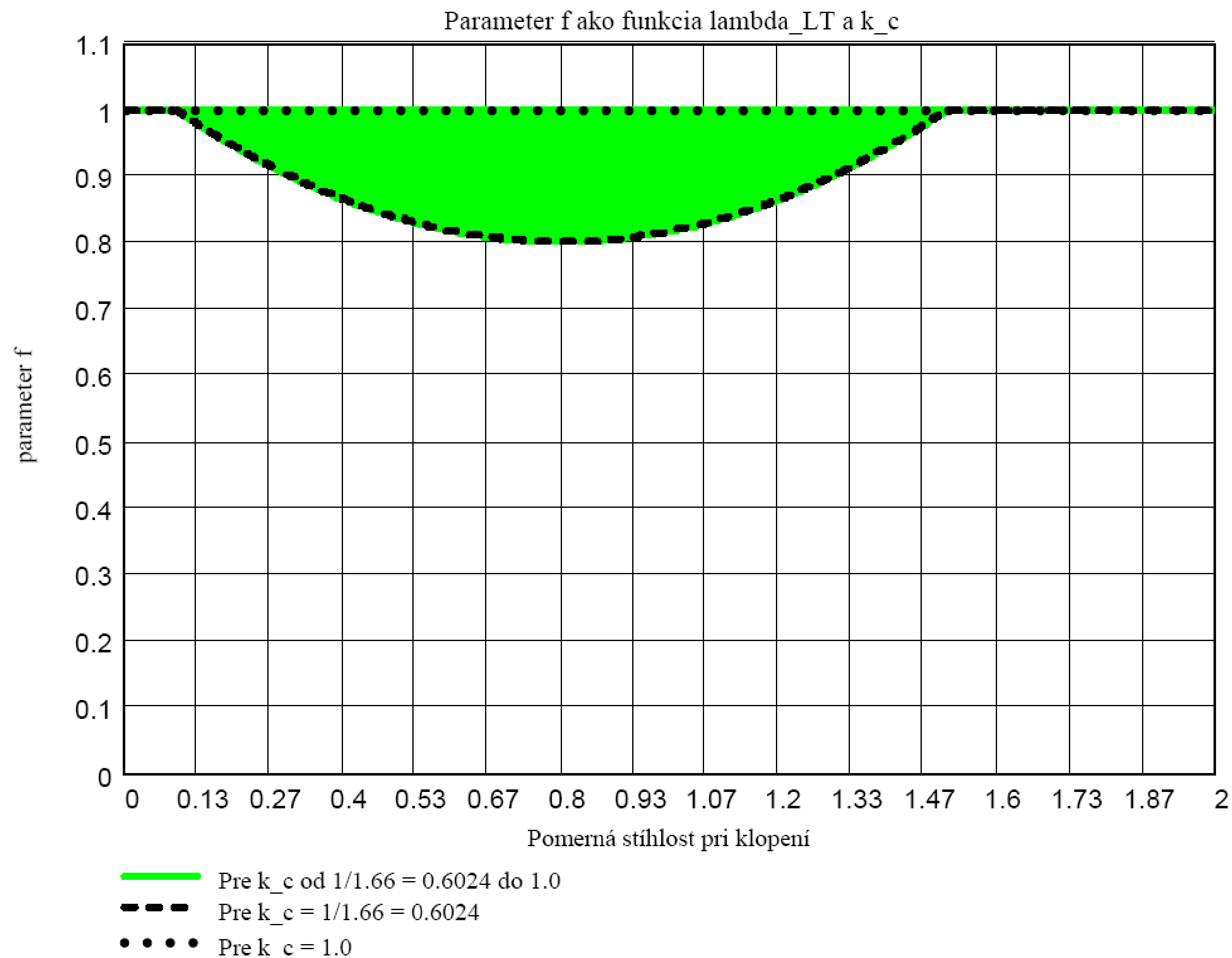
$$max[\overline{\lambda}_{LT,0}\,;\,0{,}1] \approx max[\overline{\lambda}_{LT,0}\,;\,0{,}8-\sqrt{0{,}5}\,] \le \overline{\lambda}_{LT} \le 0{,}8+\sqrt{0{,}5} \approx 1{,}5$$

$$1 \le 1/\,f \le 1{,}5$$

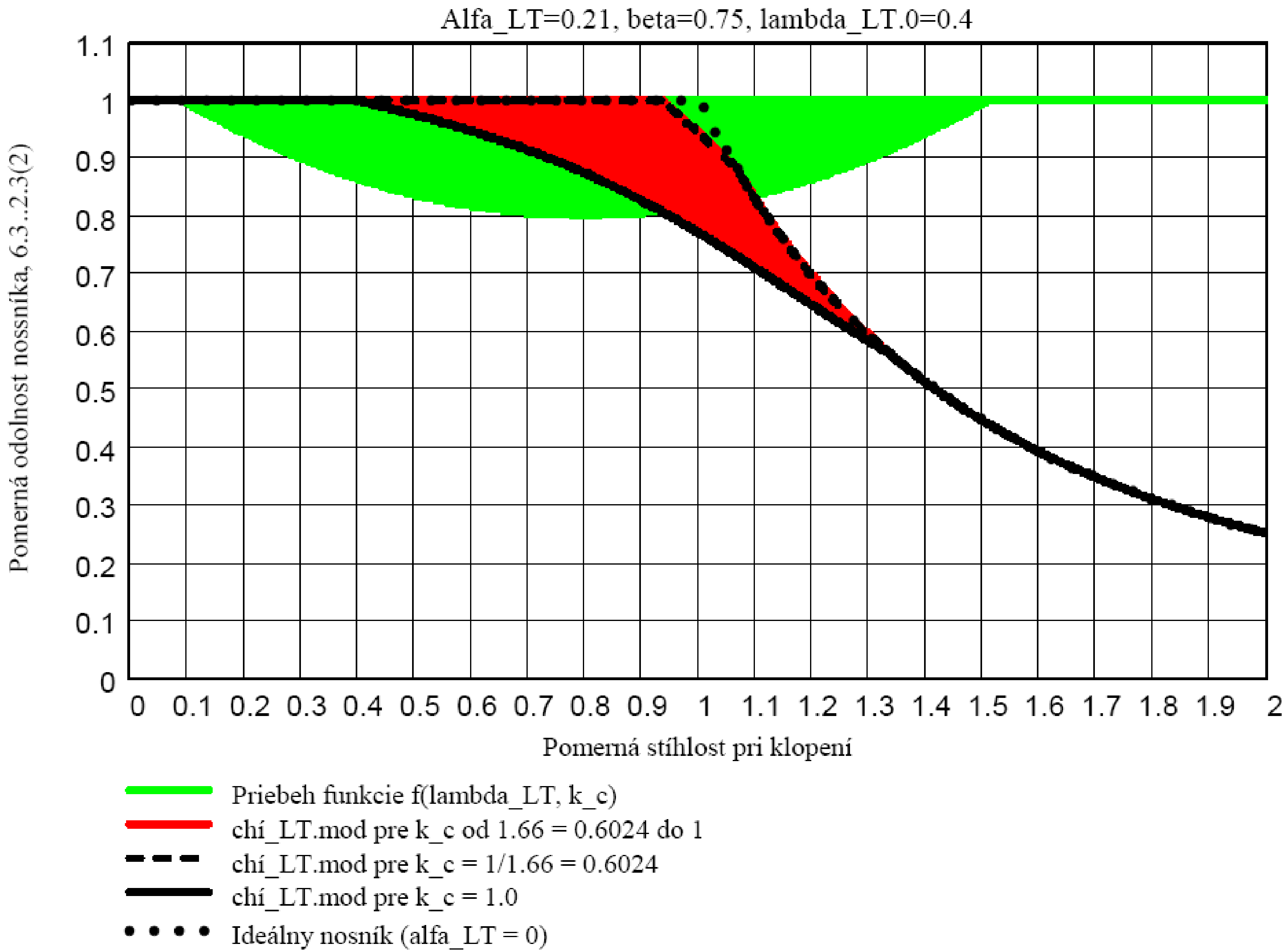
$$\overline{\lambda}_{LT,cr} = \frac{\alpha_{LT}}{2(1-\beta)} + \sqrt{\left(\frac{\alpha_{LT}}{2(1-\beta)}\right)^2 - 2\left(\frac{\alpha_{LT}}{2(1-\beta)}\right)\overline{\lambda}_{LT,0} + 1}$$

Grafické priebehy funkcií

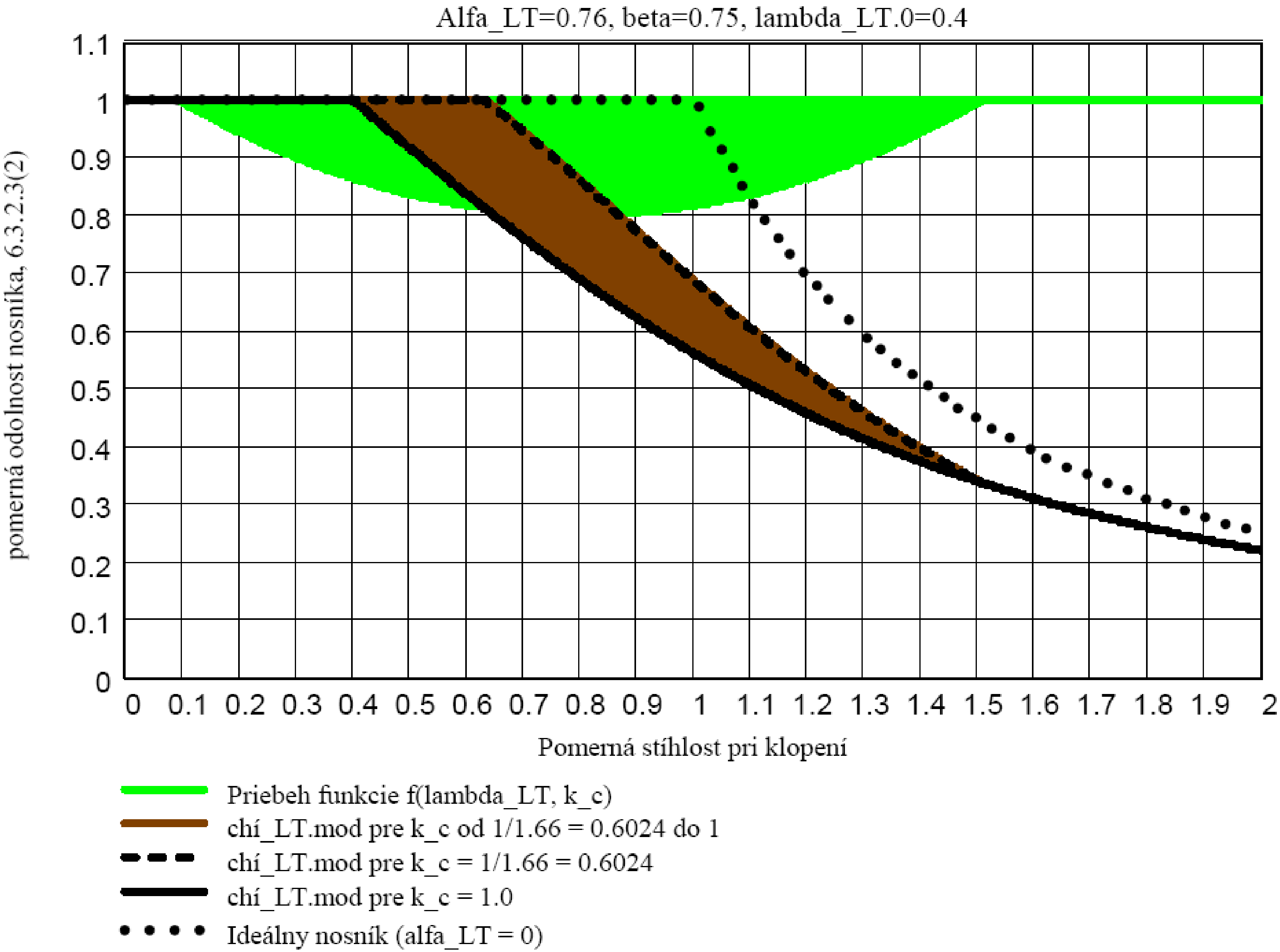
Parameter *f* ako funkcia pomernej štíhlosti $\lambda._{LT}$ a korekčného faktora k_c:



Vzperné krivky klopenia *a*: $\alpha := 0.21$ $\beta := 0.75$ $\lambda\_{LT.0} := 0.4$

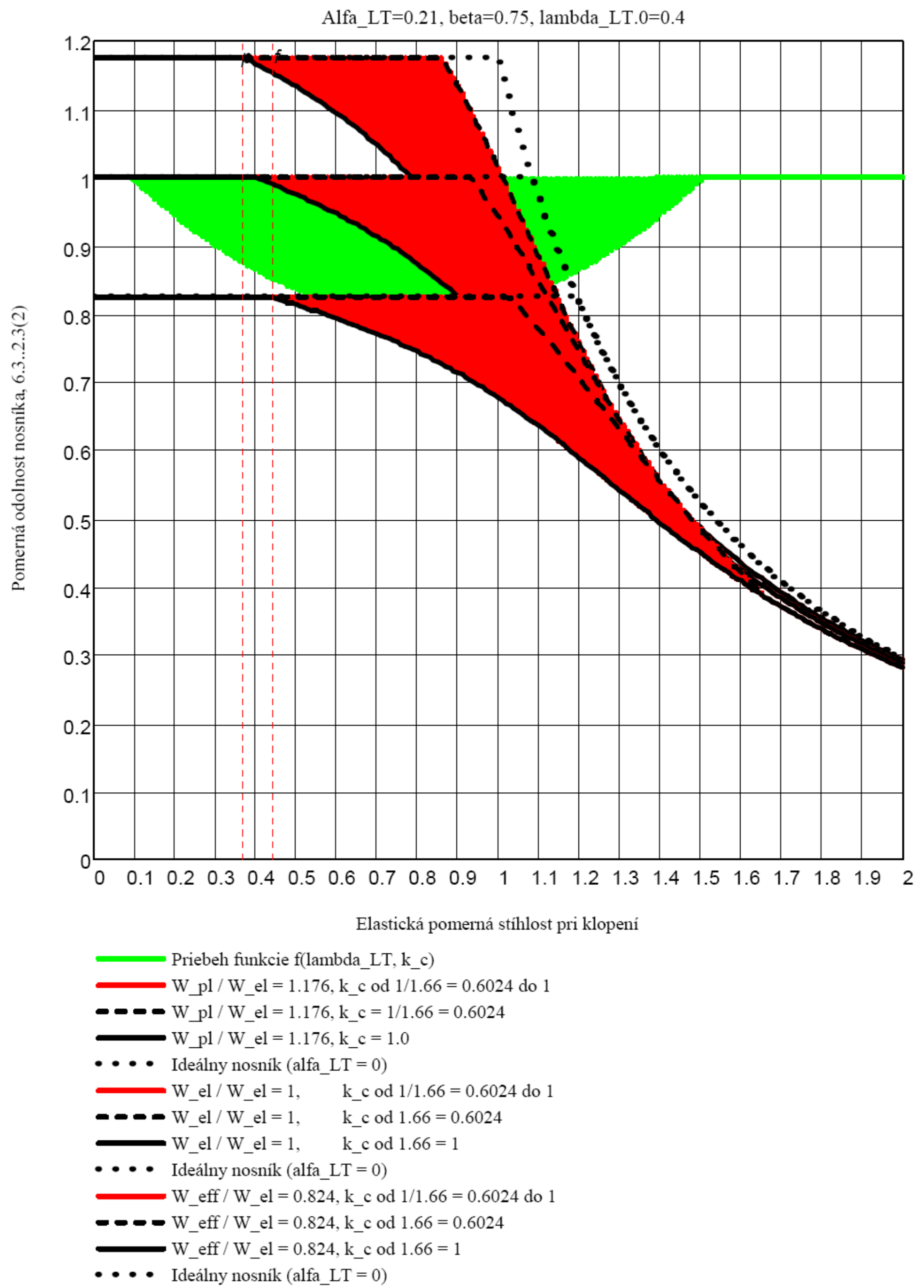


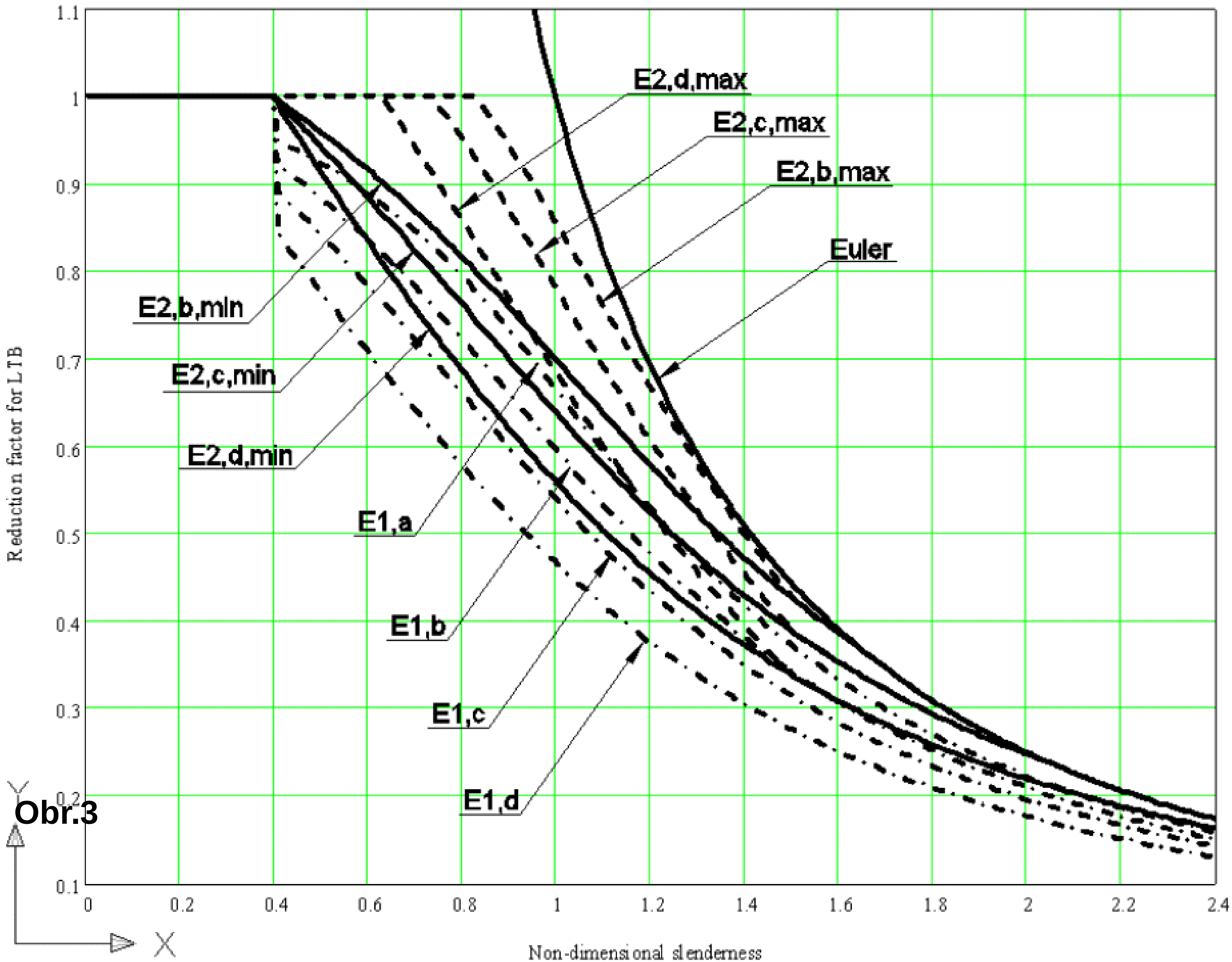
Vzperné krivky klopenia *d*: $\alpha := 0.76$ $\beta := 0.75$ $\lambda\_{LT.0} := 0.4$



Parameter β = 0.75, parameter $\lambda_{\_LT.0}$ = 0.4 $\beta := 0.75$ $\lambda_{\_LT.0} := 0.4$

Vzperné krivky klopenia *a* (α_{LT} = 0.21): $\alpha_{LT} := 0.21$





Reduction factor for LTB
Non-dimensional slenderness
E2,d,max
E2,c,max
E2,b,max
Euler
E2,b,min
E2,c,min
E2,d,min
E1,a
E1,b
E1,c
E1,d
Y
X
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1

Obr.3

6.3.2.4 Simplified assessment methods for beams with restraints in buildings

$$\bar{\lambda}_f = \frac{k_c L_c}{i_{f,z} \lambda_1} \leq \bar{\lambda}_{c0} \frac{M_{c,Rd}}{M_{y,Ed}}$$

$$M_{b,Rd} = k_{f\ell} \chi \, M_{c,Rd} \quad \text{but} \quad M_{b.Rd} \leq M_{c.Rd}$$

Theory of the 2. order of beams with imperfection

5.3.4 Member imperfections

(3) For a second order analysis taking account of lateral torsional buckling of a member in bending the imperfections may be adopted as $ke_{0,d}$, where $e_{0,d}$ is the equivalent initial bow imperfection of the weak axis of the profile considered. In general an additional torsional imperfection need not to be allowed for.

Buckling curve acc. to Table 6.1	elastic analysis e_0 / L	plastic analysis e_0 / L
a_0	1 / 350	1 / 300
a	1 / 300	1 / 250
b	1 / 250	1 / 200
c	1 / 200	1 / 150
d	1 / 150	1 / 100

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Table 6.2: Selection of buckling curve for a cross-section

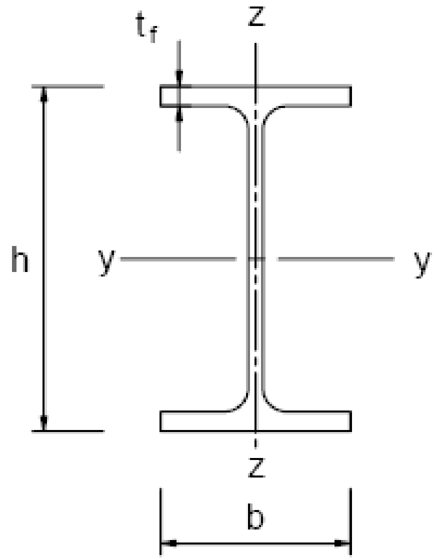
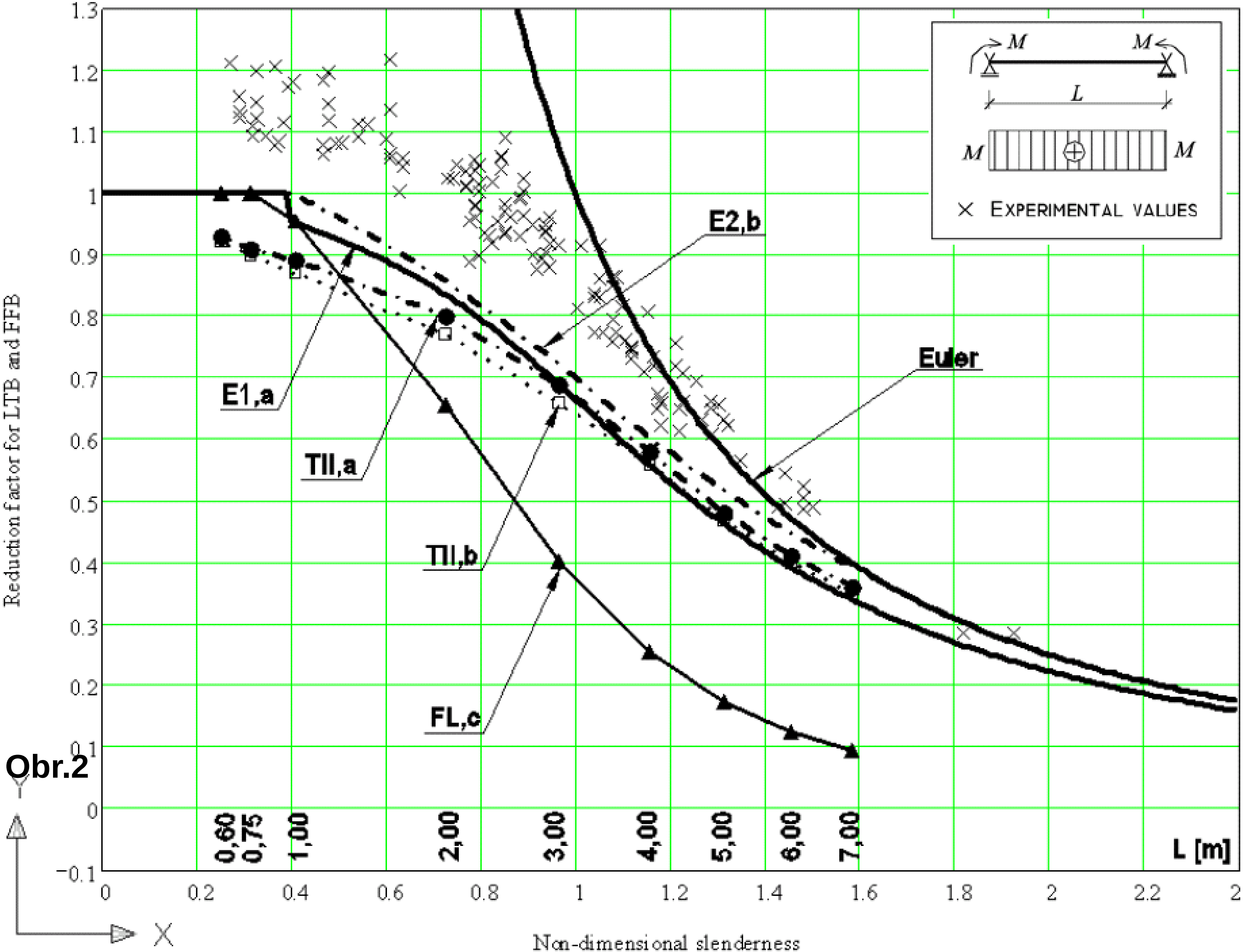
Cross section		Limits		Buckling about axis	Buckling curve	
					S 235 S 275 S 355 S 420	S 460
Rolled sections		$h/b > 1,2$	$t_f \leq 40$ mm	$y - y$ $z - z$	a b	a_0 a_0
			$40 \text{ mm} < t_f \leq 100$	$y - y$ $z - z$	b c	a a
		$h/b \leq 1,2$	$t_f \leq 100$ mm	$y - y$ $z - z$	b c	a a
			$t_f > 100$ mm	$y - y$ $z - z$	d d	c c

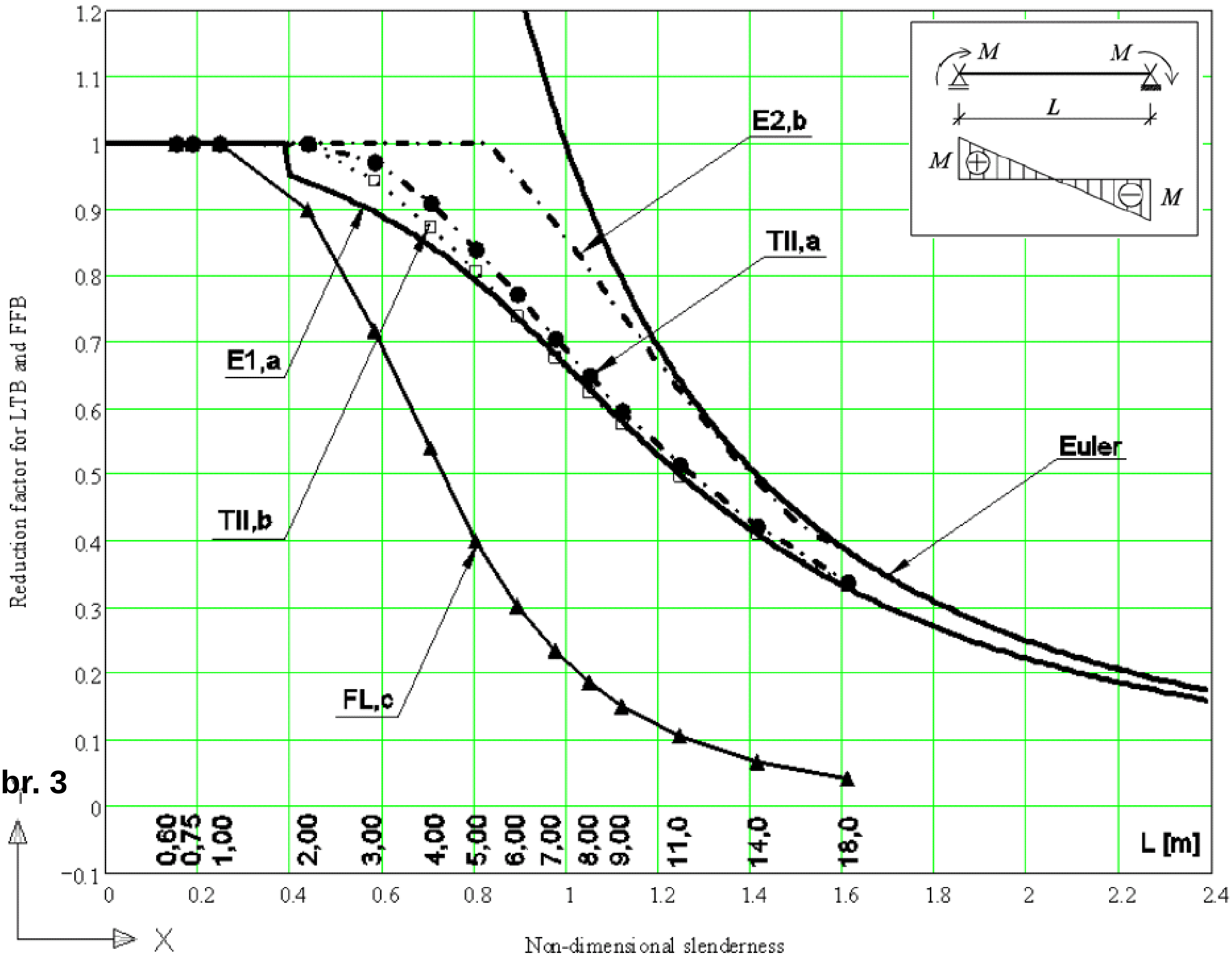
Table 6.4: Recommended values for lateral torsional buckling curves for cross-sections using equation (6.56)

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**a** **b**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**
Other cross-sections	-	**d**

Table 6.5: Recommendation for the selection of lateral torsional buckling curve for cross sections using equation (6.57)

Cross-section	Limits	Buckling curve
Rolled I-sections	$h/b \leq 2$ $h/b > 2$	**b** **c**
Welded I-sections	$h/b \leq 2$ $h/b > 2$	**c** **d**


Reduction factor for LTB and FFB
Non-dimensional slenderness
L [m]
M
L
EXPERIMENTAL VALUES
E2,b
Euler
E1,a
TII,a
TII,b
FL,c
0,60
0,75
1,00
2,00
3,00
4,00
5,00
6,00
7,00
Y
X
Obr.2



Reduction factor for LTB and FFB
Non-dimensional slenderness
L [m]
E2,b
TII,a
E1,a
TII,b
FL,c
Euler
M
L
0,60
0,75
1,00
2,00
3,00
4,00
5,00
6,00
7,00
8,00
9,00
11,0
14,0
18,0
X

Obr. 3

Reduction factors χ_{LT}

Eurocode: Perry – Robertson formula

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \quad \text{for} \quad 0.4 < \overline{\lambda}_{LT} \tag{3}$$

where $\phi_{LT} = 0.5\left(1 + \eta_{LT} + \overline{\lambda}_{LT}^2\right)$

$\eta_{LT} = \alpha_{LT}\left(\overline{\lambda}_{LT} - \overline{\lambda}_P\right)$ is the Perry factor (symbols η_{LT} and $\overline{\lambda}_P$ are not used in ENV 1993-1-1),

$\overline{\lambda}_{LT} = \sqrt{\frac{\beta_w M_{pl}}{M_{cr}}}$ is the non-dimensional slenderness.

DIN 18 800: Merchant – Rankine formula

$0.4 < \overline{\lambda}_{LT}$ $\qquad$ $\chi_{LT} = \left(\frac{1}{1 + \overline{\lambda}^{2n}}\right)^{\frac{1}{n}}$ $\qquad$ $n = 2.5$

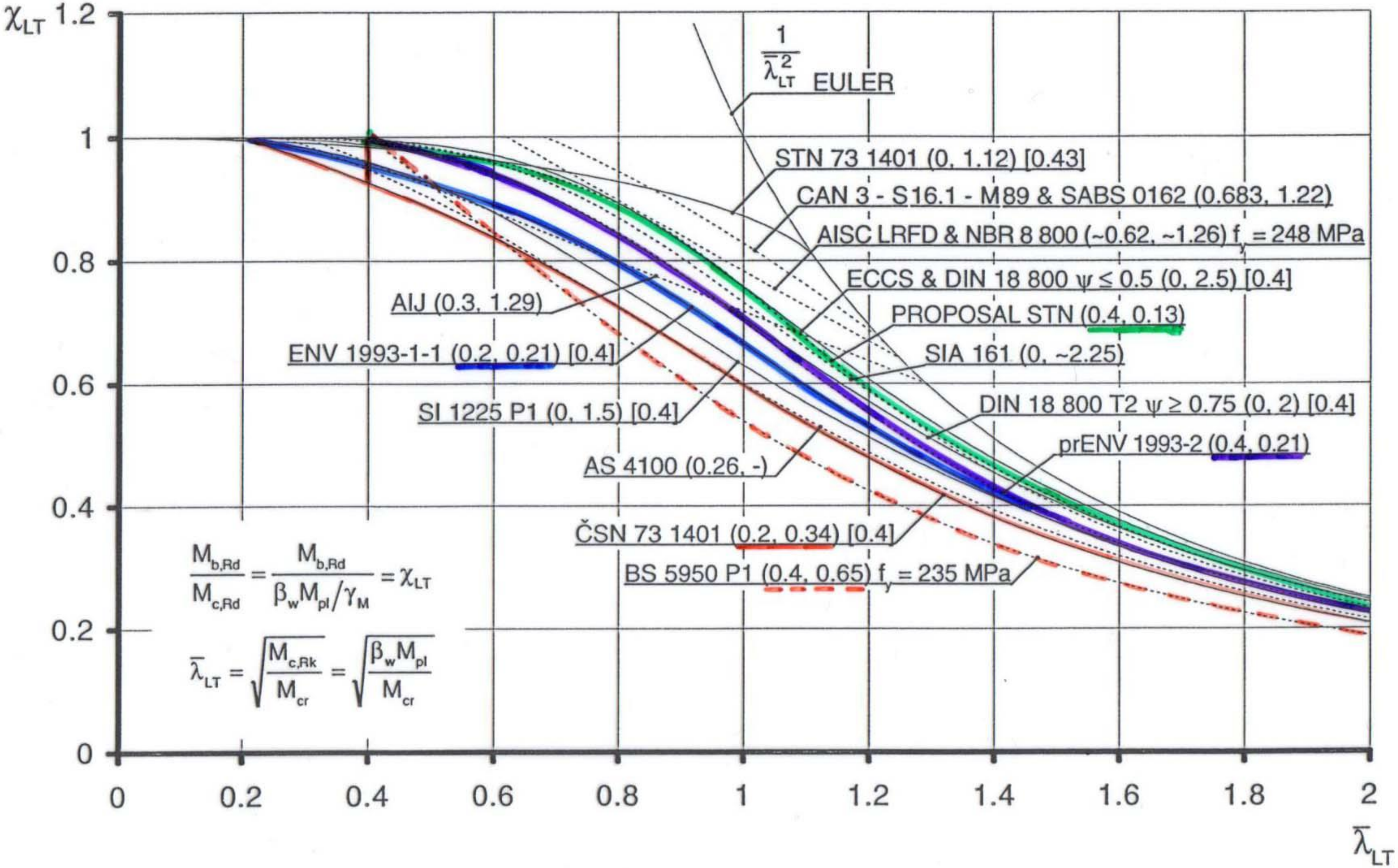


Fig. 1 Comparison of design curves for lateral torsional buckling of rolled beams. Values in brackets mean: a) ($\bar{\lambda}_p$, $\alpha_{LT} < 1$) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, $n \geq 1$) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is $\chi_{LT} = 1$. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

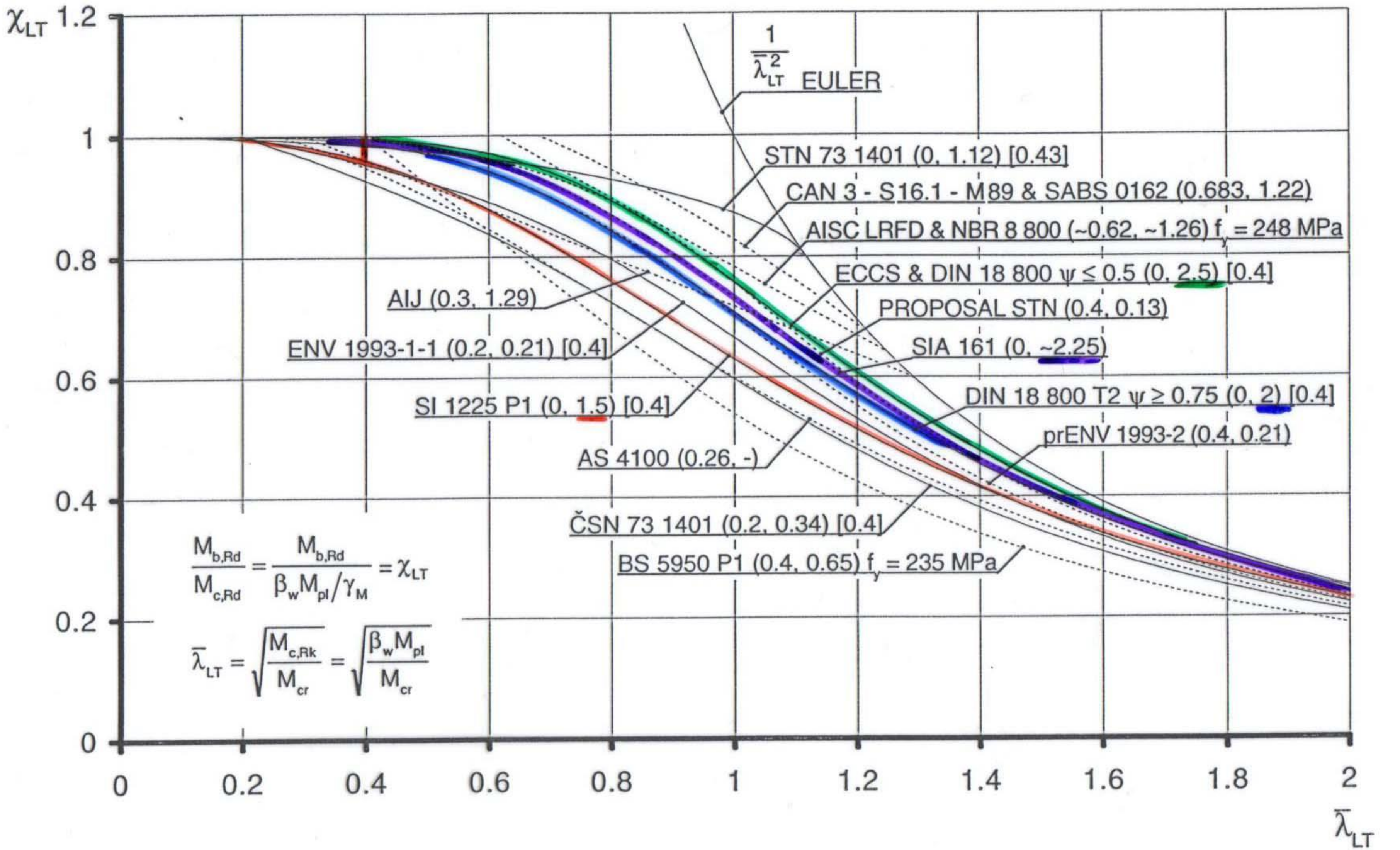


Fig. 1 Comparison of design curves for lateral torsional buckling of rolled beams. Values in brackets mean: a) ($\bar{\lambda}_p$, α_{LT}<1) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, n≥1) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is χ_{LT} = 1. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

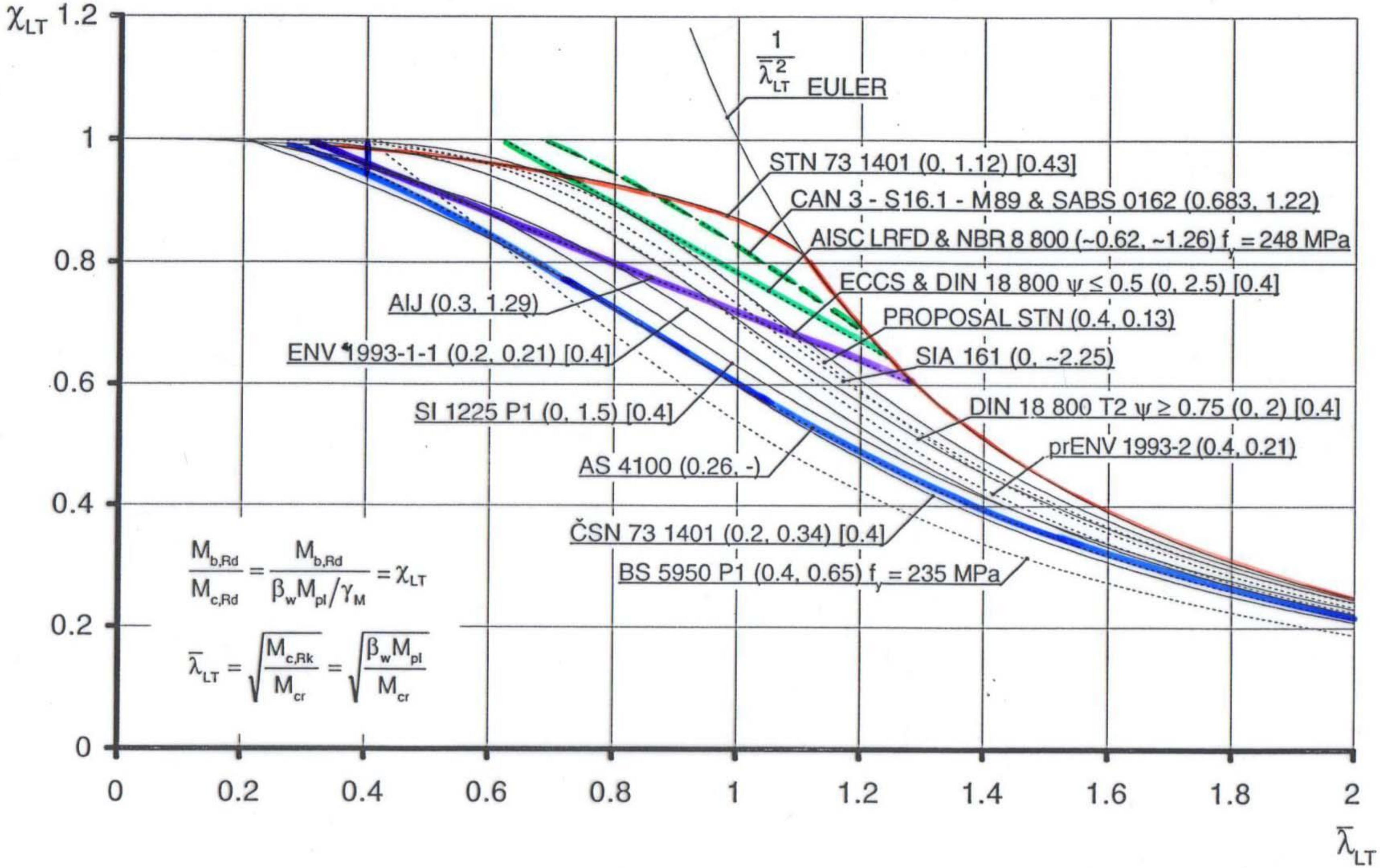


Fig. 1 Comparison of design curves for lateral torsional buckling of rolled beams. Values in brackets mean: a) ($\bar{\lambda}_p$, $\alpha_{LT}<1$) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, $n\geq1$) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is $\chi_{LT} = 1$. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

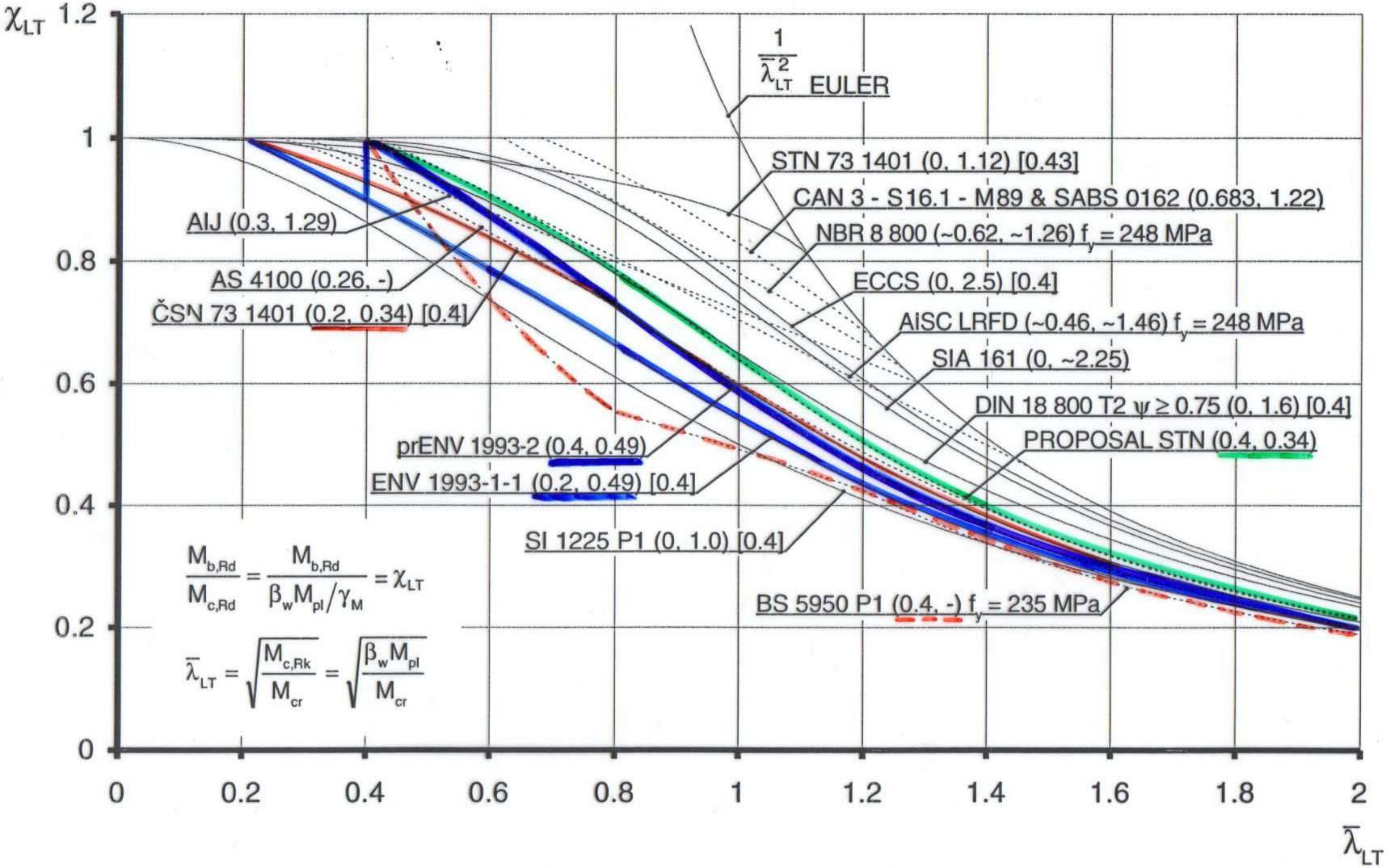


Fig. 2 Comparison of design curves for lateral torsional buckling of welded beams. Values in brackets mean: a) ($\bar{\lambda}_p$, $\alpha_{LT} < 1$) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, $n \geq 1$) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is $\chi_{LT} = 1$. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

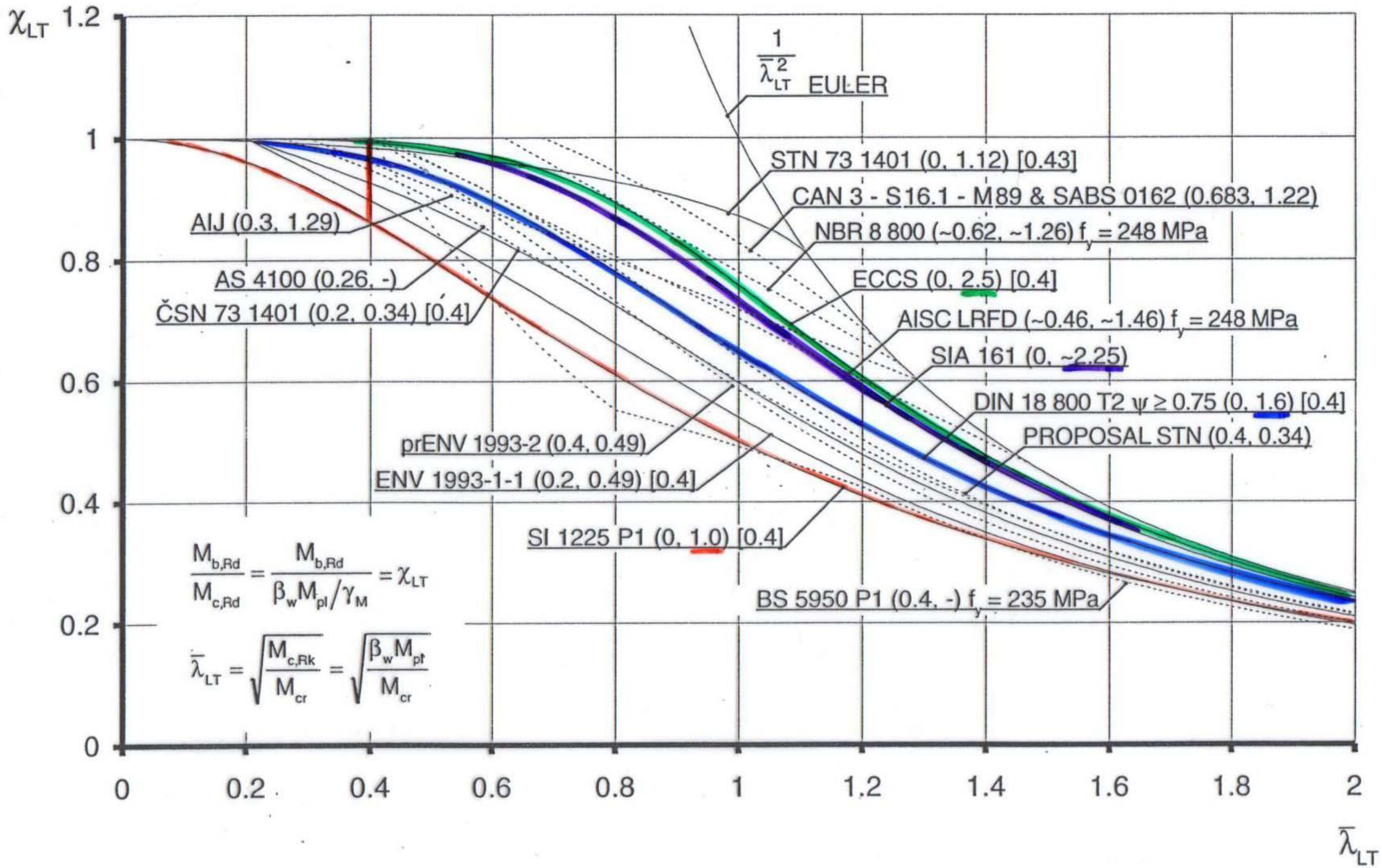


Fig. 2 Comparison of design curves for lateral torsional buckling of welded beams. Values in brackets mean: a) ($\bar{\lambda}_p$, $\alpha_{LT}<1$) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, $n \geq 1$) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is $\chi_{LT} = 1$. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

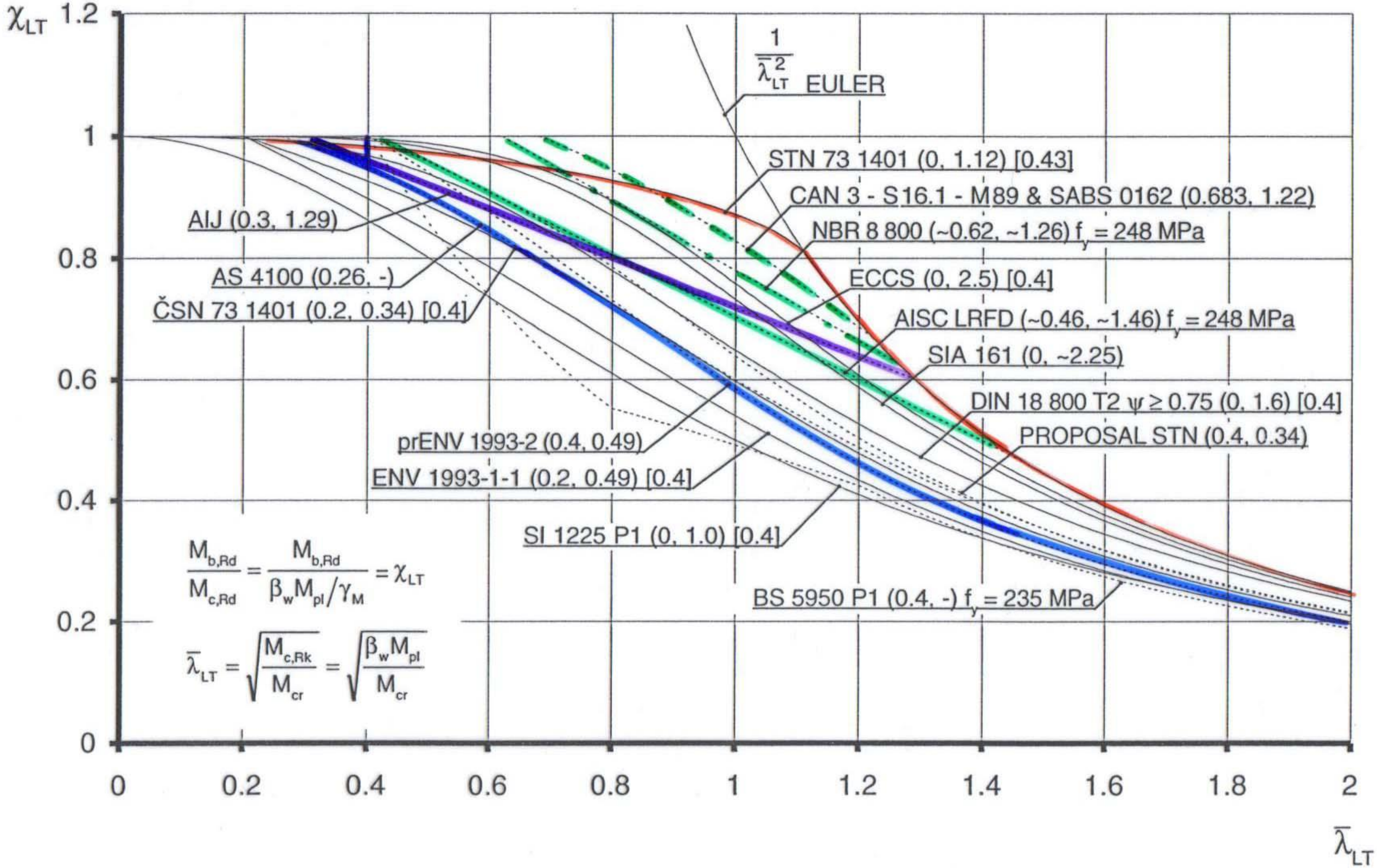


Fig. 2 Comparison of design curves for lateral torsional buckling of welded beams. Values in brackets mean: a) ($\bar{\lambda}_p$, $\alpha_{LT} < 1$) in Perry-Robertson approach, b) ($\bar{\lambda}_p$, $n \geq 1$) in Merchant-Rankine approach, c) ($\bar{\lambda}_p$, $\bar{\lambda}_{EULER}$) for AIJ, AISC, CAN, NBR and STN codes, d) [$\bar{\lambda}_{LIM}$]. For $\bar{\lambda}_{LT} \leq \bar{\lambda}_{LIM}$ is $\chi_{LT} = 1$. There is step in point $\bar{\lambda}_{LIM}$ if $\bar{\lambda}_p \neq \bar{\lambda}_{LIM}$.

EUROPEAN STANDARD

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EN 1999-1-1

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ICS 91.010.30; 91.080.10

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English Version

Eurocode 9 - Design of aluminium structures - Part 1-1: General structural rules

Eurocode 9 - Calcul des structures en aluminium - Partie 1-1: Règles générales

Eurocode 9 - Bemessung und Konstruktion von Aluminiumtragwerken - Teil 1-1: Allgemeine Bemessungsregeln

Resistance of cross-section

$$\frac{M_{\mathrm{Ed}}}{M_{\mathrm{Rd}}} \leq 1{,}0$$

$M_{\mathrm{u,Rd}} = W_{\mathrm{net}} f_{\mathrm{u}} / \gamma_{\mathrm{M2}}$ in a net section and

$M_{\mathrm{c,Rd}} = \alpha W_{\mathrm{el}} f_{o} / \gamma_{\mathrm{M1}}$ at each cross-section

Table 6.4 - Values of shape factor α

Cross-section class	Without welds	With longitudinal welds
1	W_{pl} / W_{el} *)	$W_{pl,\,haz} / W_{el}$ *)
2	W_{pl} / W_{el}	$W_{pl,haz} / W_{el}$
3	$\alpha_{3,u}$	$\alpha_{3,w}$
4	W_{eff} / W_{el}	$W_{eff,haz} / W_{el}$
*) NOTE These formulae are on the conservative side. For more refined value, recommendations are given in Annex F		

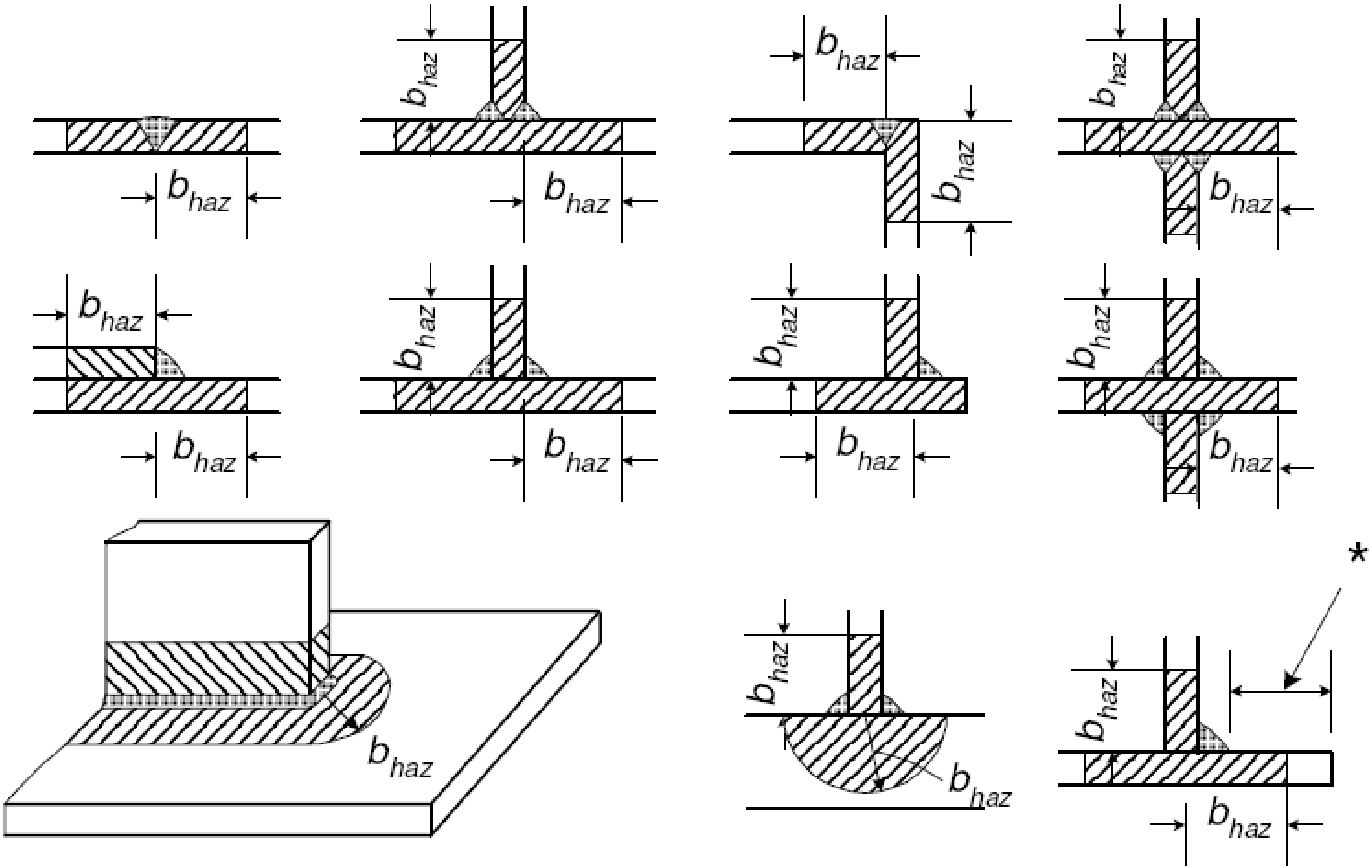


Figure 6.6 - The extent of heat-affected zones (HAZ)

6.3.2.1 Buckling resistance

$$\frac{M_{\mathrm{Ed}}}{M_{\mathrm{b,Rd}}} \leq 1{,}0$$

$$M_{\mathrm{b,Rd}} = \chi_{\mathrm{LT}} \alpha W_{\mathrm{el,y}} f_o / \gamma_{\mathrm{M1}}$$

6.3.2.2 Reduction factor for lateral torsional buckling

$$\chi_{\mathrm{LT}} = \frac{1}{\phi_{\mathrm{LT}} + \sqrt{\phi_{\mathrm{LT}}^2 - \overline{\lambda}_{\mathrm{LT}}^2}} \quad \text{but } \chi_{\mathrm{LT}} \leq 1$$

where:

$$\phi_{\mathrm{LT}} = 0{,}5\left[1 + \alpha_{\mathrm{LT}}(\overline{\lambda}_{\mathrm{LT}} - \overline{\lambda}_{0,\mathrm{LT}}) + \overline{\lambda}_{LT}^2\right]$$

α_{LT} is an imperfection factor

$\overline{\lambda}_{\mathrm{LT}}$ is the relative slenderness

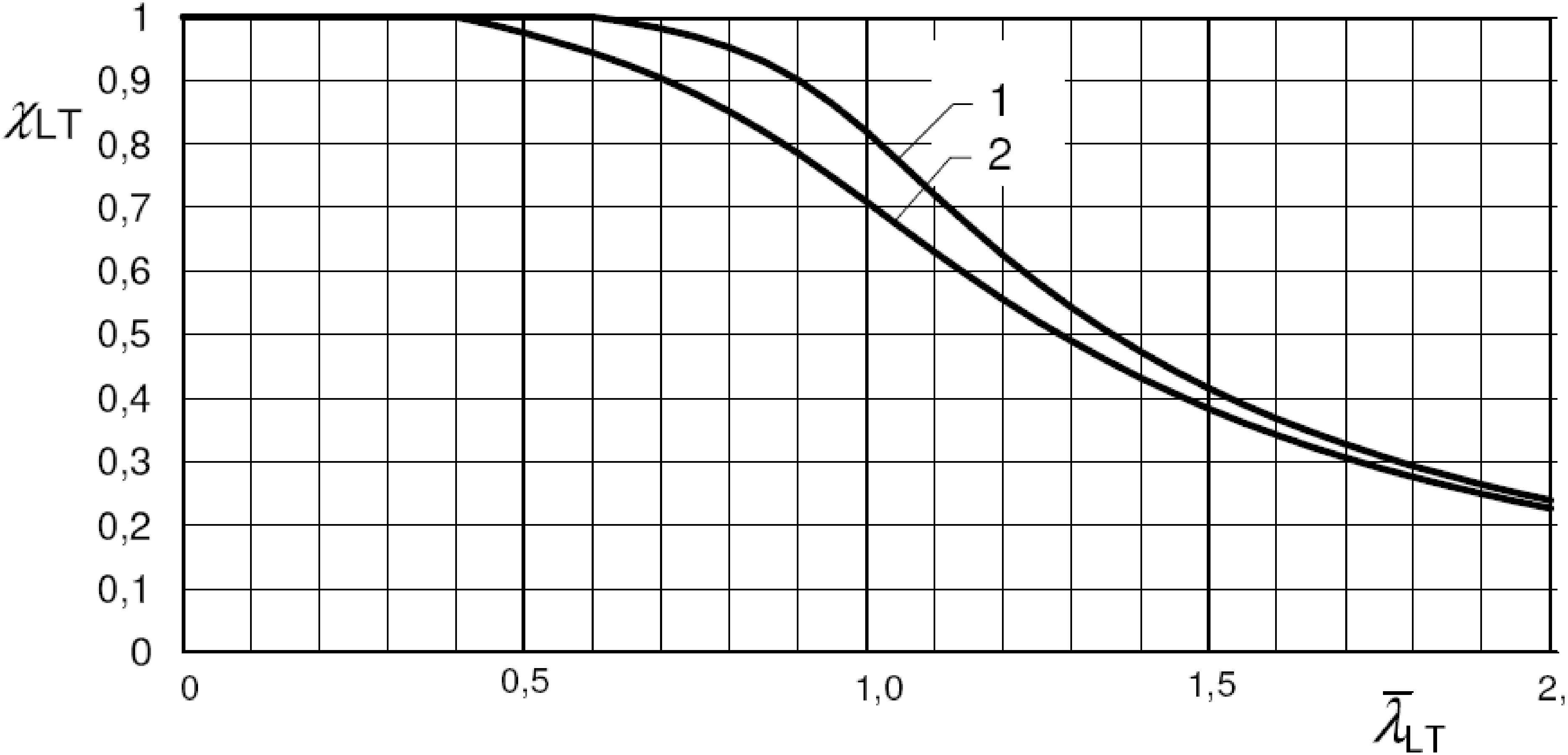
$\overline{\lambda}_{0,\mathrm{LT}}$ is the limit of the horizontal plateau

M_{cr} is the elastic critical moment for lateral-torsional buckling.

(2) The value of α_{LT} and $\overline{\lambda}_{0,LT}$ should be taken as:

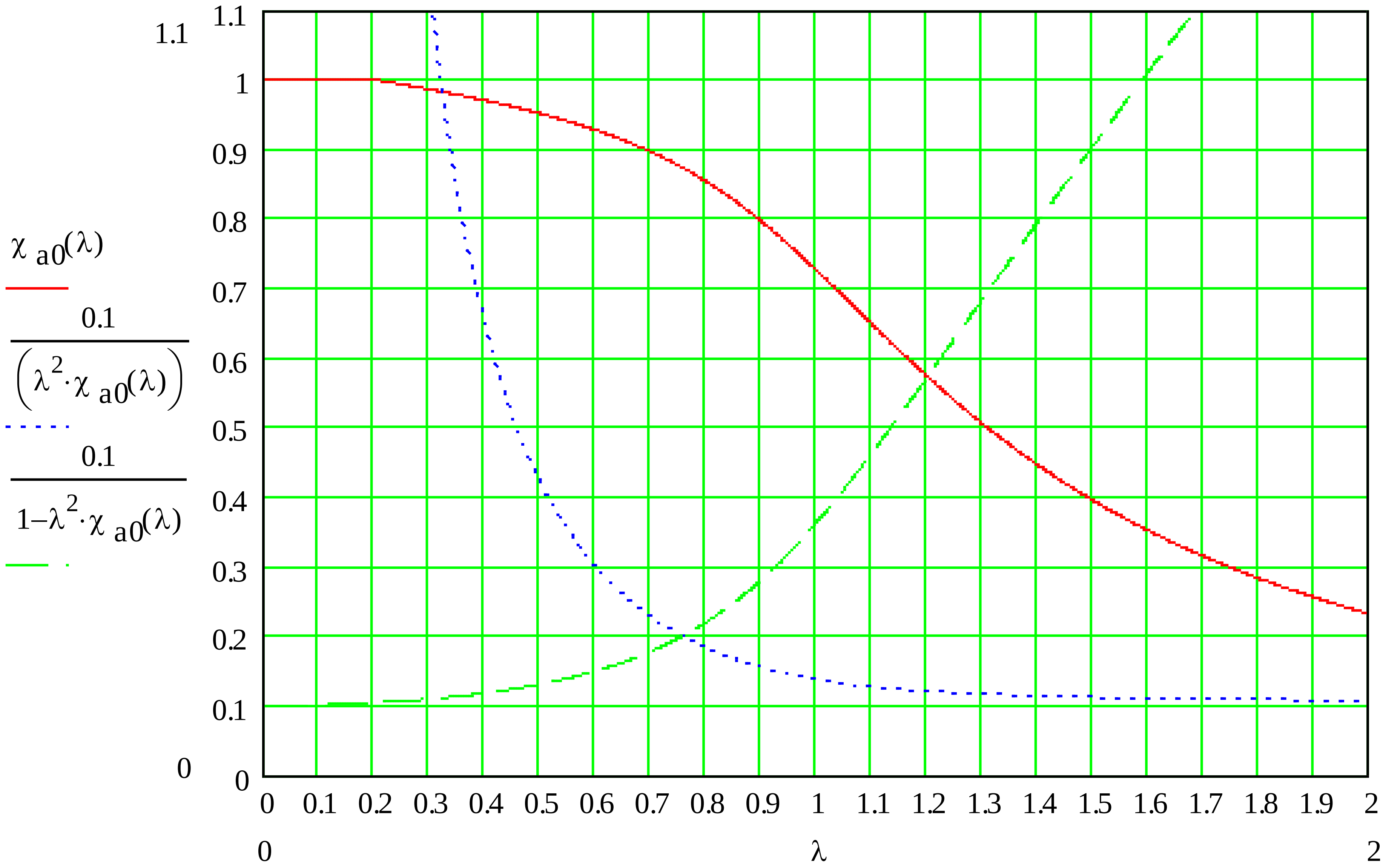
$\alpha_{LT} = 0,10$ and $\overline{\lambda}_{0,LT} = 0,6$ for class 1 and 2 cross-sections

$\alpha_{LT} = 0,20$ and $\overline{\lambda}_{0,LT} = 0,4$ for class 3 and 4 cross-sections.

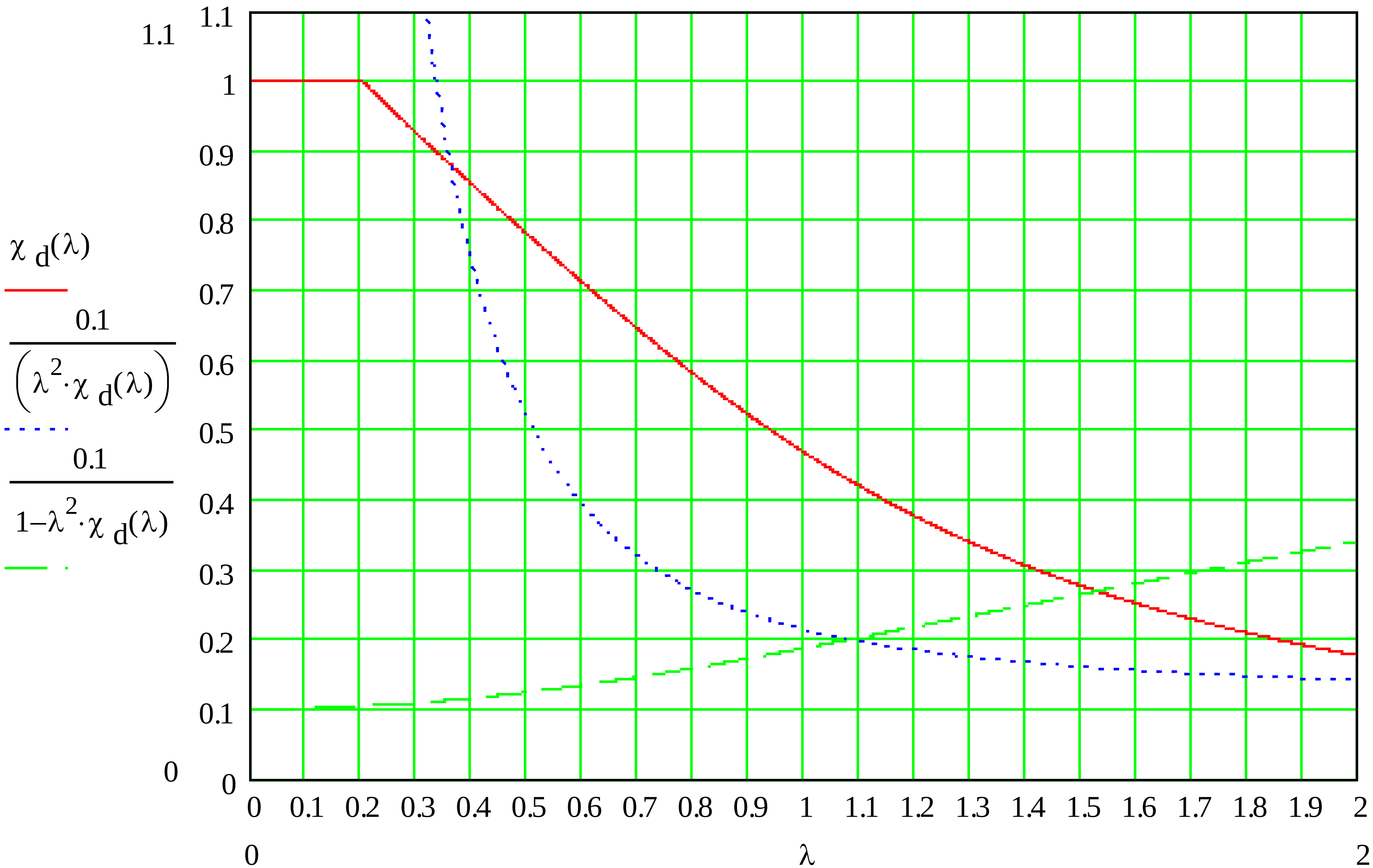


1 Class 1 and 2 cross sections,
2 Class 3 and 4 cross sections

Figure 6.13 - Reduction factor for lateral-torsional buckling



$\chi_{a0}(\lambda)$
$\frac{0.1}{\left(\lambda^2\cdot\chi_{a0}(\lambda)\right)}$
$\frac{0.1}{1-\lambda^2\cdot\chi_{a0}(\lambda)}$
1.1
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
0
λ
2



χ d(λ)
0.1 / (λ²·χ d(λ))
0.1 / (1−λ²·χ d(λ))
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
0 λ 2