Lateral torsional stability of timber beams

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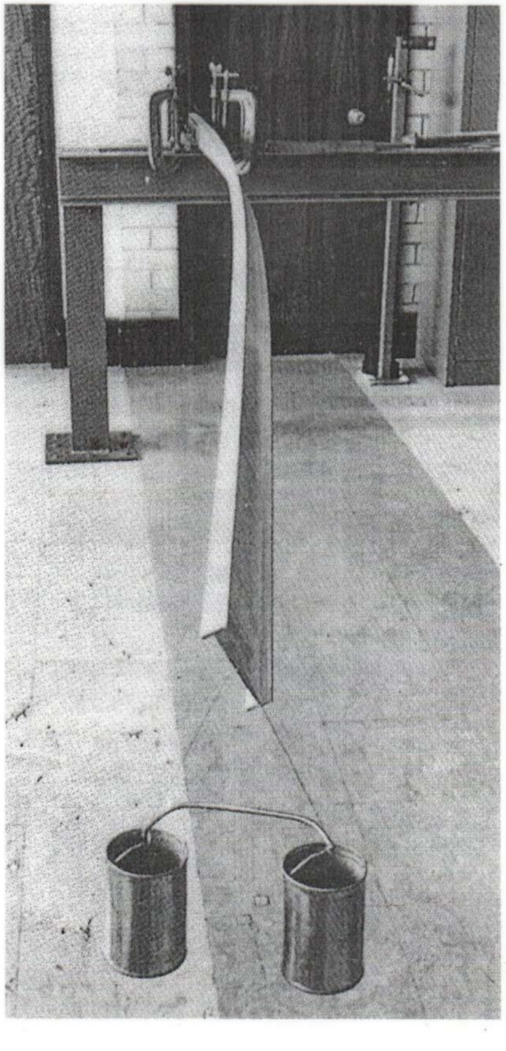
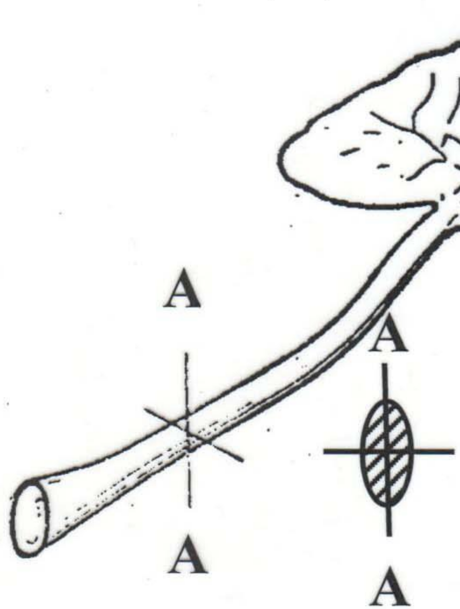
Lateral Torsional Buckling



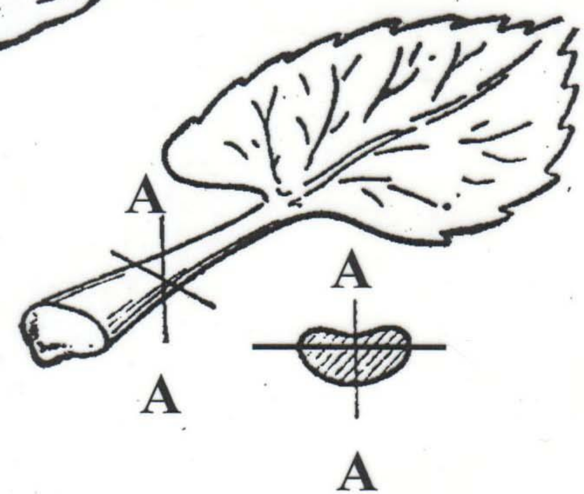


Osika

Die Espe

ASPEN

OSIKA



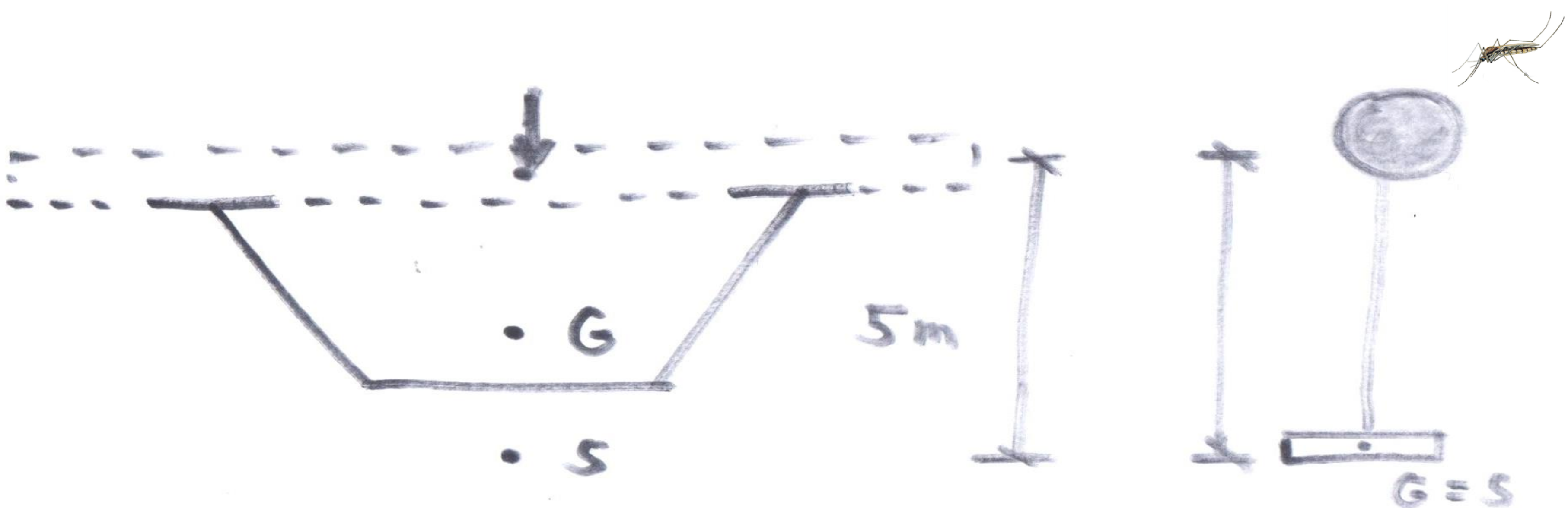
Breza

Die Birke

BIRCH

BRZOZA

LTB of member will occur even if it is loaded in the plane of minor stiffness when point of load application is too high over shear center



This phenomenon is called „mosquito wings“ effect

Structure magazine, February 2008

Avoiding Structural Failures During Construction

Part 2

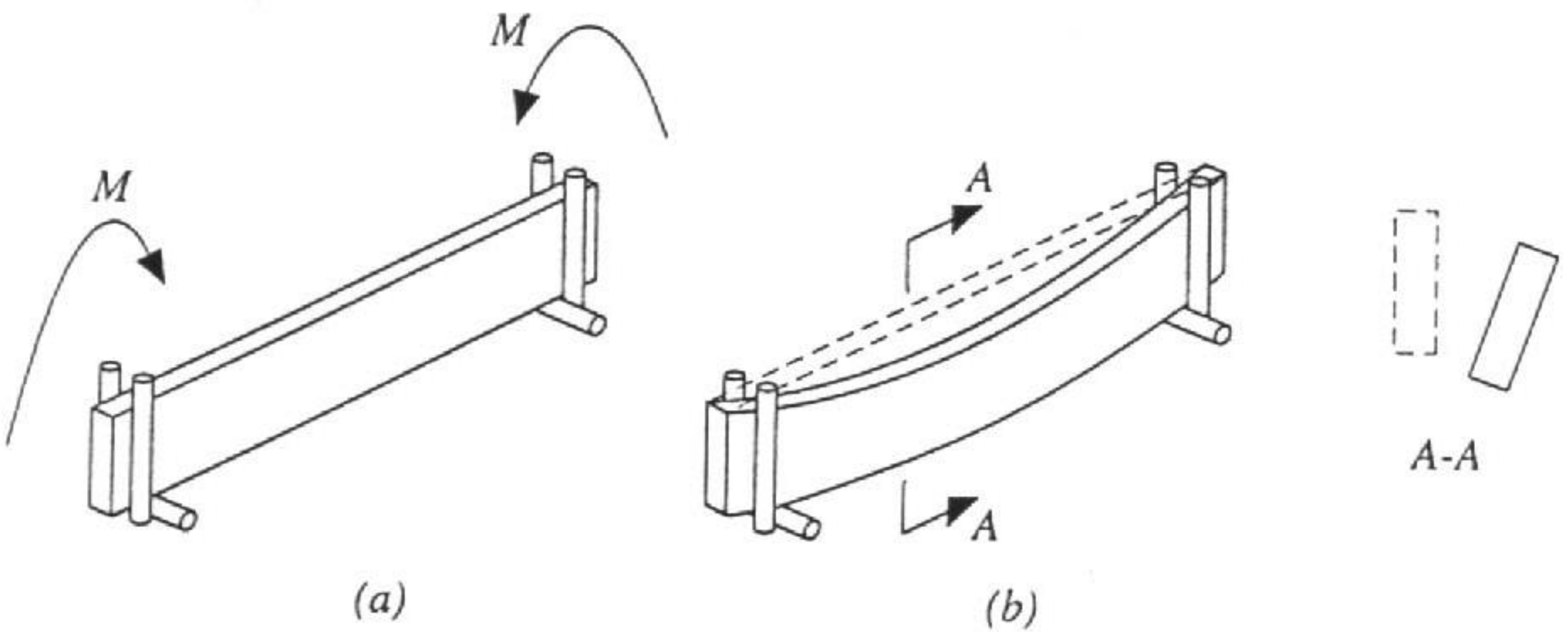
By David B. Peraza, P.E. and Dennis M. McCann, Ph.D., P.E.

Figure 4: The Marcy Pedestrian Bridge following collapse during placement of the concrete deck.

Elastic critical moment M_{cr}

„fork“ boundary conditions, uniform bending moment M,

uniform nonwarping cross-section: warping stiffness $EI_w = 0\ kNm^4$



$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L}$$

EI_z bending stiffness

GI_t torsional stiffness

L span

Taking into account all influences

$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L}$$

$$M_{cr} = \mu_{cr} \frac{\pi\sqrt{EI_z GI_t}}{L}$$

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}}\left[\sqrt{1+\kappa_{\mathrm{wt}}^2 + (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})^2} - (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})\right]$$

$$\kappa_{\mathrm{wt}} = \frac{\pi}{k_{\mathrm{w}}L}\sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{t}}}} \qquad \zeta_{\mathrm{g}} = \frac{\pi z_{\mathrm{g}}}{k_{\mathrm{z}}L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}} \qquad \zeta_{\mathrm{j}} = \frac{\pi z_{\mathrm{j}}}{k_{\mathrm{z}}L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}}$$

$$I_{\mathrm{w}} = (1-\psi_{\mathrm{f}}^2)I_{\mathrm{z}}(h_{\mathrm{s}}/2)^2 \qquad \psi_{\mathrm{f}} = \frac{I_{\mathrm{fc}} - I_{\mathrm{ft}}}{I_{\mathrm{fc}} + I_{\mathrm{ft}}} \qquad z_{\mathrm{g}} = z_{\mathrm{a}} - z_{\mathrm{s}}$$

$$z_{\mathrm{j}} = z_{\mathrm{s}} - \frac{0{,}5}{I_{\mathrm{y}}}\int_A (y^2 + z^2)\, z\, dA \qquad z_{\mathrm{j}} = 0{,}45\psi_{\mathrm{f}} h_{\mathrm{s}}\left(1 + \frac{c}{2h_{\mathrm{f}}}\right)$$

For symmetric cross-section to axis y-y is $z_j = 0$

C1, C2, C3 are coefficients depending on type of loading and boundary conditions

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}}\left[\sqrt{1+\kappa_{\mathrm{wt}}^2 + (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})^2} - (C_2\zeta_{\mathrm{g}} - C_3\zeta_{\mathrm{j}})\right]$$

(1) For beams with uniform cross-sections symmetrical about major axis, centrally symmetric and doubly symmetric cross-sections loaded perpendicular to the major axis in the plane going through the shear centre,

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}}\left[\sqrt{1+\kappa_{wt}^2 + (C_2\zeta_g)^2} - C_2\zeta_g\right]$$

(2) For end-moment loading $C_2 = 0$ and for transverse loads applied at the shear centre $z_{\mathrm{g}} = 0$. For these cases:

$$\mu_{\mathrm{cr}} = \frac{C_1}{k_{\mathrm{z}}}\sqrt{1+\kappa_{wt}^2}$$

$$\kappa_{\mathrm{wt}} = 0: \ \mu_{\mathrm{cr}} = C_1 / k_{\mathrm{z}}$$

ENV 1993-1-1, ENV 1999-1-1

Tabulka F.1.1 Hodnoty součinitelů C_1, C_2 a C_3 v závislosti na velikosti součinitele k. Zatížení koncovými momenty

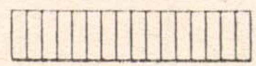
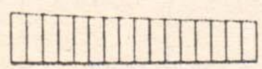
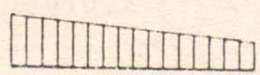
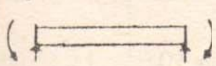
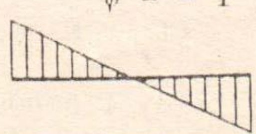
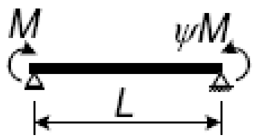
Zatížení a podmínky podepření	Průběh ohybového momentu	Hodnoty k	Hodnoty součinitelů		
			C_1	C_2	C_3
	$\psi = +1$ 	1,0 0,7 0,5	1,000 1,000 1,000	–	1,000 1,113 1,144
	$\psi = +3/4$ 	1,0 0,7 0,5	1,141 1,270 1,305	–	0,998 1,565 2,283
	$\psi = +1/2$ 	1,0 0,7 0,5	1,323 1,473 1,514	–	0,992 1,556 2,271
M ψ M 	$\psi = -1$ 	1,0 0,7 0,5	2,752 3,063 3,149	–	0,000 0,000 0,000

Table I.1 - Values of factors C_1 and C_3 corresponding to various end moment ratios ψ, values of buckling length factor k_z and cross-section parameters ψ_f and κ_{wt}.
End moment loading of the simply supported beam with buckling length factors $k_y = 1$ for major axis bending and $k_w = 1$ for torsion

Loading and support conditions. Cross-section monosymmetry factor ψ_f	Bending moment diagram. End moment ratio ψ. M-side, ψM-side	k_z 2)	Values of factors: C_1 1) $C_{1,0}$	C_1 1) $C_{1,1}$	C_3: $\psi_f = -1$	C_3: $-0{,}9 \le \psi_f \le 0$	C_3: $0 \le \psi_f \le 0{,}9$	C_3: $\psi_f = 1$
 $k_y = 1$, $k_w = 1$ Beam M-side: $\psi_f \ge 0$, $\psi_f \le 0$	M_{cr} $\psi = +1$	1,0	1,000	1,000	1,000			
		0,7L	1,016	1,100	1,025		1,000	
		0,7R	1,016	1,100	1,025		1,000	
		0,5	1,000	1,127	1,019			
	M_{cr} $\psi = +3/4$	1,0	1,139	1,141	1,000			
		0,7L	1,210	1,313	1,050		1,000	
		0,7R	1,109	1,201	1,000			
		0,5	1,139	1,285	1,017			
	M_{cr} $\psi = +1/2$	1,0	1,312	1,320	1,150	1,000		
		0,7L	1,480	1,616	1,160		1,000	
		0,7R	1,213	1,317	1,000			
		0,5	1,310	1,482	1,150	1,000		
	M_{cr} $\psi = -1$	1,0	2,555	2,733	2,00	$-\psi_f$		-2,00
		0,7L	1,921	2,103	1,55	0,380	-0,580	-1,55
		0,7R	1,921	2,103	1,55	0,580	-0,380	-1,55
		0,5	2,223	2,390	1,88	$0{,}125 - 0{,}7\psi_f$	$-0{,}125 - 0{,}7\psi_f$	-1,88

ENV 1993-1-1, ENV 1999-1-1

Tabulka F.1.2 Hodnoty součinitelů C_1, C_2 a C_3 v závislosti na velikosti součinitele k. Případy příčného zatížení

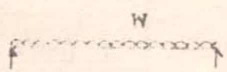
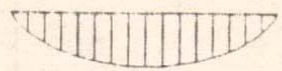
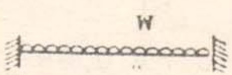
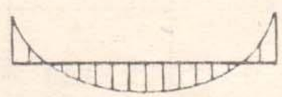
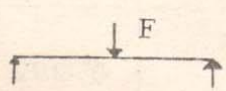
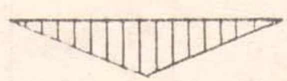
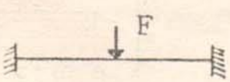
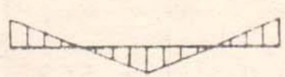
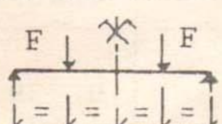
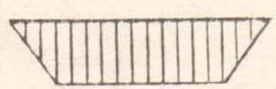
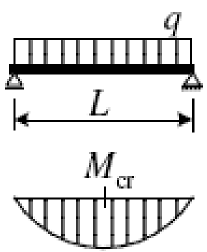
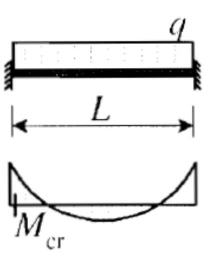
Zatížení a podmínky podepření	Průběh ohybového momentu	Hodnoty k	Hodnoty součinitelů		
			C_1	C_2	C_3
		1,0 0,5	1,132 0,972	0,459 0,304	0,525 0,980
		1,0 0,5	1,285 0,712	1,562 0,652	0,753 1,070
		1,0 0,5	1,365 1,070	0,553 0,432	1,730 3,050
		1,0 0,5	1,565 0,938	1,267 0,715	2,640 4,800
		1,0 0,5	1,046 1,010	0,430 0,410	1,120 1,890

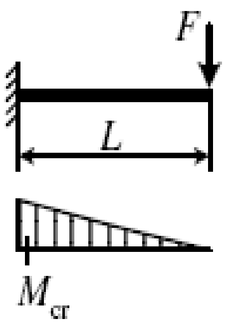
Table I.2 - Values of factors C_1, C_2 and C_3 corresponding to various transverse loading cases, values of buckling length factors k_y, k_z, k_w, cross-section monosymmetry factor ψ_f and torsion parameter κ_{wt}.

Loading and support conditions	Buckling length factors			Values of factors							
				C_1 [1)]		C_2			C_3		
	k_y	k_z	k_w	$C_{1,0}$	$C_{1,1}$	$\psi_f = -1$	$-0,9 \le \psi_f \le 0,9$	$\psi_f = 1$	$\psi_f = -1$	$-0,9 \le \psi_f \le 0,9$	$\psi_f = 1$
	1	1	1	1,127	1,132	0,33	0,459	0,50	0,93	0,525	0,38
	1	1	0,5	1,128	1,231	0,33	0,391	0,50	0,93	0,806	0,38
	1	0,5	1	0,947	0,997	0,25	0,407	0,40	0,84	0,478	0,44
	1	0,5	0,5	0,947	0,970	0,25	0,310	0,40	0,84	0,674	0,44
						$\psi_f = -1$	$-0,5 \le \psi_f \le 0,5$	$\psi_f = 1$	$\psi_f = -1$	$-0,5 \le \psi_f \le 0,5$	$\psi_f = 1$
	0,5	1	1	2,576	2,608	1,00	1,562	0,15	1,00	-0,859	-1,99
	0,5	0,5	1	1,490	1,515	0,56	0,900	0,08	0,61	-0,516	-1,20
	0,5	0,5	0,5	1,494	1,746	0,56	0,825	0,08	0,61	0,002712	-1,20

Cantilever

$$M_{cr} = \mu_{cr} \frac{\pi \sqrt{EI_z GI_t}}{L}$$

Table I.3 - Relative non-dimensional critical moment μ_{cr} for cantilever $(k_{\mathrm{y}} = k_{\mathrm{z}} = k_{\mathrm{w}} = 2)$ loaded by concentrated end load F.

Loading and support conditions	$\frac{\pi}{L}\sqrt{\frac{EI_{\mathrm{w}}}{GI_{\mathrm{t}}}} = k_w \kappa_{wt} = \kappa_{wt0}$	$\frac{\pi z_{\mathrm{g}}}{L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}} = k_z \zeta_g = \zeta_g$	(T) (C) (C) (T) $\frac{\pi z_{\mathrm{j}}}{L}\sqrt{\frac{EI_{\mathrm{z}}}{GI_{\mathrm{t}}}} = k_z \zeta_j = \zeta_{j0}$ (C) (T) (T) (C)						
			-4	-2	-1	0	1	2	4
	0	4	0,107	0,156	0,194	0,245	0,316	0,416	0,759
		2	0,123	0,211	0,302	0,463	0,759	1,312	4,024
		0	0,128	0,254	0,478	1,280	3,178	5,590	10,730
		-2	0,129	0,258	0,508	1,619	3,894	6,500	11,860
		-4	0,129	0,258	0,511	1,686	4,055	6,740	12,240
	0,5	4	0,151	0,202	0,240	0,293	0,367	0,475	0,899
		2	0,195	0,297	0,393	0,560	0,876	1,528	5,360
		0	0,261	0,495	0,844	1,815	3,766	6,170	11,295
		-2	0,329	0,674	1,174	2,423	4,642	7,235	12,595
		-4	0,364	0,723	1,235	2,529	4,843	7,540	13,100
	1	4	0,198	0,257	0,301	0,360	0,445	0,573	1,123
		2	0,268	0,391	0,502	0,691	1,052	1,838	6,345
		0	0,401	0,750	1,243	2,431	4,456	6,840	11,920
		-2	0,629	1,326	2,115	3,529	5,635	8,115	13,365
		-4	0,777	1,474	2,264	3,719	5,915	8,505	13,960

a) For $z_j = 0$, $z_g = 0$ and $\kappa_{wt0} \leq 8$: $\mu_{cr} = 1{,}27 + 1{,}14\,\kappa_{wt0} + 0{,}017\,\kappa_{wt0}^2$.

b) For $z_j = 0$, $-4 \leq \zeta_g \leq 4$ and $\kappa_{wt} \leq 4$, μ_{cr} may be calculated also from formulae (I.7) and (I.8), where the following approximate values of the factors C_1, C_2 should be used for the cantilever under tip load F:

$$C_1 = 2{,}56 + 4{,}675\,\kappa_{wt} - 2{,}62\,\kappa_{wt}^2 + 0{,}5\kappa_{wt}^3, \quad \text{if} \quad \kappa_{wt} \leq 2$$

$$C_1 = 5{,}55 \quad \text{if} \quad \kappa_{wt} > 2$$

$$C_2 = 1{,}255 + 1{,}566\kappa_{wt} - 0{,}931\,\kappa_{wt}^2 + 0{,}245\kappa_{wt}^3 - 0{,}024\kappa_{wt}^4, \quad \text{if} \quad \zeta_g \geq 0$$

$$C_2 = 0{,}192 + 0{,}585\kappa_{wt} - 0{,}054\kappa_{wt}^2 - (0{,}032 + 0{,}102\,\kappa_{wt} - 0{,}013\kappa_{wt}^2)\,\zeta_g, \quad \text{if} \quad \zeta_g < 0$$

EN 1999-1-1
steel + alumunium

EN 1995-1-1
timber

$$M_{cr} = \mu_{cr} \frac{\pi\sqrt{EI_z GI_t}}{L}$$

$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L_{ef}}$$

$$L_{ef} = L \; / \; \mu_{cr}$$

Procedure of (Baláž & Koleková 2000a, b, 2002a, b)

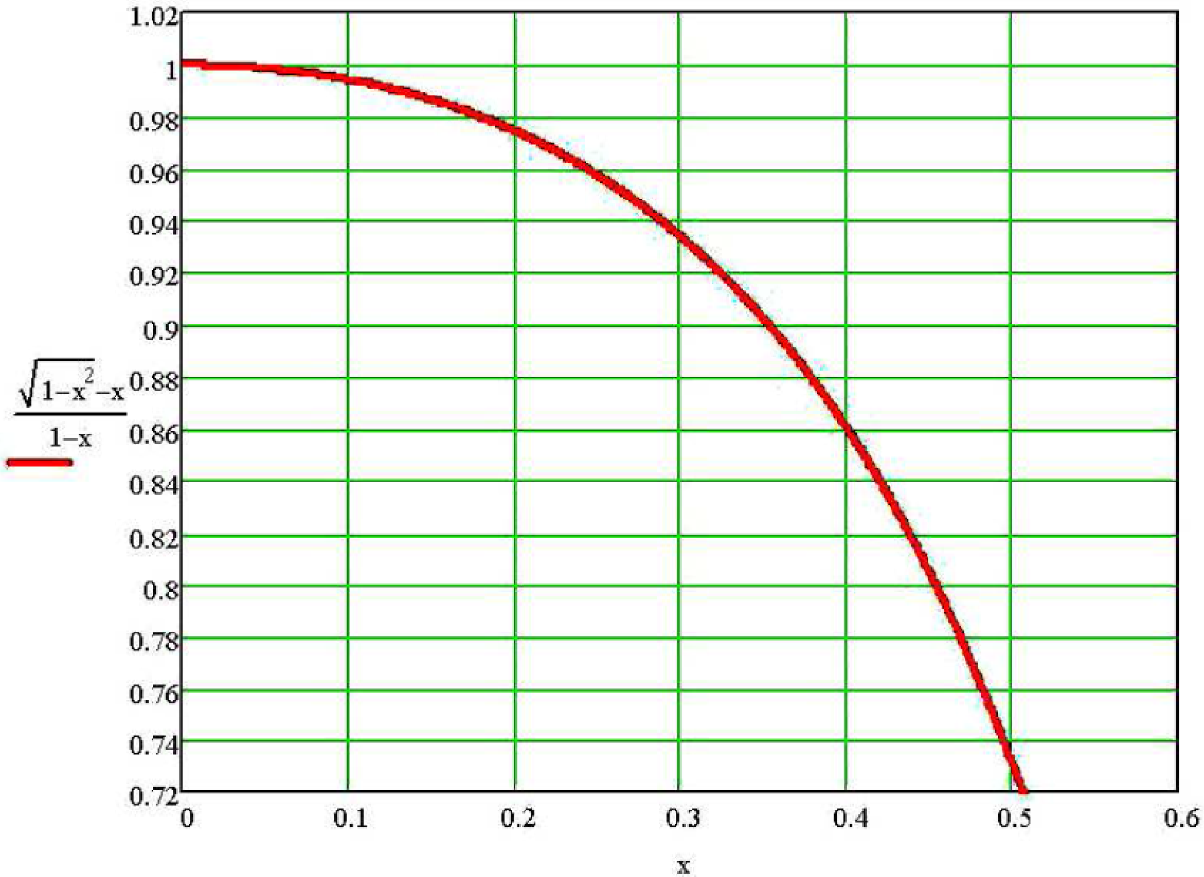
$$M_{cr} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}^2+(C_2\zeta_g)^2} - C_2\zeta_g\right]\frac{\pi\sqrt{EI_z GI_t}}{L}$$

Procedure used in German standard DIN 1052: 2004 Annex E.3
German National Annex to DIN EN 1995-1-1: 2005

$$M_{cr} = a_1\left[1 - a_2\frac{a_z}{L}\sqrt{\frac{EI_z}{GI_t}}\right]\frac{\pi\sqrt{EI_z GI_t}}{L} = \frac{C_1}{k_z}\left[1 - C_2\zeta_g\right]\frac{\pi\sqrt{EI_z GI_t}}{L}$$

$$a_1 = C_1 / k_z;\; a_2 = \pi C_2 / k_z;\; a_z = z_g$$

$$\frac{M_{cr,DIN\,1052}}{M_{cr,Baláž,Koleková,EN\,1999-1-1}} = \frac{\left(\frac{L}{l_{ef}}\right)_{DIN\,1052}}{\mu_{cr}} = \frac{\frac{1}{1-x}}{\frac{1}{\sqrt{1-x^2}-x}} = \frac{\sqrt{1-x^2}-x}{1-x}$$



$$x = C_2 \zeta_g$$

$$x = C_2 \frac{\pi z_g}{k_z L} \sqrt{\frac{EI_z}{GI_t}}$$

1.4 *Solution of system of differential equations using Bessel functions*

System of differential equations for beam in bending with rectangular cross-section was solved using Bessel functions in (Baláž, Koleková, 2015) and in (Voľmir, 1965). Solution leads for $M_y(x) = -Fx$ to the equation

$$\zeta_g \sqrt{\frac{\mu_{cr}}{\pi}} \Gamma(0.25) \sum_{k=0}^{n} \frac{(-1)^k}{k!\Gamma(k+1.25)} \left(\frac{\pi\mu_{cr}}{4}\right)^{2k+0.25} - \Gamma(0.75) \sum_{k=0}^{n} \frac{(-1)^k}{k!\Gamma(k+0.75)} \left(\frac{\pi\mu_{cr}}{4}\right)^{2k-0.25} = 0 \quad (21)$$

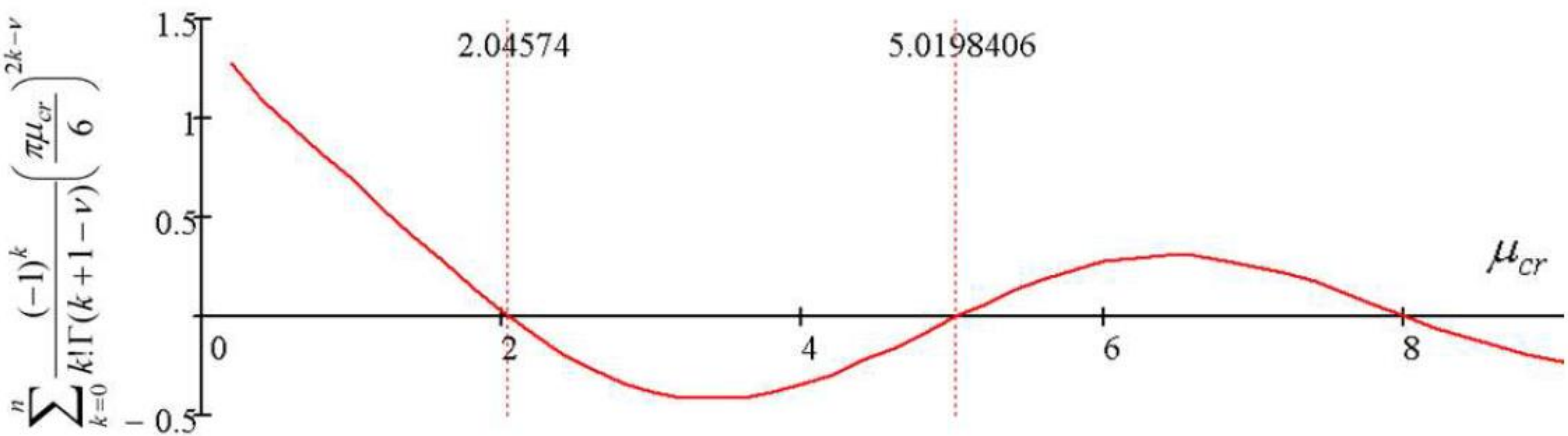


Figure 2. Graphical interpretation of equation (21). The least root is the solution: $\mu_{cr} = 2.04574$.

Table 6.1 – Effective length as a ratio of the span

Beam type	**Loading type**	ℓ_{ef}/ℓ [a]
Simply supported	Constant moment Uniformly distributed load Concentrated force at the middle of the span	1,0 0,9 0,8
Cantilever	Uniformly distributed load Concentrated force at the free end	0,5 0,8

[a] The ratio between the effective length ℓ_{ef} and the span ℓ is valid for a beam with torsionally restrained supports and loaded at the centre of gravity. If the load is applied at the compression edge of the beam, ℓ_{ef} should be increased by $2h$ and may be decreased by $0,5h$ for a load at the tension edge of the beam.

$$M_{cr} = \frac{\pi\sqrt{EI_z GI_t}}{L_{ef}}$$

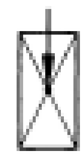
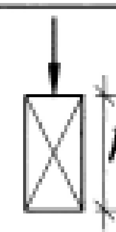
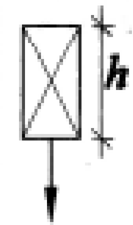
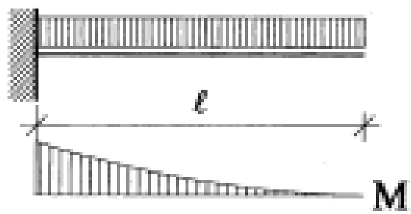
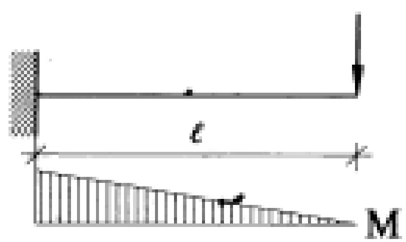
Procedure given in Eurocode EN 1995-1-1: 2004

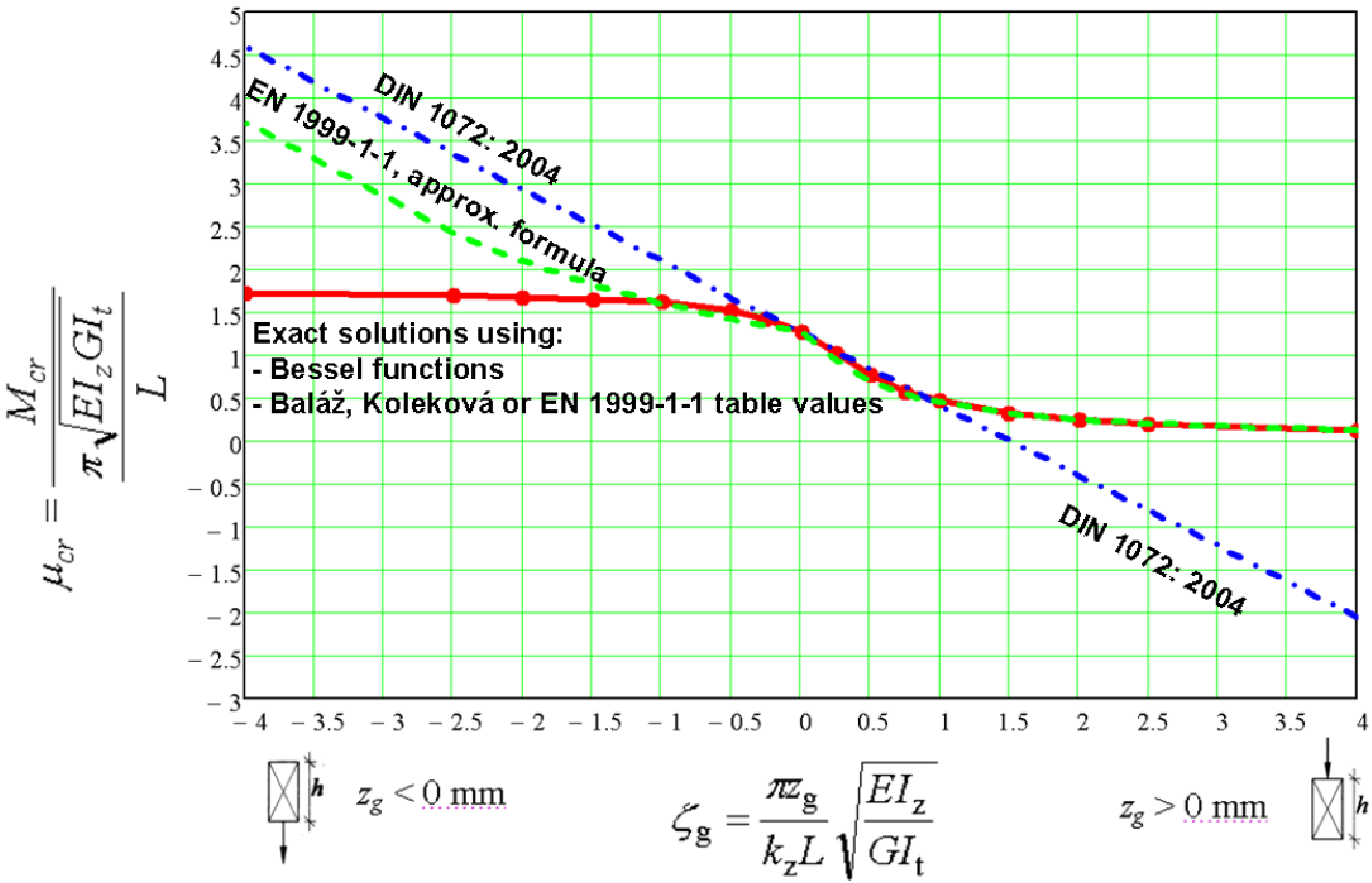
$$M_{cr} = \frac{1}{0.8+\dfrac{2h}{L}}\frac{\pi\sqrt{EI_z GI_t}}{L} \quad \text{if} \quad z_g = \frac{h}{2}; \quad M_{cr} = \frac{1}{0.8-\dfrac{h}{2L}}\frac{\pi\sqrt{EI_z GI_t}}{L} \quad \text{if} \quad z_g = -\frac{h}{2}$$

1.3 *Procedure given in Eurocode EN 1995-1-1: 2004*

According to EN 1995-1-1: 2004 effective length $L_{ef} = l_{ef}$, where for cantilever with concentrated load F at the free end $l_{ef} / L = 0.8$. If the load F is applied at the compression edge of the beam, l_{ef} should be increased by $2h$ and may be decreased by $0.5h$ for a load at the tension edge of the beam, where h is height of the rectangular cross-section. This wording is not correct in the case of cantilever (see (31)). The wording should be replaced by the following one: for loads applied above (below) the shear centre l_{ef} should be increased by $2h$ (decreased by $0.5h$).

Table 8 Effective length l_{ef} (Colling, 2014)

		ℓ_{ef}		
				
4		$0{,}5 \cdot \ell$	$-h/2$	$+2 \cdot h$
5		$0{,}8 \cdot \ell$	$-h/2$	$+2 \cdot h$



F at the free end of the cantilver

Table 10 Comparisons of L_{ef}/L values for elastic critical moment M_{cr}, see formula (5), defined by various publications. Evaluation of rules for $a_z =$ 0 m.

Boundary conditions of the beam	Type of loading	Ratio L_{ef}/L according to different codes. Quantity „a_z" denotes distance of point of load application from shear center S.						
		Kirby, Nethercot, 1979, Slovak NAD to STN P ENV 1995-1-1: 2002 ($L_{ef}/L = m$) $a_z =$ 0 m;	Czech NAD to ČSN P ENV 1995-1-1: 1996; ($L_{ef}/L = m$)			Eurocode EN 1995-1-1: 2003; ($L_{ef}/L = k_{l,ef}$); if $a_z = +h/2$: $L_{ef} = L_{ef} + 2h$; $a_z = -h/2$: $L_{ef} = L_{ef} - h/2$	German code DIN 1052: 2004; ($L_{ef}/L = k_{l,ef}$) $\left[a_1(1-a_2\frac{a_z}{L}\sqrt{\frac{EI_z}{GI_t}})\right]^{-1}$ for $a_z =$ 0 m after with values from Table 9:	Baláž, Koleková, 2002b; Eurocode EN 1999-1-1: 2007; ($L_{ef}/L = 1/\mu_{cr}$), a_z=0; remark: „a_z" is in the above references denoted as „z_g"
			$a_z = +h/2$, if point of load application is above S	$a_z =$ 0 m, if if point of load application is in S	$a_z = -h/2$, if if point of load application is below S			
beam simply supported on both ends $k_y = 1.0$ $k_z = 1.0$ $k_w = 1.0$	end moments M and $\psi.M$, $\psi = 1$	1	———	1	———	1	1	1
	end moments M and $\psi.M$, $\psi = 0$	0.57	———	–	———	–	–	1 / 1.77 = 0.565
	end moments M and $\psi.M$, $\psi = -1$	0.43	———	–	———	–	–	1 / 2.555 = 0.391
	point load F at midspan	0.74	0.8	0.75	0.7	0.8	1 / 1.35 = 0.741	1 / 1.348 = 0.742
	point load F at quarter-span	0.69	0.74	0.69	0.64	–	–	–
	point load F at „x"-section, $\xi = x/L$ $\alpha = 1.35-1.4\xi(1-\xi)$	–	0.8/α	0.75/α	0.7/α	–	–	–
	two point loads F and F at quarters-span	0.95	–	–	–	–	–	1 / 1.038 = 0.963
	uniformly dist. load q	0.88	0.95	0.9	0.85	0.9	1 / 1.13 = 0.885	1 / 1.127 = 0.887
beam fixed on both ends	$k_y = 0.5$ $k_z = 0.5$ $k_w = 1.0$ point load in mid. F	0.39	–	–	–	–	1 / 6.81 = 0.147	1 / 6.072 = 0.165
	uniformly dist. load q	0.59	–	–	–	–	1 / 5.12 = 0.192	1 / 5.152 = 0.194
interior span of continuous beam	$k_y = 0.5$ $k_z = 1.0$ $k_w = 1.0$ point load in mid. F	–	–	–	–	–	1 / 1.7 = 0.588	1 / 1.683 = 0594
	uniformly dist. load q	–	–	–	–	–	**1 / 1.3 = 0.769**	1 / 2.576 = 0.388
cantilever $k_y = k_z = k_w = 2$	end moment M	–	———	2	———	–	–	-
	point load F at free-end	–	–	**1.7**	–	0.8	1 / 1.27 = 0.787	1 / 1.28 = 0.781
	uniformly distributed load q	–	–	**1.2**	–	0.5	1 / 2.05 = 0.488	1 / 2.054 = 0.487

The values in bold and boxes are incorrect. The value 0.769 is valid for section at midspan and not at the support. The correct value at the support is 0.769 / 2 = 0.385.

CONCLUSION

Harmonization and unification of rules of metal (steel, aluminium) and timber Eurocodes, if it is reasonable, will be good for everybody especially for designers in practice.

Shortening of Eurocodes is desirable.

Thank for agreeing with us