CRITICAL AXIAL FORCE OF TORSIONAL-FLEXURAL BUCKLING FOR VARIOUS BOUNDARY CONDITIONS BY GOLDENVEJZER'S APPROXIMATE METHOD




Prof. Ivan BALÁŽ   Assoc. Prof. Yvona KOLEKOVÁ   Ing. Michal KOVÁČ   Ing. Tomáš ŽIVNER

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

SLOVAK REPUBLIC,   EUROPEAN UNION


Flag


Coat of arms

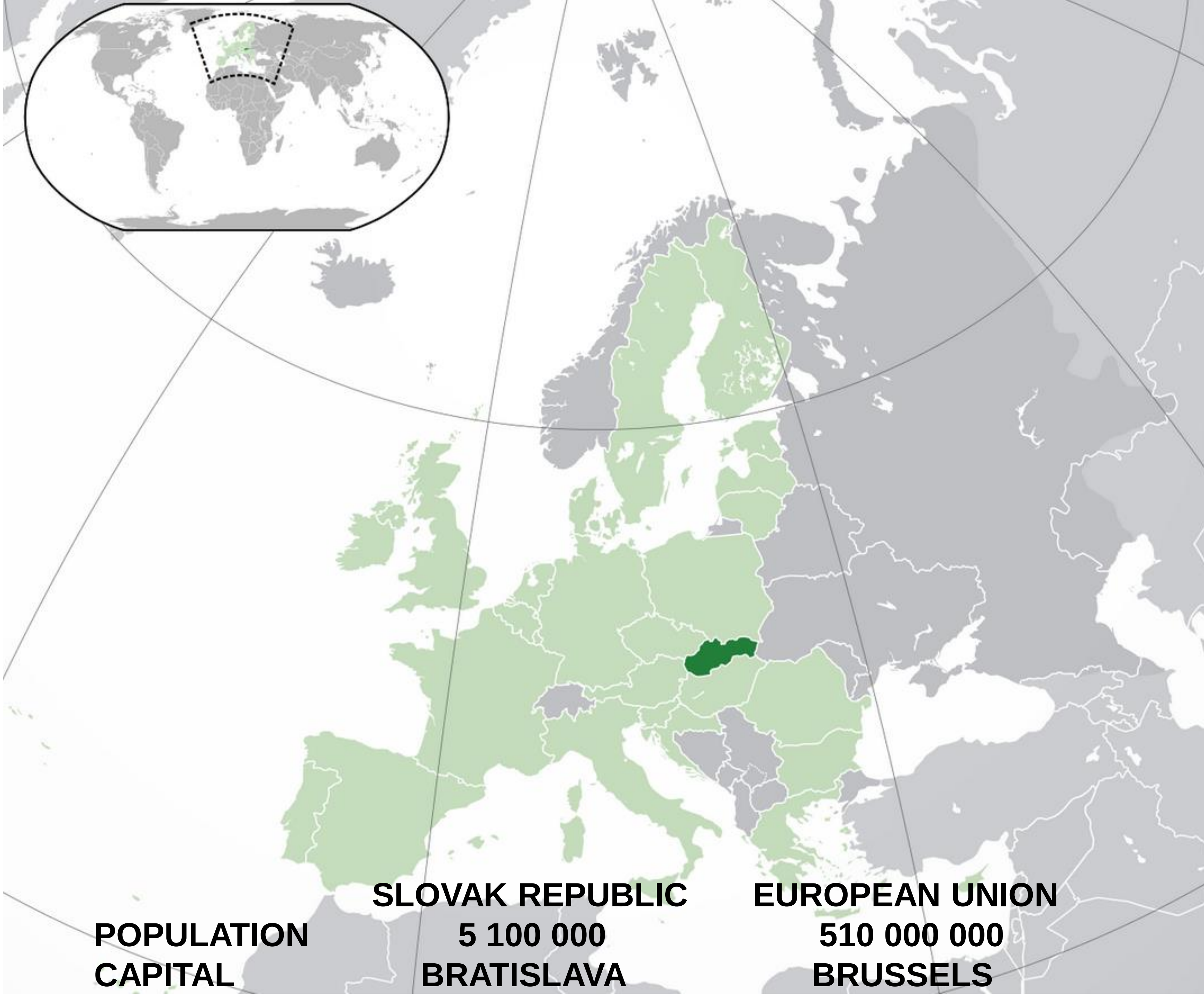

Flag

HONG KONG


Flag

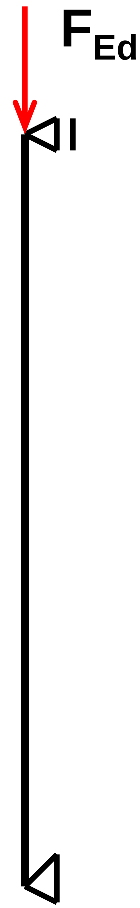
Emblem

ICSAS 2016

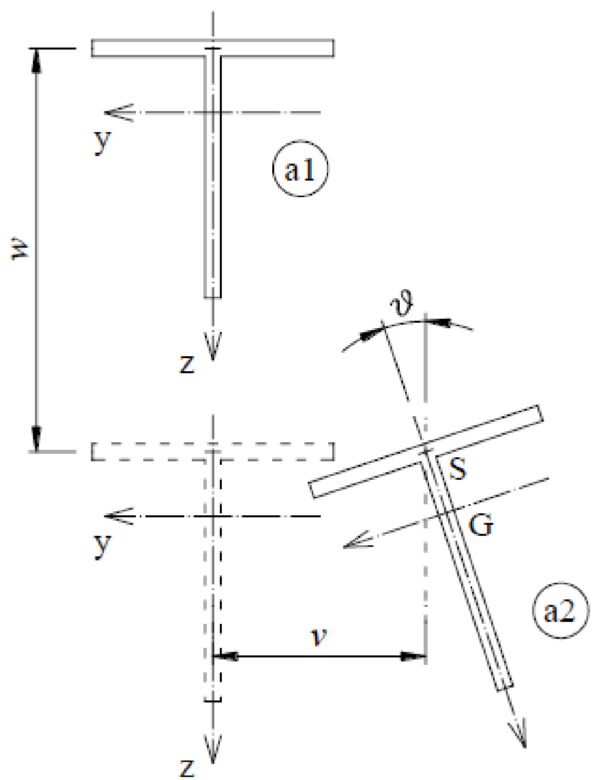


	SLOVAK REPUBLIC	EUROPEAN UNION
POPULATION	5 100 000	510 000 000
CAPITAL	BRATISLAVA	BRUSSELS

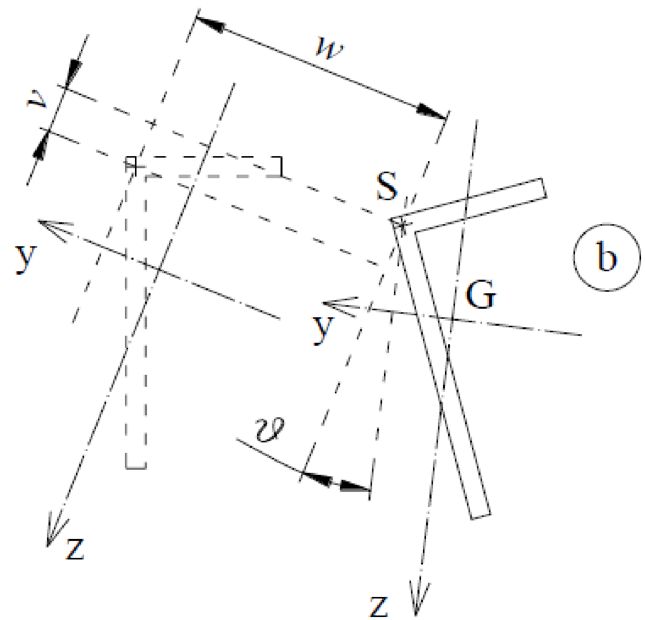
CRITICAL FORCE $N_{cr,TF}$ OF TORSIONAL-FLEXURAL BUCKLING FOR ANY COMBINATIONS OF BENDING AND TORSION BOUNDARY CONDITIONS



Monosymmetric section



Section without axis of symmetry

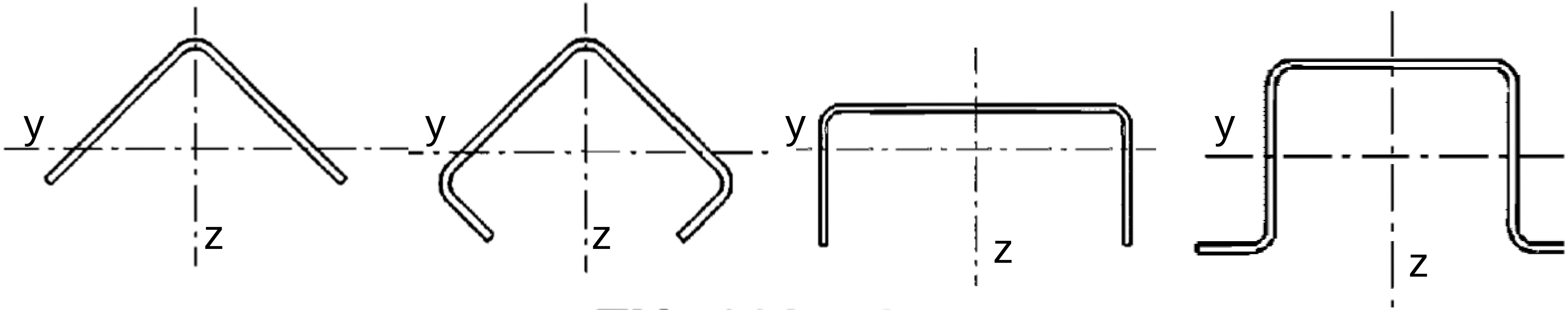


ANY COMBINATIONS OF BOUNDARY CONDITIONS IN BENDING AND IN TORSION

FBBCs = Flexural Buckling Boundary Conditions. Buckling length factors k_y , k_z		
Non-sway FBBCs	1	0.5
	2	0.7
	3	0.7
	4	1.0
Sway FBBCs	5	1.0
	6	1.0
	7	2.0
	8	2.0
	9	2.0
	10	2.0

TBBCs = Torsional Buckling Boundary Conditions Buckling length factors k_w									
Non-sway TBBCs				Sway TBBCs					
I	II	III	IV	V	VI	VII	VII	IX	X
0.5	0.7	0.7	1.0	1.0	1.0	2.0	2.0	2.0	2.0

Monosymmetric section $N_{cr,TFz}$



EN 1993-1-3

Eurocode 3 - Design of steel structures - Part 1-3: General rules - Supplementary rules for cold-formed members and sheeting

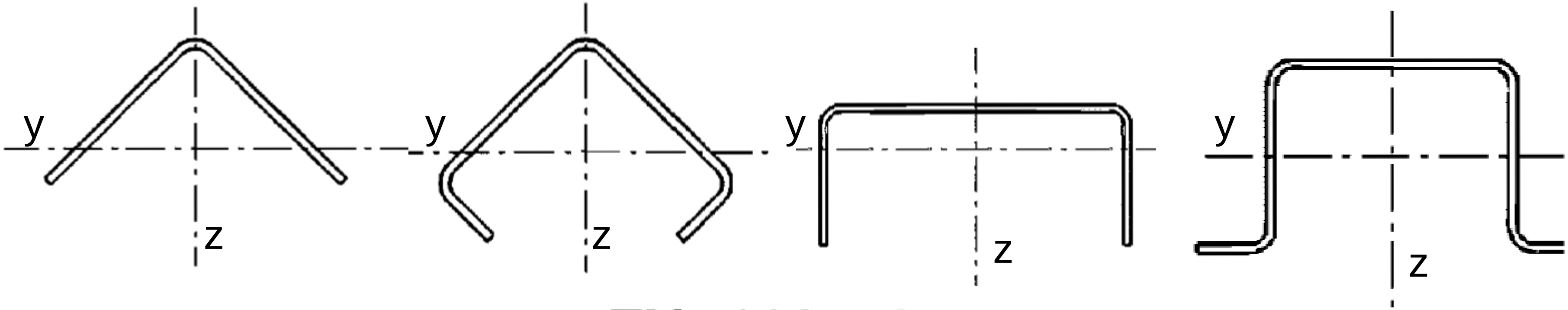
Ncr,TFz

$$\frac{1}{2(1-1\,z_s^2/i_s^2)}\left[(N_{cr,z}+N_{cr,T})\mp\sqrt{(N_{cr,z}+N_{cr,T})^2-4N_{cr,z}N_{cr,T}(1-1\,z_s^2/i_s^2)}\right]$$

$$N_{cr,z}=\frac{\pi^2 EI_z}{k_z^2 L^2} \qquad N_{cr,T}=\frac{1}{i_s^2}\left[GI_t+\frac{\pi^2 EI_w}{k_w^2 L^2}\right] \qquad i_s^2=\frac{I_y+I_z}{A}+y_s^2+z_s^2$$

NOTE: valid only when bending and torsion boundary conditions are the same.

Monosymmetric section $N_{cr,TFz}$



EN 1993-1-3

$$\frac{1}{2(1-1\ z_s^2/i_s^2)}\left[(N_{cr,z}+N_{cr,T}) \mp \sqrt{(N_{cr,z}+N_{cr,T})^2 - 4N_{cr,z}N_{cr,T}(1-1\ z_s^2/i_s^2)}\right]$$

EN 1999-1-1

Eurocode 9: Design of aluminium structures - Part 1-1: General structural rules

$$\frac{1}{2(1-\alpha_{zw} z_s^2/i_s^2)}\left[(N_{cr,z}+N_{cr,T}) \mp \sqrt{(N_{cr,z}+N_{cr,T})^2 - 4N_{cr,z}N_{cr,T}(1-\alpha_{zw} z_s^2/i_s^2)}\right]$$

1940, 1958

Vlasov V.Z., *Thin-walled elastic beams*, Moscow

1941

Goľdenvejzer A. L., “Stability of thin-walled members under axial force depending on boundary conditions”, Moscow. (In Russian).

1954

Březina V., Boundary conditions in stability of members in compression. Prague.

1962

Březina V., *Buckling resistance of metal members and beams*, Prague. (In Czech).

1982

Chalupa A. et al., *Design of steel structures, Commentary to ČSN 73 1401*, Prague. (In Czech).

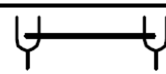
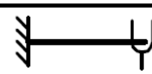
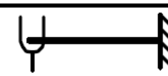
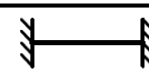
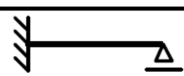
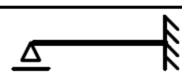
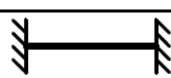
1941 Gol’denvejzer A. L.

Eigenfunctions of vibration modes (EFVMs)

$$w^{IV}(\xi) - \lambda^4 w(\xi) = 0$$

$$\text{where:}\quad \lambda = L\sqrt[4]{\frac{\mu\omega_0^2}{EI}};\quad \xi = \frac{x}{L};\quad 0 \le \xi \le 1$$

Table 1: Factors α_{zw} resulted from vibration eigenfunctions

Flexural BCs, *Buckling length factors:* k_y , k_z	Torsional BCs, Buckling length factors k_w			
	 1.0	 0.7	 0.7	 0.5
 1.0	**1**	**0.817**	**0.817**	**0.780**
 0.7	**0.817**	**1**		**0.765**
 0.7	**0.817**		**1**	**0.765**
 0.5	**0.780**	**0.765**	**0.765**	**1**

1954

Březina V.

Eigenfunctions of buckling modes (EFBMs)

$$w^{IV}(\xi) + \varepsilon^2 w^{II}(\xi) = 0$$

where: $\varepsilon = \sqrt{\frac{N}{EI}}L; \quad \xi = \frac{x}{L}; \quad 0 \le \xi \le 1$

Table 1: Factors α_{zw} resulted from vibration eigenfunctions

Flexural BCs, *Buckling length factors:* k_y , k_z	Torsional BCs, Buckling length factors k_w			
	1.0	0.7	0.7	0.5
1.0	**1**			**0.721**
0.7				
0.7				
0.5	**0.721**			**1**

1962 Březina V.

Eigenfunctions of vibration modes (EFVMs)

$$w^{IV}(\xi) - \lambda^4 w(\xi) = 0$$

where: $\lambda = L\sqrt[4]{\frac{\mu\omega_0^2}{EI}}; \quad \xi = \frac{x}{L}; \quad 0 \leq \xi \leq 1$

He calculated α_{zw} values for all 100 combinations of bending and torsion boundary conditions. Many of them are incorrect.

Tab. 3 Factors α_{yw} (TBBC, FBBCy) or α_{zw} (TBBC, FBBCz) calculated using fundamental functions EFVMs

FBBCs = Flexural Buckling Boundary Conditions. Buckling length factors k_y , k_z		TBBCs = Torsional Buckling Boundary Conditions Buckling length factors k_w									
		Non-sway TBBCs				Sway TBBCs					
		I 0.5	II 0.7	III 0.7	IV 1.0	V 1.0	VI 1.0	VII 2.0	VII 2.0	IX 2.0	X 2.0
Non-sway FBBCs	1 0.5	**1**	**0.765**	**0.765**	**0.78**	0.008	0.008	0.127	0.127	-0.421	-0.421
	2 0.7	**0.765**	**1**	0.472	**0.817**	0.425	0.008	0.794	0.117	1.776	-0.683
	3 0.7	**0.765**	0.472	**1**	**0.817**	0.008	0.425	0.117	0.794	-0.683	1.776
	4 1.0	**0.78**	**0.817**	**0.817**	**1**	0.106	0.106	0.721	0.721	1.805	1.805
Sway FBBCs	5 1.0	0.008	0.425	0.008	0.106	1	0.97	0.78	-0.378	1.348	-4.583
	6 1.0	0.008	0.008	0.425	0.106	0.97	1	-0.378	0.78	-4.583	1.348
	7 2.0	0.127	0.794	0.117	0.721	0.78	-0.378	1	0.405	1.707	1.707
	8 2.0	0.127	0.117	0.794	0.721	-0.378	0.78	0.405	1	1.707	1.707
	9 2.0	-0.421	1.776	-0.683	1.805	1.348	-4.583	1.707	1.707	1	16.783
	10 2.0	-0.421	-0.683	1.776	1.805	-4.583	1.348	1.707	1.707	16.783	1

Tab. 4 Factors α_{yw} (TBBC, FBBCy) or α_{zw} (TBBC, FBBCz) calculated using fundamental functions EFBMs

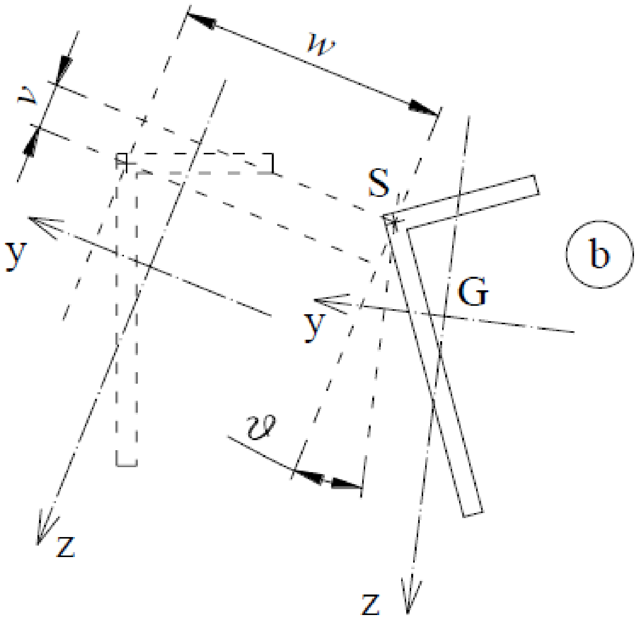
FBBCs = Flexural Buckling Boundary Conditions. Buckling length factors k_y , k_z		TBBCs = Torsional Buckling Boundary Conditions. Buckling length factors k_w									
		Non-sway TBBCs				Sway TBBCs					
		I 0.5	II 0.7	III 0.7	IV 1.0	V 1.0	VI 1.0	VII 2.0	VIII 2.0	IX 2.0	X 2.0
Non-sway FBBCs	1 0.5	1	0.66	0.66	0.721	0	0	0.115	0.115	-0.422	-0.422
	2 0.7	0.66	1	0.308	0.758	0.367	0.042	0.808	0.104	1.494	-0.601
	3 0.7	0.66	0.308	1	0.758	0.042	0.367	0.104	0.808	-0.601	1.494
	4 1.0	0.721	0.758	0.758	1	0	0	0.721	0.721	1.318	1.318
Sway FBBCs	5 1.0	0	0.367	0.042	0	1	1	0.721	-0.36	1.318	-2.637
	6 1.0	0	0.042	0.367	0	1	1	-0.36	0.721	-2.637	1.318
	7 2.0	0.115	0.808	0.104	0.721	0.721	-0.36	1	0.405	1.483	1
	8 2.0	0.115	0.104	0.808	0.721	-0.36	0.721	0.405	1	1	1.483
	9 2.0	-0.422	1.494	-0.601	1.318	1.318	-2.637	1.483	1	1	5.428
	10 2.0	-0.422	-0.601	1.494	1.318	-2.637	1.318	1	1.483	5.428	1

EN 1999-1-1:2007+A1

Table I.6 - Values of α_{yw} or α_{zw} for combinations of bending and torsion boundary conditions

Bending boundary condition k_y or k_z	Torsion boundary condition, k_w								
	1,0	0,7	0,7	0,5	2,0	2,0	1,0	1,0	2,0
1,0	1	0,817	0,817	0,780	a)	a)	a)	a)	a)
0,7	0,817	1	a)	0,766	a)	a)	a)	a)	a)
0,7	0,817	a)	1	0,766	a)	a)	a)	a)	a)
0,5	0,780	0,766	0,766	1	a)	a)	a)	a)	a)
2,0	a)	a)	a)	a)	1	a)	a)	a)	a)
2,0	a)	a)	a)	a)	a)	1	a)	a)	a)
1,0	a)	a)	a)	a)	a)	a)	1	a)	a)
1,0	a)	a)	a)	a)	a)	a)	a)	1	a)
2,0	a)	a)	a)	a)	a)	a)	a)	a)	1
a) conservatively, use α_{yw} = 1 and α_{zw} = 1									

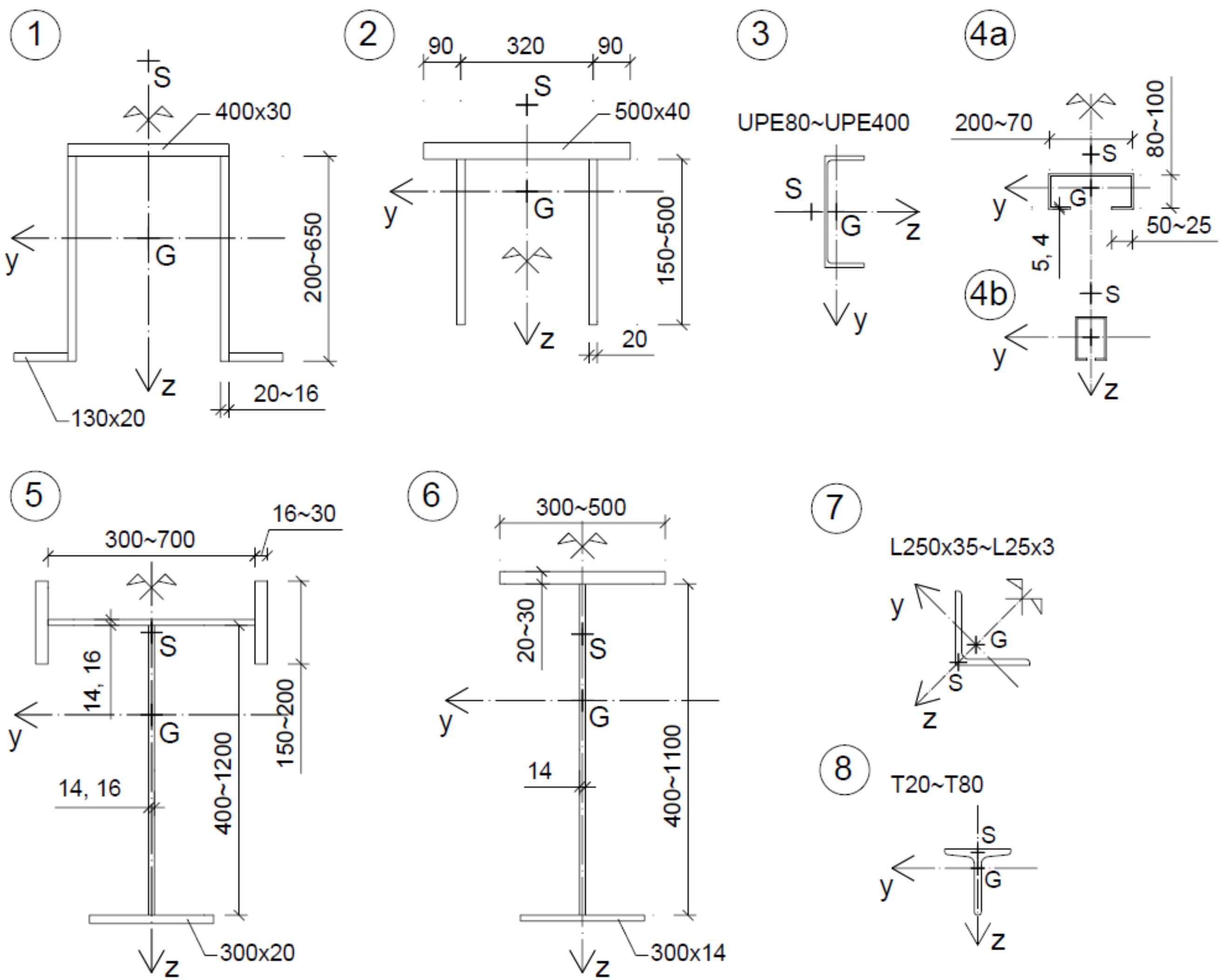
Table is valid also for sections without axis of symmetry



$$(N_{cr,y}-N_{cr})(N_{cr,z}-N_{cr})(N_{cr,T}-N_{cr})i_s^2 -$$

$$-\alpha_{zw} z_s^2 N_{cr}^2 (N_{cr,y}-N_{cr}) - \alpha_{yw} y_s^2 N_{cr}^2 (N_{cr,z}-N_{cr}) = 0$$

Large parametrical study using FEM



Tab. 4 Factors α_{yw} (TBBC, FBBCy) or α_{zw} (TBBC, FBBCz) calculated using fundamental functions EFBMs

FBBCs = Flexural Buckling Boundary Conditions. Buckling length factors k_y , k_z		TBBCs = Torsional Buckling Boundary Conditions. Buckling length factors k_w									
		Non-sway TBBCs				Sway TBBCs					
		I 0.5	II 0.7	III 0.7	IV 1.0	V 1.0	VI 1.0	VII 2.0	VIII 2.0	IX 2.0	X 2.0
Non-sway FBBCs	1 0.5	1	0.66	0.66	0.721	0	0	0.115	0.115	-0.422	-0.422
	2 0.7	0.66	1	0.308	0.758	0.367	0.042	0.808	0.104	1.494	-0.601
	3 0.7	0.66	0.308	1	0.758	0.042	0.367	0.104	0.808	-0.601	1.494
	4 1.0	0.721	0.758	0.758	1	0	0	0.721	0.721	1.318	1.318
Sway FBBCs	5 1.0	0	0.367	0.042	0	1	1	0.721	-0.36	1.318	-2.637
	6 1.0	0	0.042	0.367	0	1	1	-0.36	0.721	-2.637	1.318
	7 2.0	0.115	0.808	0.104	0.721	0.721	-0.36	1	0.405	1.483	1
	8 2.0	0.115	0.104	0.808	0.721	-0.36	0.721	0.405	1	1	1.483
	9 2.0	-0.422	1.494	-0.601	1.318	1.318	-2.637	1.483	1	1	5.428
	10 2.0	-0.422	-0.601	1.494	1.318	-2.637	1.318	1	1.483	5.428	1

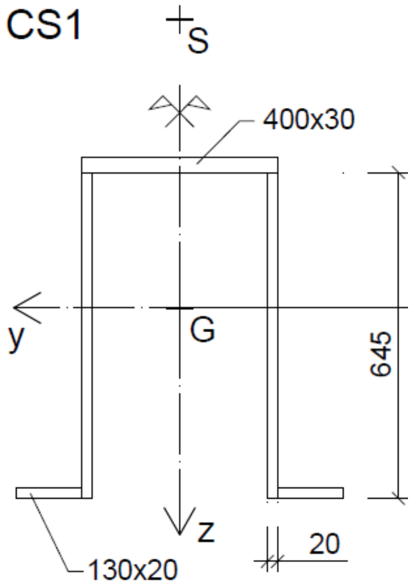
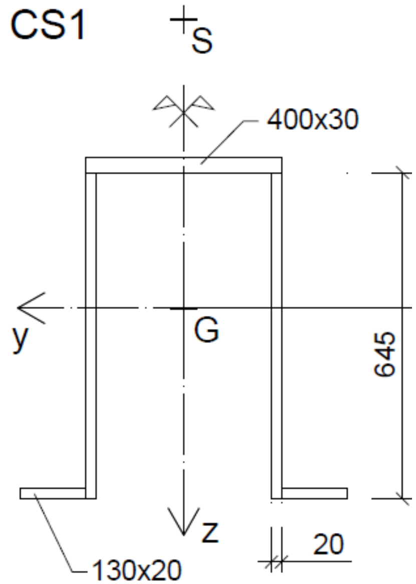


Table 3: Intervals of errors in (2) for regular α_{zw} (from table 2) for CS1 with z_s/i_s=0.880

	0	0	-3	-1	-3	-1	-2	-1	2	20	2	20	**-60**	0	**-60**	0	-3	10	-3	10
	-5	0	0	0	-18	-1	-5	-1	1	18	-1	**-87**	-14	7	**-45**	0	**-42**	0	-1	12
	-5	0	-18	-1	0	0	-5	-1	-1	**-87**	1	18	**-45**	0	-14	7	-1	12	-42	0
	-6	0	-8	0	-8	0	0	0	0	**-89**	0	**-89**	**-82**	0	**-82**	0	**-40**	0	**-40**	0
	20	31	**31**	**36**	-8	2	-16	-6	0	0	21	**46**	8	12	23	20	0	0	6	20
	20	31	-8	2	**31**	**36**	-16	-6	21	**46**	0	0	23	20	8	12	6	20	0	0
	-26	-1	2	16	-9	-3	-20	-4	1	9	-7	3	0	0	0	0	-2	**-89**	-6	0
	-26	-1	-9	-3	2	16	-20	-4	-7	3	1	9	0	0	0	0	-6	0	-2	**-89**
	11	20	-4	-2	20	23	-1	-1	0	0	6	20	**-59**	**-85**	-2	-1	0	0	-9	0
	11	20	20	23	-4	-2	-1	-1	6	20	0	0	-2	-1	**-59**	**-85**	-9	0	0	0



When α_{zw} = 1.0

Table 4: Intervals of errors in (3) for CS1 with z_s/i_s=0.880

	0	0	1	4	1	4	3	6	1	15	1	15	3	28	3	28	1	15	1	15
	0	3	0	0	2	10	2	5	0	13	1	15	2	19	2	19	0	13	1	15
	0	3	2	10	0	0	2	5	1	15	0	13	2	19	2	19	1	15	0	13
	0	3	0	3	0	3	0	0	0	11	0	11	0	16	0	16	0	11	0	11
	16	26	25	**30**	23	27	25	28	0	0	3	13	3	6	3	6	0	0	3	13
	16	26	23	27	25	**30**	25	28	3	13	0	0	3	6	3	6	3	13	0	0
	5	**35**	10	**32**	10	**32**	17	**37**	0	3	0	3	0	0	0	0	0	3	0	3
	5	**35**	10	**32**	10	**32**	17	**37**	0	3	0	3	0	0	0	0	0	3	0	3
	16	26	25	**30**	23	27	25	28	0	0	3	13	3	6	3	6	0	0	3	13
	16	26	23	27	25	**30**	25	28	3	13	0	0	3	6	3	6	3	13	0	0

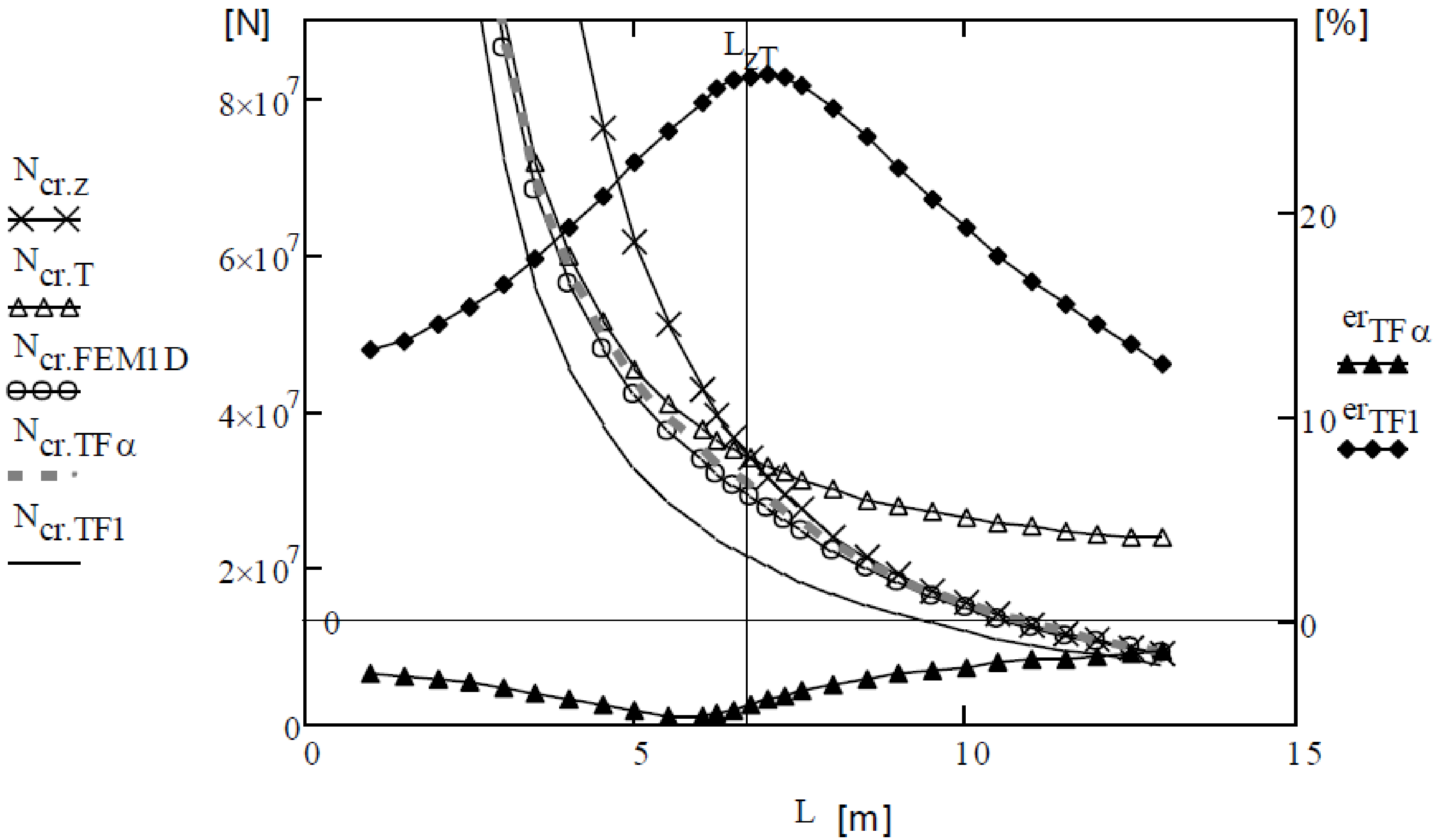


Fig. 4 Left vertical axis: elastic critical forces $N_{cr.z}$, $N_{cr.T}$, $N_{cr.TF\alpha}$, $N_{cr.TF1}$, and reference value $N_{cr.FEM1D}$ as functions of length L of member (Fig.2). Right vertical axis: Errors $er_{TF\alpha}$, er_{TF1} in critical forces $N_{cr.TF\alpha}$, $N_{cr.TF1}$ comparing with "exact" $N_{cr.FEM1D}$ value

Tab. 3 Factors α_{yw} (TBBC, FBBCy) or α_{zw} (TBBC, FBBCz) calculated using fundamental functions EFVMs

FBBCs = Flexural Buckling Boundary Conditions. Buckling length factors k_y, k_z		TBBCs = Torsional Buckling Boundary Conditions Buckling length factors k_w									
		Non-sway TBBCs				Sway TBBCs					
		I 0.5	II 0.7	III 0.7	IV 1.0	V 1.0	VI 1.0	VII 2.0	VII 2.0	IX 2.0	X 2.0
Non-sway FBBCs	1 0.5	**1**	**0.765**	**0.765**	**0.78**	Max. error 30 % on safe side when $\alpha_{zw} = 1$					
	2 0.7	**0.765**	**1**	0.472	**0.817**						
	3 0.7	**0.765**	0.472	**1**	**0.817**						
	4 1.0	**0.78**	**0.817**	**0.817**	**1**						
Sway FBBCs	5 1.0	Max. error 30 % on safe side when $\alpha_{zw} = 1$				Max. error 15 % on safe side when $\alpha_{zw} = 1$					
	6 1.0										
	7 2.0										
	8 2.0										
	9 2.0										
	10 2.0										

New generation of EN 1999-1-1 will contains the following table with improved α_{zw} values smaller than 1.0 giving errors in $N_{cr,TFz}$ in interval -3% - +5%.

The errors in relative slenderness λ will be smaller and in reduction factor χ and in member resistance $N_{b,Rd}$ even smaller.

Table I.6 - Values of α_{yw} or α_{zw} for combinations of bending and torsion boundary conditions

FBCs = Flexural Boundary Conditions 1 – 10. (Buckling length factors k_y and k_z)		TBCs = Torsional Boundary Conditions I – X. (Buckling length factors k_w)									
		Non-sway TBCs				Sway TBCs					
		I (0,5)	II (0,7)	III (0,7)	IV (1,0)	V (1,0)	VI (1,0)	VII (2,0)	VIII (2,0)	IX (2,0)	X (2,0)
Non-sway FBCs	1 (0,5)	1	0,9	0,9	0,9	0,7	0,7	0,8	0,8	0,8	0,8
	2 (0,7)	0,9	1	0,8	0,9	0,6	0,6	0,7	0,8	0,8	0,7
	3 (0,7)	0,9	0,8	1	0,9	0,6	0,6	0,8	0,7	0,7	0,8
	4 (1,0)	0,9	0,9	0,9	1	0,4	0,4	0,7	0,7	0,7	0,7
Sway FBCs	5 (1,0)	0,1	0,1	0,1	0,1	1	1	0,9	0,9	0,9	0,9
	6 (1,0)	0,1	0,1	0,1	0,1	1	1	0,9	0,9	0,9	0,9
	7 (2,0)	0,2	0,2	0,2	0,2	0,9	0,9	1	0,6	0,6	1
	8 (2,0)	0,2	0,2	0,2	0,2	0,9	0,9	0,6	1	1	0,6
	9 (2,0)	0,2	0,2	0,2	0,2	0,9	0,9	0,6	1	1	0,6
	10 (2,0)	0,2	0,2	0,2	0,2	0,9	0,9	1	0,6	0,6	1

NOTE Conservatively $\alpha_{yw} = 1$ or $\alpha_{zw} = 1$ may be used for any combinations of bending 1 – 10 and torsion I – X boundary conditions.

THE END