FACULTY OF ENGINEERING
AND ARCHITECTURE

ELECTRONICS AND INFORMATION SYSTEMS DEPARTMENT
LIQUID CRYSTALS AND PHOTONICS RESEARCH GROUP

LIQUID CRYSTALS AND LIGHT EMITTING MATERIALS FOR PHOTONIC APPLICATIONS

Kristiaan Neyts April 2018 Lecture series at WAT in Warsaw

GHENT
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nb-photonics
Center for Nano- and Biophotonics
Ghent University

Liquid Crystals
and Photonics

OVERVIEW

Electrical and optical properties of materials (6h)

Polarizability of dielectric materials
Light propagation
Conductors and semiconductors
Light propagation in anisotropic media
Polarized light
Spontaneous and stimulated emission

Liquid Crystals and Photonics

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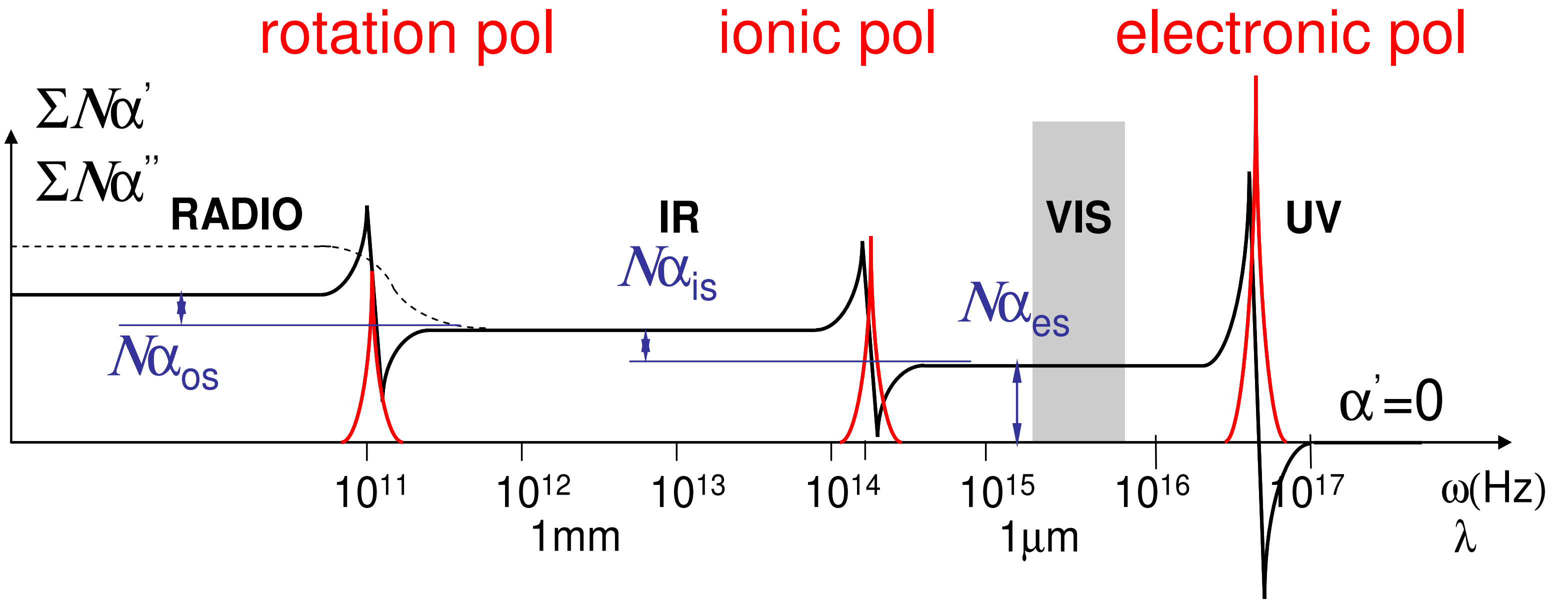
GENERAL DIELECTRIC BEHAVIOR

different resonance/relaxation processes

sum of all polarizabilities per unit volume:



PROPAGATION OF PLANE MONOCHROMATIC WAVES

Maxwell equations
in a medium

$$\boxed{\begin{array}{l} \nabla\cdot\vec{E}=0 \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B}=\varepsilon\mu_0\dfrac{\partial\vec{E}}{\partial t} \end{array}} \Longrightarrow$$

wave equation in vacuum

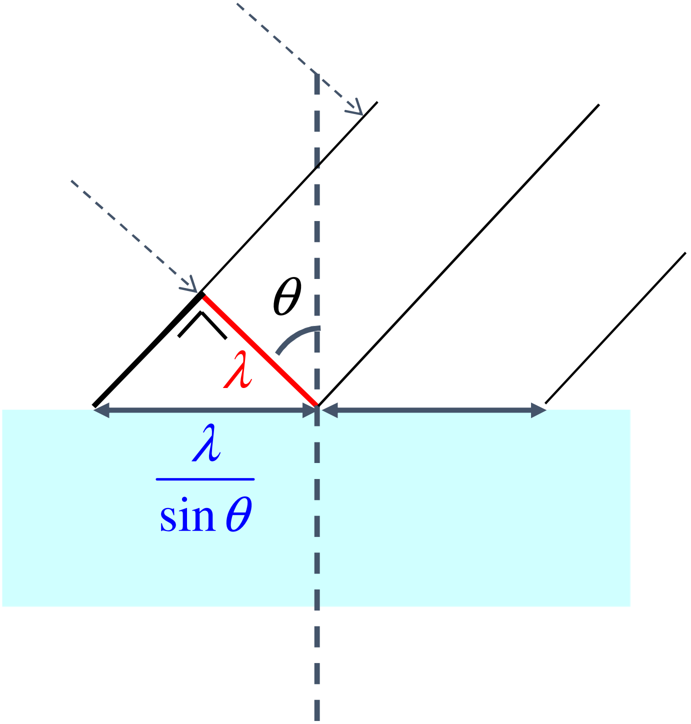
$$\nabla\times\left(\nabla\times\vec{E}\right)=\nabla\times\left(-\frac{\partial\vec{B}}{\partial t}\right)=-\varepsilon\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{E}-\varepsilon\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=0}$$

solution $\vec{E}_0\cos\left(\omega t-\vec{k}\cdot\vec{r}\right)=\vec{E}_0\operatorname{Re}\left[e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]$

$$\Longrightarrow k^2-\varepsilon\mu_0\omega^2=0 \qquad k=\frac{\omega}{c}\sqrt{\frac{\varepsilon}{\varepsilon_0}}=\frac{\omega}{c}n$$

REFRACTION OF LIGHT



Light incident in air:

wavelength λ

incidence angle θ

there is a translation symmetry:

the problem is invariant

for a horizontal shift over $\frac{\lambda}{\sin\theta}$

the same invariance

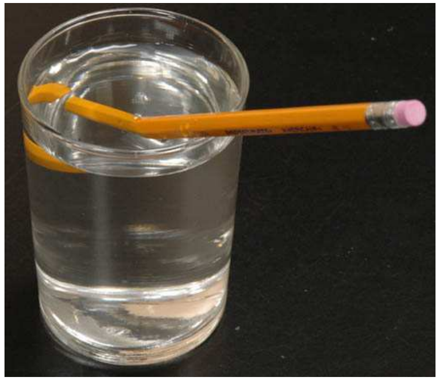
must be present in the medium

REFRACTION OF LIGHT

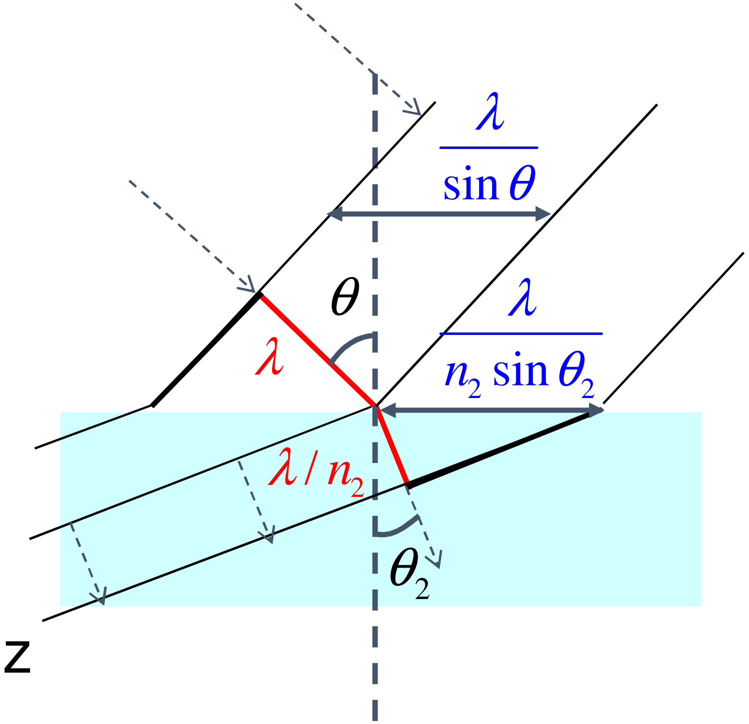
Light incident from air
on a surface with angle θ

$$\frac{\lambda}{\sin\theta} = \frac{\lambda}{n_2 \sin\theta_2}$$

$$\sin\theta = n_2 \sin\theta_2$$



Law of Snell

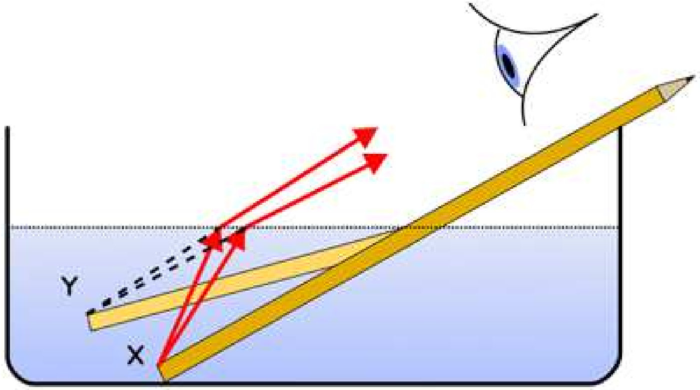


REFRACTION OF LIGHT

the wisdom of the spear-fisher

“objects are deeper than they appear”

national Geographic



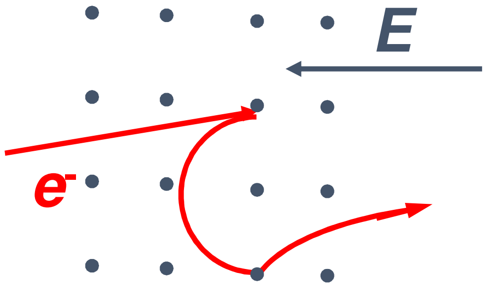
http://www.freeendlessinfo.com/2012/05/why-does-water-always-seem-shallower-than-it-is/#.VCzVXRb6Svk

http://plus.maths.org/content/light-bends-wrong-way

C.1. The 1D-Drude Free-Electron Model for Metals

most electrons accelerate under influence of the field

some electrons collide with ions,
then thermal distribution (v~0)
average time between collisions τ



conductivity depends on the frequency:

$$J = \sigma E \Longrightarrow \boxed{\sigma(\omega) = \frac{Ne^2}{m\left(i\omega + 1/\tau\right)}}$$

unit 1/Ωm

MICROSCOPIC THEORY FOR CONDUCTORS

formula for dielectric constant with conductivity

$$\left.\begin{aligned} \varepsilon &= \varepsilon_0\left(1+\chi-i\frac{\sigma}{\omega\varepsilon_0}\right) \\ \sigma(\omega) &= \frac{Ne^2}{m\left(i\omega+1/\tau\right)} \end{aligned}\right\}$$

$$\boxed{\varepsilon = \varepsilon_0\left(1+\chi+\frac{Ne^2}{m\varepsilon_0}\cdot\frac{1}{-\omega^2+i\omega/\tau}\right)}$$

$\uparrow$ free electron oscillation

$\uparrow$ damping absorption (collisions)

The term $i\omega/\tau$ (absorption)

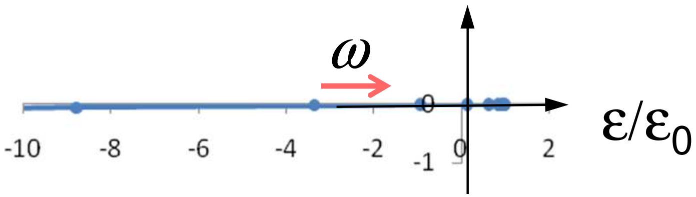
$<< \omega^2$ in the visible spectrum (high ω)

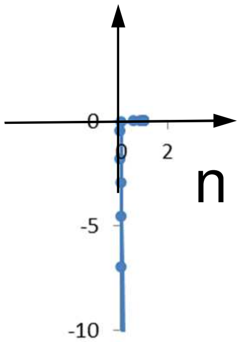
becomes important in the IR

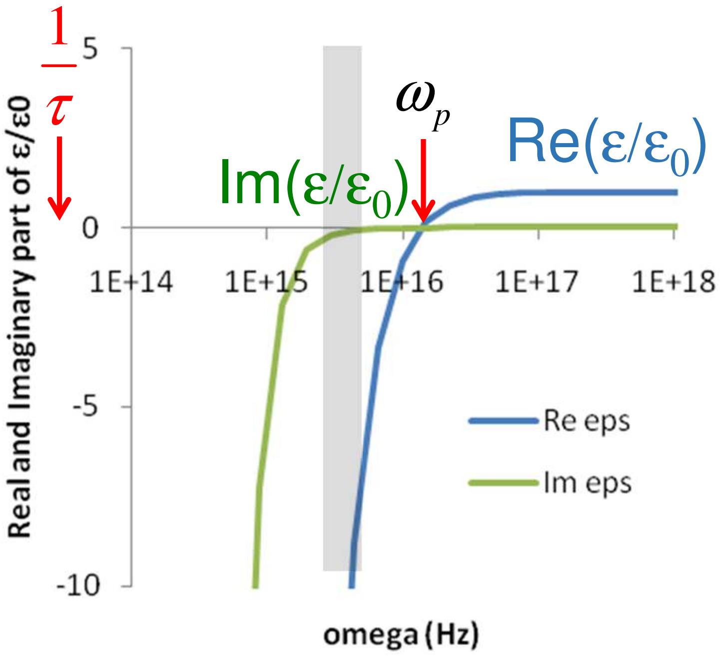
MICROSCOPIC THEORY FOR CONDUCTORS

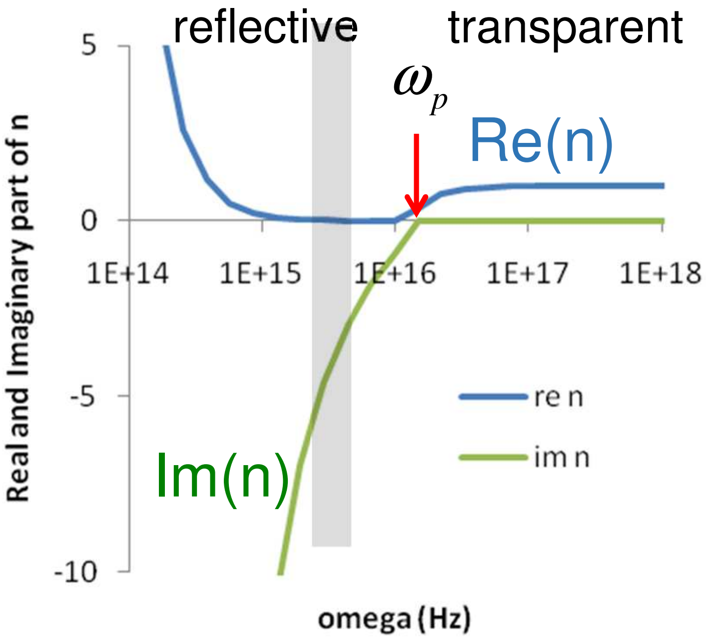
Example:

Silver







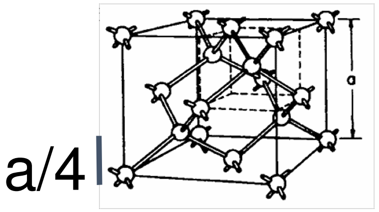


BAND STRUCTURE IN SEMICONDUCTORS

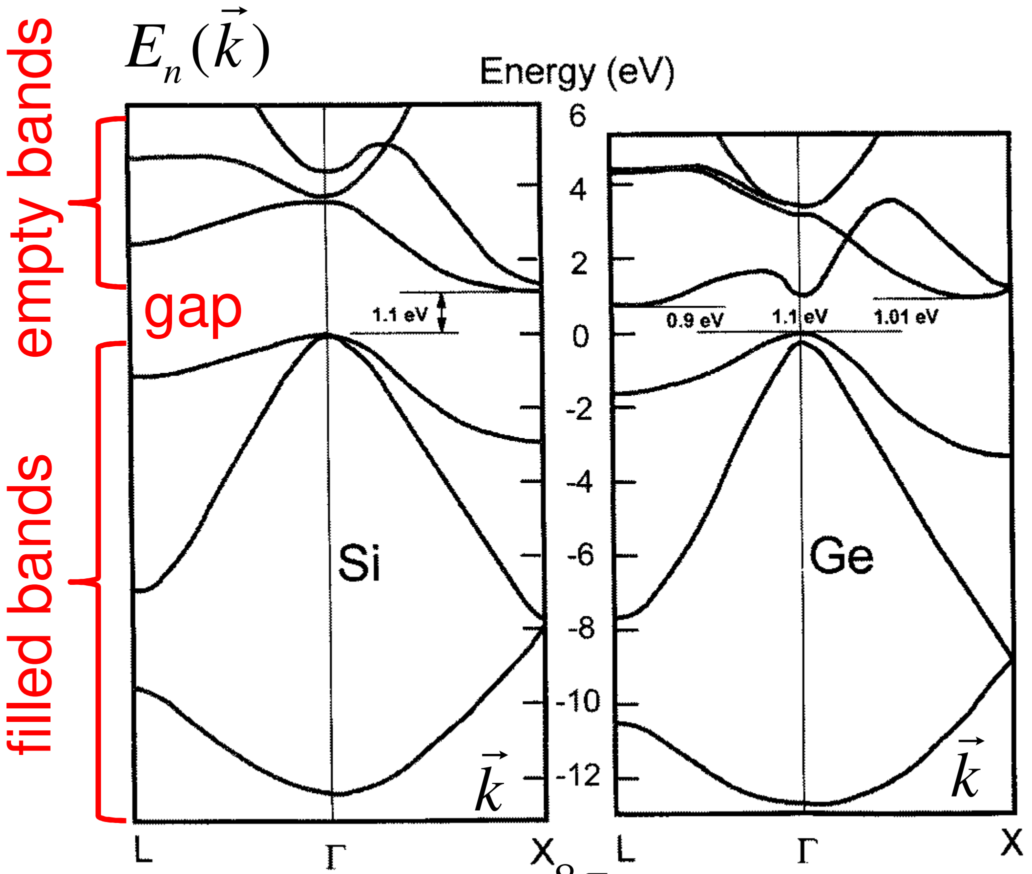
Only certain states allowed: energy versus *n*, *k*

Si, Ge: diamond structure,

similar to FCC



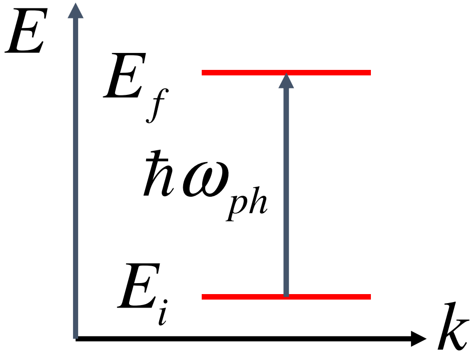
a=0.5 nm



each point in the diagram is an electron state

$$k_X = \frac{8\pi}{a} \approx 50nm^{-1}$$

INTERBAND TRANSITIONS



Conservation of energy

$$E_f = E_i + \hbar\omega_{ph}$$

Conservation of momentum

$$\hbar\vec{k}_f = \hbar\vec{k}_i + \hbar\vec{k}_{ph}$$

wave vector of a photon: order 10^7m^{-1}

dimension of Brillouin zone: 10^{10}m^{-1}

$$\hbar\vec{k}_{ph} << \hbar\vec{k}_i, \hbar\vec{k}_f$$

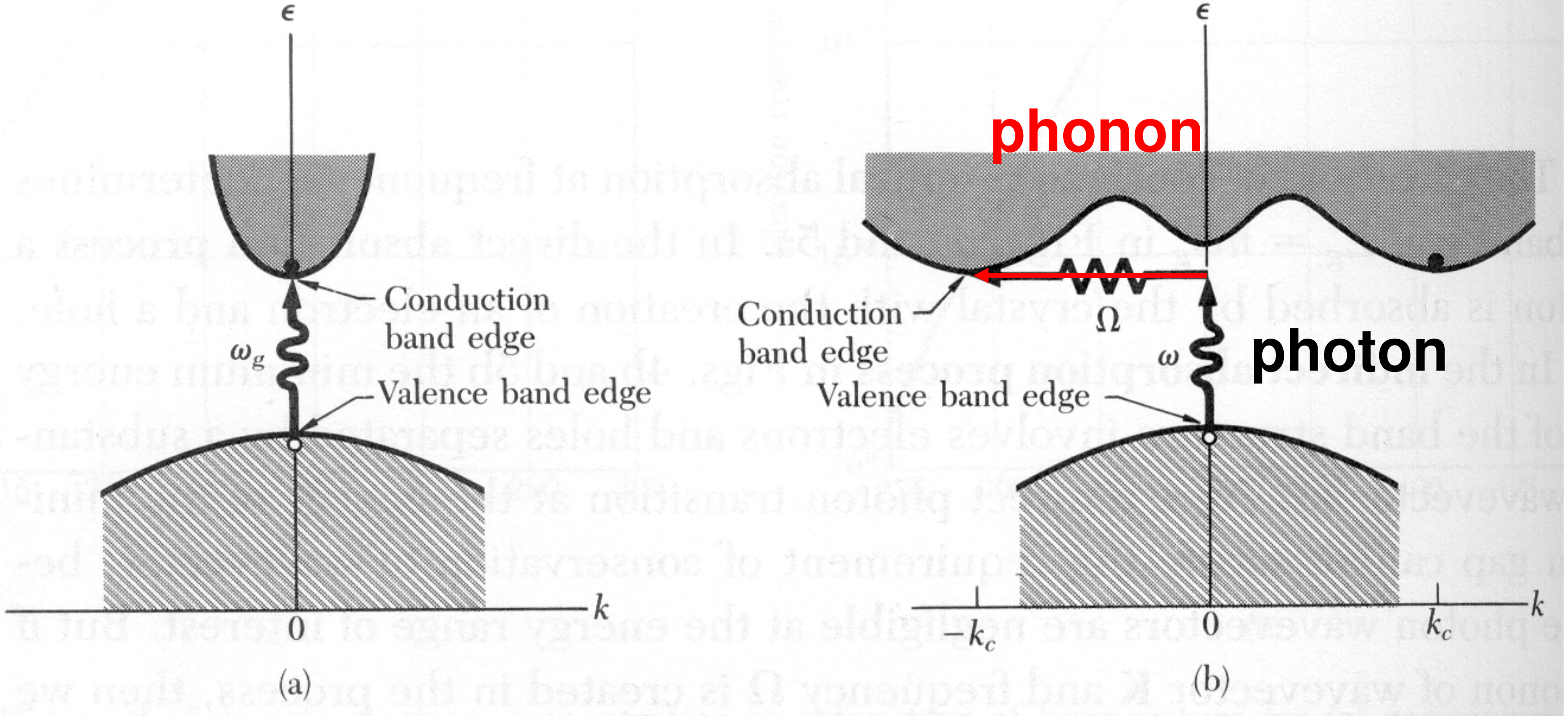
thus: absorption of a photon does not change the k-vector of the electron

vertical line in the energy diagram

$$\hbar\vec{k}_f \approx \hbar\vec{k}_i$$

INTERBAND TRANSITIONS

Direct and indirect band-gap



direct band-gap

indirect band-gap

ABSORPTION OF LIGHT

Most materials: no absorption in the visible region

electrons absorb in the UV; ions absorb in the IR

some exceptions:

Semiconductor electrons

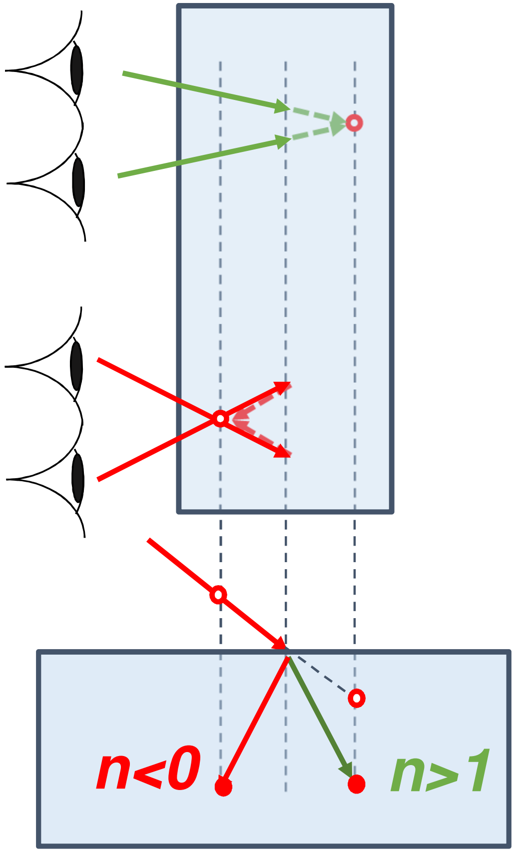
band-gap absorption

Impurity electrons
with large orbitals

Organic molecules

with large orbitals

NEGATIVE REFRACTIVE INDEX MATERIALS



top view

normal refraction

object seems
to be higher

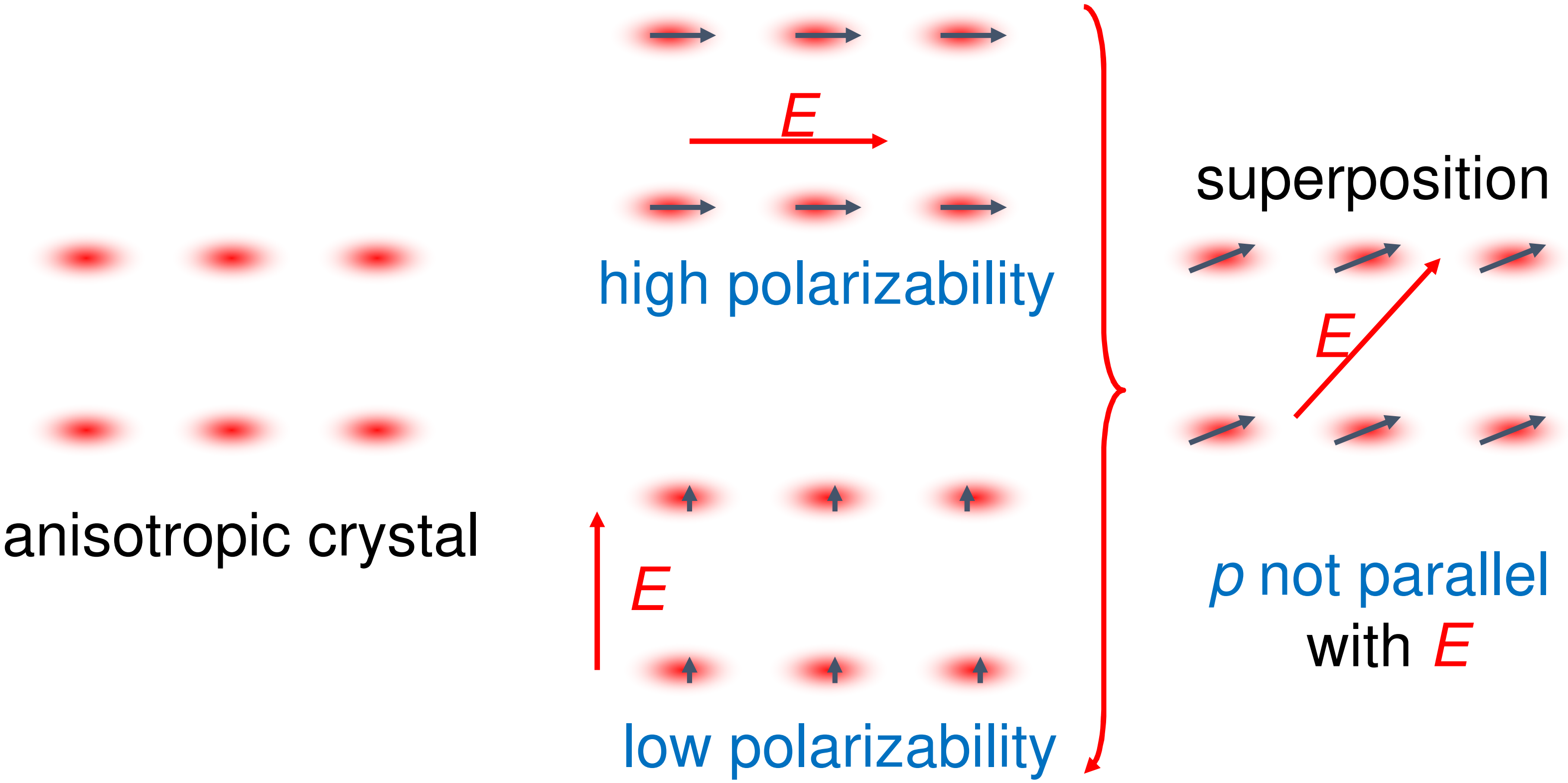
negative refraction

object seems
to be above the medium

side view

ANISOTROPIC POLARIZABILITY

Anisotropic polarizability in a crystalline structure



anisotropic crystal

high polarizability

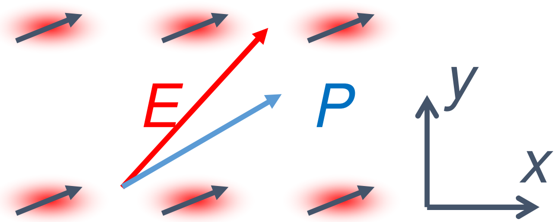
low polarizability

superposition

p not parallel
with *E*

ANISOTROPIC POLARIZATION

Tensor relation between *P* and *E*



P not parallel with *E*

for special choice of *xy* axes:

$$P_x = \chi_{xx}\varepsilon_0 E_x$$

$$P_y = \chi_{yy}\varepsilon_0 E_y$$

general choice of *xy* axes:

Tensor notation: $\vec{P} = \overline{\overline{\chi}}\varepsilon_0 \vec{E}$

Shortened notation: $P_i = \chi_{ij}\varepsilon_0 E_j$

ANISOTROPIC POLARIZATION

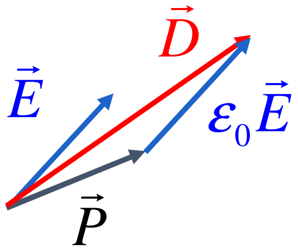
Tensor relation between *D* and *E*

Tensor notation: $$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \overline{\overline{\varepsilon}} \vec{E}$$

Shortened notation: $$D_i = \varepsilon_{ij} E_j$$

D, *P* normally not parallel with *E*

D lies between *E* and *P*



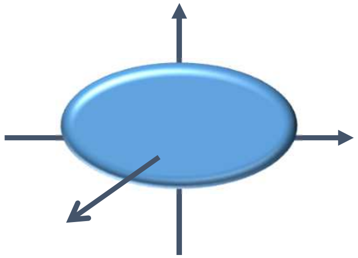
ANISOTROPIC POLARIZATION

Alternatively, matrix algebra teaches us:

"a symmetric matrix can always be diagonalized by an appropriate coordinate transformation"

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} \rightarrow \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

The new axes are called the principle axes of the ellipsoid (or the matrix)



$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} = \varepsilon_0 \begin{pmatrix} n_1^2 & 0 & 0 \\ 0 & n_2^2 & 0 \\ 0 & 0 & n_3^2 \end{pmatrix}$$

principle indices of refraction

ANISOTROPIC POLARIZATION

Introduction of the impermeability tensor $\overline{\overline{\eta}}$

Inverse of the dielectric tensor (without dimension)

$$\mathbf{D} = \overline{\overline{\varepsilon}}\,\mathbf{E} \qquad \overline{\overline{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_1^2 & 0 & 0 \\ 0 & n_2^2 & 0 \\ 0 & 0 & n_3^2 \end{pmatrix}$$

$$\mathbf{E} = \frac{1}{\varepsilon_0}\overline{\overline{\eta}}\,\mathbf{D} \qquad \overline{\overline{\eta}} = \begin{pmatrix} \frac{1}{n_1^2} & 0 & 0 \\ 0 & \frac{1}{n_2^2} & 0 \\ 0 & 0 & \frac{1}{n_3^2} \end{pmatrix}$$

THE INDEX ELLIPSOID

Now we use the impermeability tensor $\overline{\overline{\eta}} = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix}$

to eliminate *E* (instead of *D*) by $\mathbf{E} = \frac{1}{\varepsilon_0} \overline{\overline{\eta}} \mathbf{D}$

the energy density becomes:

$$\tfrac{1}{2} \mathbf{D} \cdot \mathbf{E} = \frac{1}{2\varepsilon_0} \mathbf{D} \overline{\overline{\eta}} \mathbf{D}$$

$$\Rightarrow \quad \varepsilon_0 \mathbf{D} \cdot \mathbf{E} = \eta_{11} D_x^2 + \eta_{22} D_y^2 + \eta_{33} D_z^2 + 2\eta_{12} D_x D_y + 2\eta_{23} D_y D_z + 2\eta_{31} D_z D_x$$

This is an ellipsoid, we normalize by: $X_i = \frac{D_i}{\sqrt{\varepsilon_0 \mathbf{D} \cdot \mathbf{E}}}$

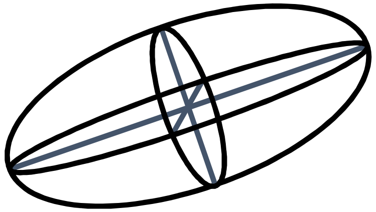
$$\mathbf{X} \overline{\overline{\eta}} \mathbf{X} = 1$$

FRESNEL'S EQUATION

The normalized equation is the index ellipsoid $\mathbf{X}\bar{\bar{\eta}}\mathbf{X}=1$

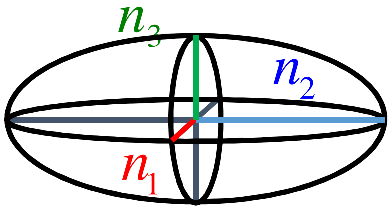
$$\eta_{11}X^2+\eta_{22}Y^2+\eta_{33}Z^2+2\eta_{12}XY+2\eta_{23}YZ+2\eta_{31}ZX=1$$

the symmetric impermeability tensor
is represented by a quadric
Short notation: $X_iX_j\eta_{ij}=1$



By rotation (coordinate transformation) this can be reduced to:

$$\frac{X^2}{n_1^2}+\frac{Y^2}{n_2^2}+\frac{Z^2}{n_3^2}=1$$



$$\bar{\bar{\eta}}=\begin{pmatrix}\frac{1}{n_1^2} & 0 & 0\\ 0 & \frac{1}{n_2^2} & 0\\ 0 & 0 & \frac{1}{n_3^2}\end{pmatrix}$$

THE INDEX ELLIPSOID

Light propagation in an anisotropic medium?

The wave equation relates *E* and *D*

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2}$$

Let us now eliminate *E* :

$$\mathbf{E} = \frac{1}{\varepsilon_0} \bar{\bar{\eta}} \mathbf{D}$$

$$\bar{\bar{\eta}} = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix}$$

$$\nabla \times (\nabla \times \bar{\bar{\eta}} \mathbf{D}) = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2}$$

THE INDEX ELLIPSOID

Plane wave solution for *D*?

$$\mathbf{D} = \mathbf{D}_0 \, \mathrm{e}^{\mathrm{i}\left(\omega t - \mathbf{k}\cdot\mathbf{r}\right)}$$

$$\nabla \times \left(\nabla \times \overline{\overline{\eta}} \mathbf{D} \right) = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2}$$

substitution of the proposed periodic solution:

$$-\mathbf{k} \times \left(\mathbf{k} \times \overline{\overline{\eta}} \mathbf{D} \right) = \varepsilon_0 \mu_0 \omega^2 \mathbf{D}$$

$$\downarrow$$

$$\mathbf{s} \times \left(\mathbf{s} \times \overline{\overline{\eta}} \mathbf{D} \right) = -\frac{1}{n^2} \mathbf{D}$$

$$\mathbf{k} = k\mathbf{e}_k = \frac{\omega}{c} n \cdot \mathbf{s} = \sqrt{\varepsilon_0 \mu_0} \, \omega n \cdot \mathbf{s}$$

s: unit vector along **k**

THE INDEX ELLIPSOID

$$\mathbf{s}\times\left(\mathbf{s}\times\bar{\bar{\eta}}\mathbf{D}\right)=-\frac{1}{n^2}\mathbf{D}$$

$$\mathbf{s}\cdot\left(\mathbf{s}\cdot\bar{\bar{\eta}}\mathbf{D}\right)-\bar{\bar{\eta}}\mathbf{D}\underbrace{\left(\mathbf{s}\cdot\mathbf{s}\right)}_{1}=-\frac{1}{n^2}\mathbf{D}$$

Choose the *z*-axis along the *k*-vector, then $\mathbf{s}=\mathbf{e}_z$

$$\mathbf{e}_z\cdot\left(\bar{\bar{\eta}}\mathbf{D}\right)_z-\bar{\bar{\eta}}\mathbf{D}=-\frac{1}{n^2}\mathbf{D}$$

$$\mathbf{e}_z\cdot\left(\bar{\bar{\eta}}\mathbf{D}\right)_z-\mathbf{e}_x\cdot\left(\bar{\bar{\eta}}\mathbf{D}\right)_x-\mathbf{e}_y\cdot\left(\bar{\bar{\eta}}\mathbf{D}\right)_y-\mathbf{e}_z\cdot\left(\bar{\bar{\eta}}\mathbf{D}\right)_z=-\frac{1}{n^2}\mathbf{D}$$

$$\Rightarrow\quad\begin{pmatrix}\eta_{11} & \eta_{12} & \eta_{13}\\ \eta_{21} & \eta_{22} & \eta_{23}\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}D_x\\ D_y\\ D_z\end{pmatrix}=\frac{1}{n^2}\begin{pmatrix}D_x\\ D_y\\ D_z\end{pmatrix}$$

THE INDEX ELLIPSOID

$$\begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \frac{1}{n^2}\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \longrightarrow$$

Third equation says: $D_z=0$

$\downarrow$

$\mathbf{D} \perp \mathbf{s}$

$$\begin{pmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{pmatrix}\begin{pmatrix} D_x \\ D_y \end{pmatrix} = \frac{1}{n^2}\begin{pmatrix} D_x \\ D_y \end{pmatrix}$$

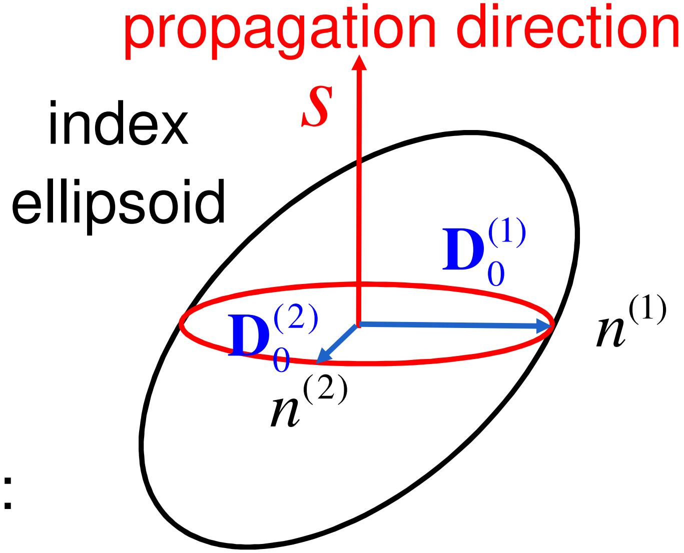
This is an eigenvalue problem for a 2x2 matrix: $\begin{pmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{pmatrix}$

solution:

- two eigenvalues $1/n^2$
- two eigenmodes for the vector $\begin{pmatrix} D_x \\ D_y \end{pmatrix}$

THE INDEX ELLIPSOID

$$\mathbf{D} = \mathbf{D}_0\, \mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n \mathbf{s}\cdot\mathbf{r}\right)}$$



Conclusion:

to find the wave solutions with *k* along *s*:

- use the index ellipsoid
- intersect with a plane perpendicular to *s*
- the result is an ellipse
- the principle axes of the ellipse give the eigenmodes for *D*
- the lengths of the intersections give the refractive indices

PROPERTIES OF EIGENMODES

$$\boxed{\begin{array}{l}\mathbf{D}=\mathbf{D}_0\,\mathrm{e}^{\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\\ \mathbf{E}=\mathbf{E}_0\,\mathrm{e}^{\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\\ \mathbf{H}=\mathbf{H}_0\,\mathrm{e}^{\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\end{array}}$$

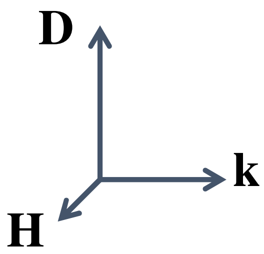
From the laws of Maxwell, we find:

$$\nabla\times\mathbf{E}=-\frac{\partial\mu_0\mathbf{H}}{\partial t}\qquad\rightarrow -i\mathbf{k}\times\mathbf{E}=-i\omega\mu_0\mathbf{H}\qquad\rightarrow \mathbf{H}\perp\mathbf{E};\ \mathbf{H}\perp\mathbf{k}$$

$$\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}\qquad\rightarrow -i\mathbf{k}\times\mathbf{H}=i\omega\mathbf{D}\qquad\rightarrow \mathbf{D}\perp\mathbf{H};\ \mathbf{D}\perp\mathbf{k}$$

PROPERTIES OF EIGENMODES

$$\mathbf{H} \perp \mathbf{k};\mathbf{D} \perp \mathbf{H};\mathbf{D} \perp \mathbf{k}$$



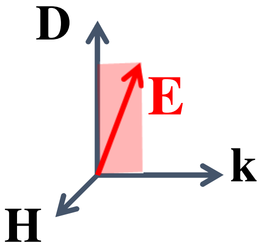
Thus *D*, *H* and *k* form a set of mutually perpendicular vectors

E is perpendicular to *H*, and lies therefore in the *D*,*k* plane

For anisotropic materials:

E is normally not parallel to *D*

E is normally not perpendicular to *k*



OPTICAL CLASSIFICATION OF ANISOTROPIC MEDIA

Three optical classes

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_1^2 & 0 & 0 \\ 0 & n_2^2 & 0 \\ 0 & 0 & n_3^2 \end{pmatrix}$$

Biaxial crystals: $n_1 \neq n_2 \neq n_3$

2 optical axes $n_1 < n_2 < n_3$

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$$

Uniaxial crystals: $n_1 = n_2 = n_o$

1 optical axis $n_3 = n_e$

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix}$$

Isotropic materials: $n_1 = n_2 = n_3 = n$

all directions optical axes

Liquid Crystals and Photonics

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OPTICAL CLASSIFICATION OF ANISOTROPIC MEDIA

Relation and optical class and symmetry elements
of a crystal?

Cubic crystal: three axes *x*,*y*,*z* are equivalent
ellipsoid with three equivalent axes is a sphere
therefore $n_1 = n_2 = n_3 = n$
the dielectric tensor is isotropic

Point group of a crystal:
set of rotational/mirror symmetries of a crystal

OPTICAL CLASSIFICATION OF ANISOTROPIC MEDIA

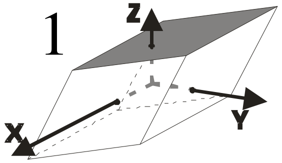
point groups of crystals

Crystal System	Class Symbol		Elements of Symmetry
	International	Schoenflies	
Triclinic	1	C_1	E
	$\bar{1}$	C_i	EI
Monoclinic	m	C_s	$E\sigma_h$
	2	C_2	EC_2
	2/m	C_{2h}	$EC_2I\sigma_h$
Orthorhombic	2mm	C_{2v}	$EC_2\sigma_v'\sigma_v''$
	222	D_2	$EC_2C_2'C_2''$
	mmm	D_{2h}	$EC_2C_2'C_2''I\sigma_h\sigma_v'\sigma_v''$
Tetragonal	4	C_4	$E2C_4C_2$
	$\bar{4}$	S_4	$E2S_4C_2$
	4/m	C_{4h}	$E2C_4C_2I2S_4\sigma_h$
	4mm	C_{4v}	$E2C_4C_22\sigma_v'2\sigma_v''$
	$\bar{4}$2m	D_{2d}	$EC_2C_2'C_2''\sigma_v'2S_4\sigma_v''$
	422	D_4	$E2C_4C_22C_2'2C_2''$
	4/mmm	D_{4h}	$E2C_4C_22C_2'2C_2''I2S_4\sigma_h2\sigma_v'2\sigma_v''$

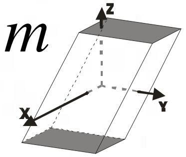
Biaxial

Uniaxial

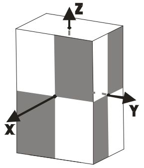
1

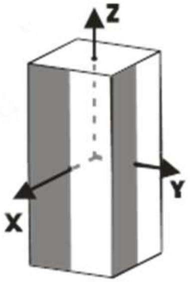


$2/m$



222



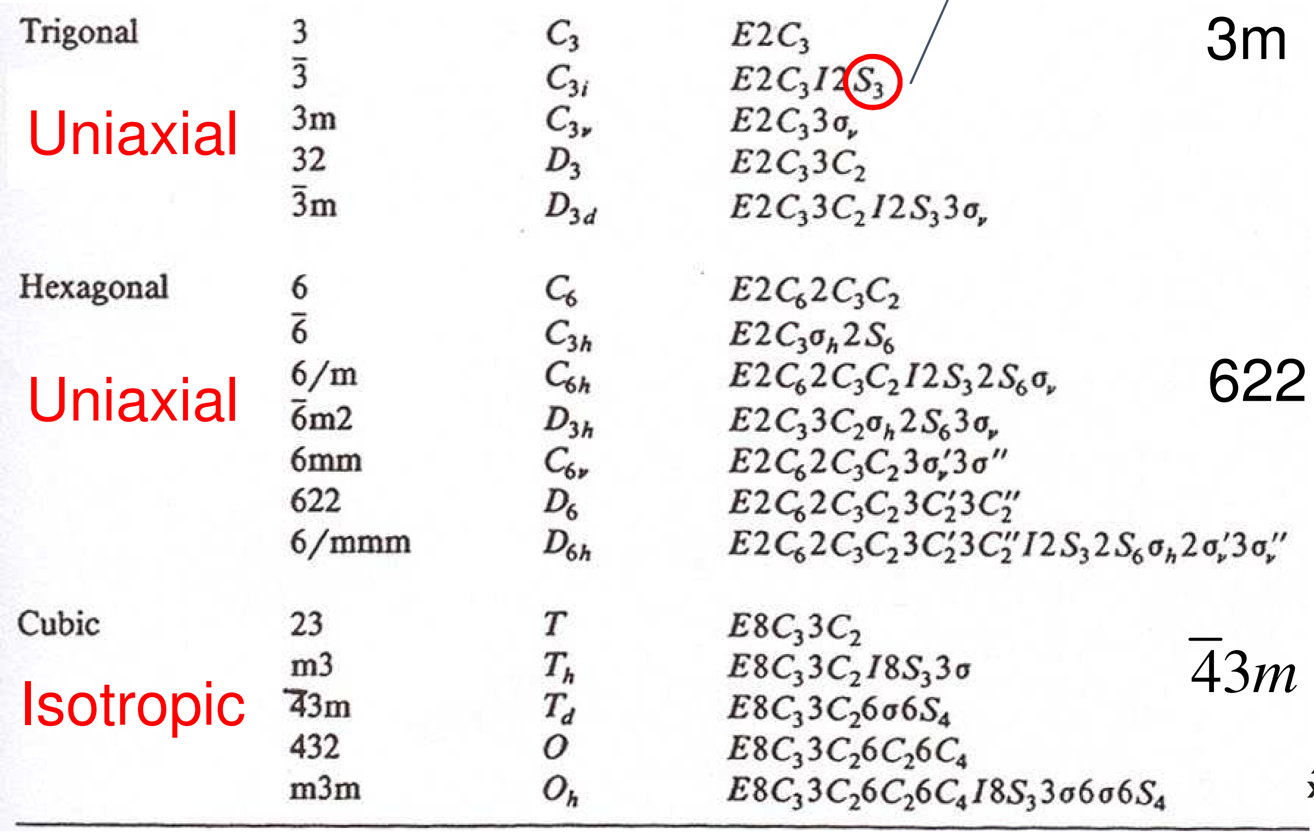


$4/m$

OPTICAL CLASSIFICATION OF ANISOTROPIC MEDIA

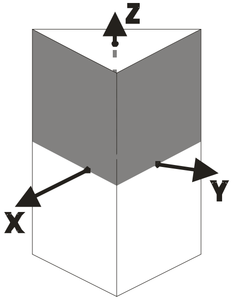
Overview of all point groups

In literature: often this is called S_6 rotation and mirror image

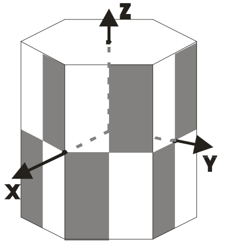


Trigonal	3	C_3	$E2C_3$
	$\bar{3}$	C_{3i}	$E2C_3I2S_3$
Uniaxial	3m	C_{3v}	$E2C_33\sigma_v$
	32	D_3	$E2C_33C_2$
	$\bar{3}$m	D_{3d}	$E2C_33C_2I2S_33\sigma_v$
Hexagonal	6	C_6	$E2C_62C_3C_2$
	$\bar{6}$	C_{3h}	$E2C_3\sigma_h2S_6$
Uniaxial	6/m	C_{6h}	$E2C_62C_3C_2I2S_32S_6\sigma_v$
	$\bar{6}$m2	D_{3h}	$E2C_33C_2\sigma_h2S_63\sigma_v$
	6mm	C_{6v}	$E2C_62C_3C_23\sigma_v'3\sigma''$
	622	D_6	$E2C_62C_3C_23C_2'3C_2''$
	6/mmm	D_{6h}	$E2C_62C_3C_23C_2'3C_2''I2S_32S_6\sigma_h2\sigma_v'3\sigma_v''$
Cubic	23	T	$E8C_33C_2$
	m3	T_h	$E8C_33C_2I8S_33\sigma$
Isotropic	$\bar{4}$3m	T_d	$E8C_33C_26\sigma6S_4$
	432	O	$E8C_33C_26C_26C_4$
	m3m	O_h	$E8C_33C_26C_26C_4I8S_33\sigma6\sigma6S_4$

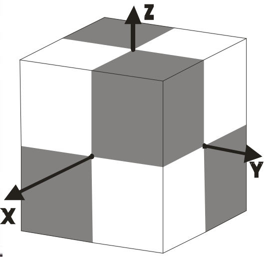
3m



622



$\overline{4}3m$



LIGHT PROPAGATION IN UNIAXIAL CRYSTALS

dielectric constant

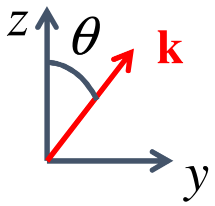
$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$$

index ellipsoid $$\frac{X^2+Y^2}{n_o^2}+\frac{Z^2}{n_e^2}=1$$

rotational symmetry around the *z*-axis

we can assume that the *k* vector is in the *yz* plane

wave vector makes an angle θ with the *z*-axis



$$\mathbf{k} = \frac{\omega}{c} n\left(\sin\theta\,\mathbf{e}_y + \cos\theta\,\mathbf{e}_z\right)$$

$$\begin{cases} k_x = 0 \\ k_y = \dfrac{\omega}{c} n \sin\theta \\ k_z = \dfrac{\omega}{c} n \cos\theta \end{cases}$$

LIGHT PROPAGATION IN UNIAXIAL CRYSTALS

Index ellipsoid:

$$\frac{X^2+Y^2}{n_o^2}+\frac{Z^2}{n_e^2}=1$$

k-vector ~ $\mathbf{s}=\sin\theta\,\mathbf{e}_y+\cos\theta\,\mathbf{e}_z$

plane perpendicular to the *k*-vector

intersection with index ellipsoid is an ellipse

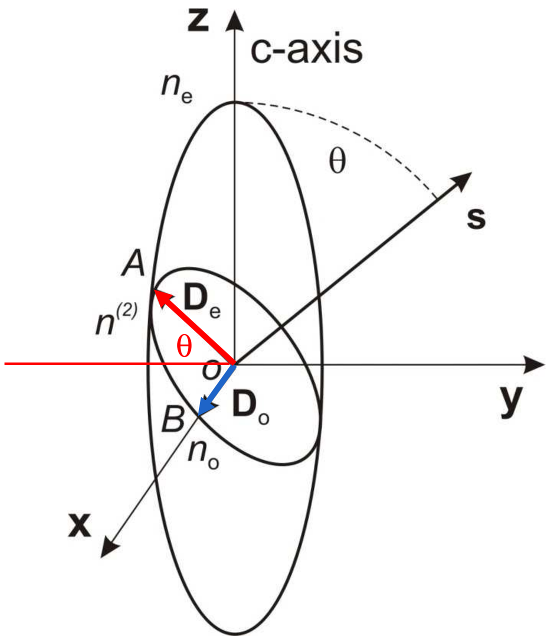
two eigenmodes:

- ordinary mode: D_o along x with n_o
- extra-ordinary mode: $n^{(2)}$ or n_{eff}

D_e in the yz plane, $\perp$ s

$$\begin{cases} X=0 \\ Y=-n^{(2)}\cos\theta \\ Z=n^{(2)}\sin\theta \end{cases}$$

$$\frac{n^{(2)2}\cos^2\theta}{n_o^2}+\frac{n^{(2)2}\sin^2\theta}{n_e^2}=1$$



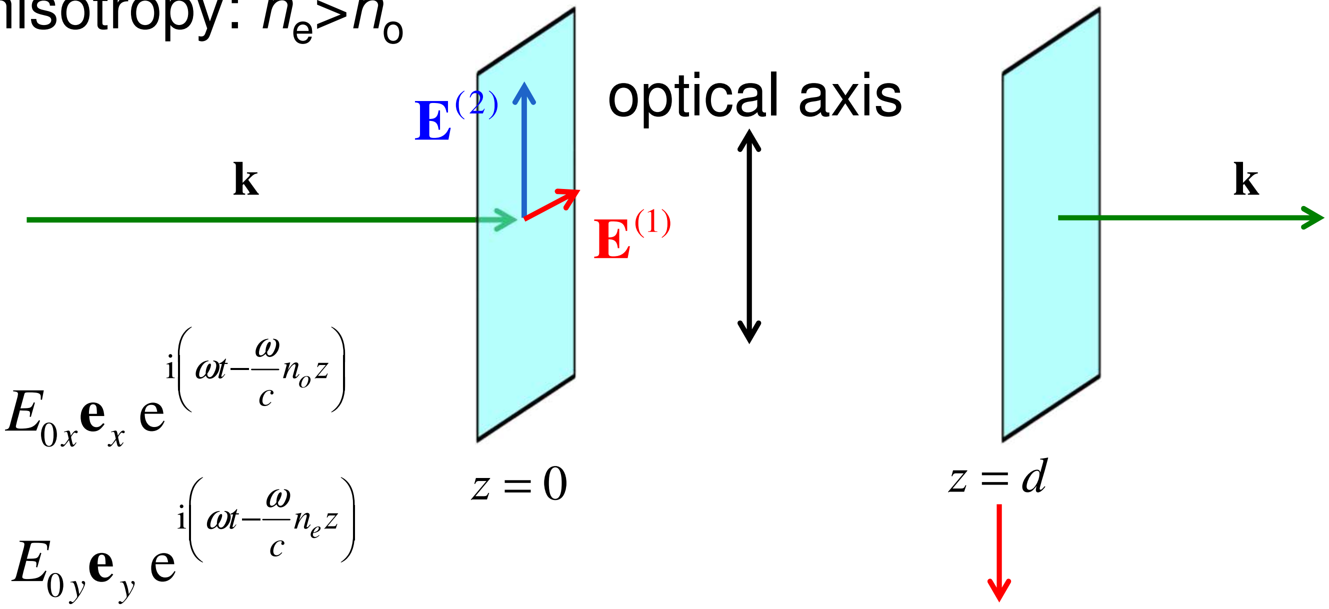
Liquid Crystals and Photonics

GHENT UNIVERSITY

WAVELENGTH RETARDATION PLATES

Uniaxial material, optic axis in the plane (A-plate)

positive anisotropy: $n_e > n_o$



$$\mathbf{E}^{(1)} = E_{0x}\mathbf{e}_x\,\mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n_o z\right)}$$

$$\mathbf{E}^{(2)} = E_{0y}\mathbf{e}_y\,\mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n_e z\right)}$$

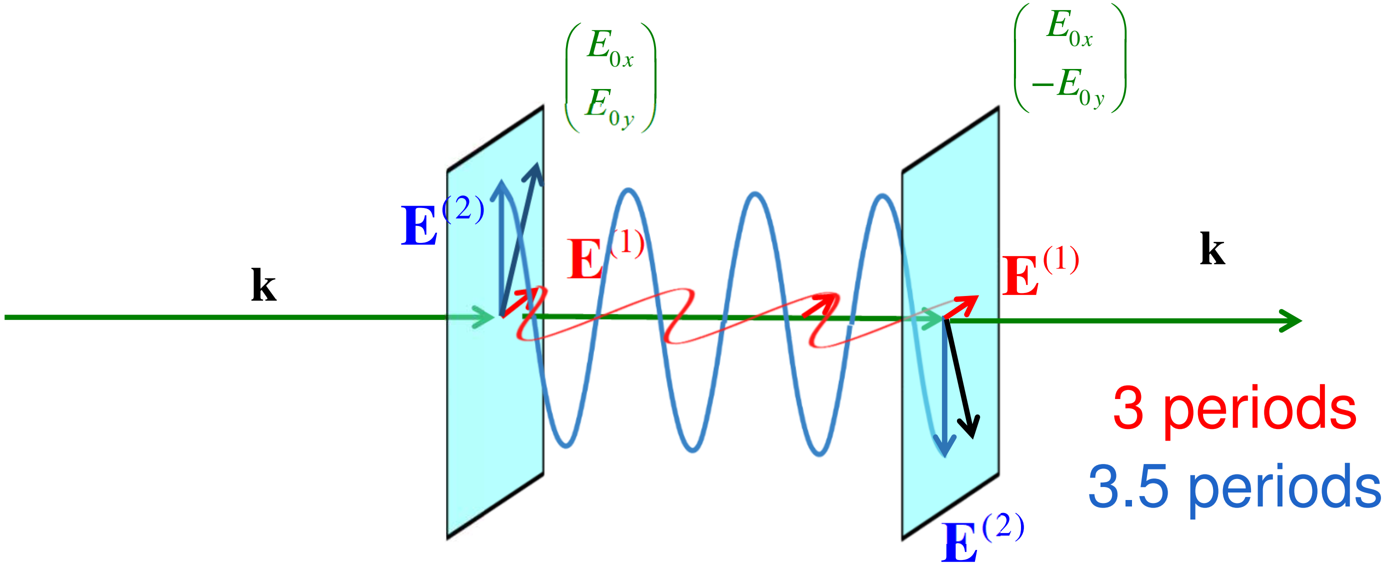
phase delay φ_1-φ_2: $\frac{\omega}{c}\left(n_e - n_o\right)d$

optical path length difference: $\left(n_e - n_o\right)d$

WAVELENGTH RETARDATION PLATES

Example: half-wave plate

the wave **E**$_2$ is delayed by a phase π



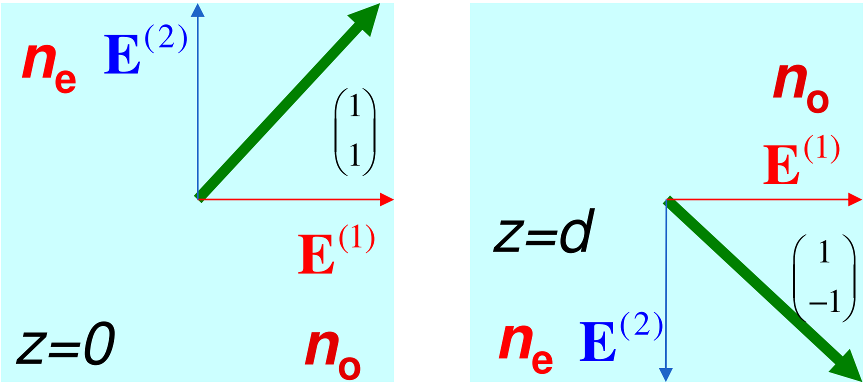
$$\frac{\omega}{c}(n_e - n_o)d = \frac{2\pi}{\lambda}(n_e - n_o)d = \pi$$

WAVELENGTH RETARDATION PLATES

z=0: linearly polarized at 45°

phase delay π

mirror polarization plane



seen from the destination

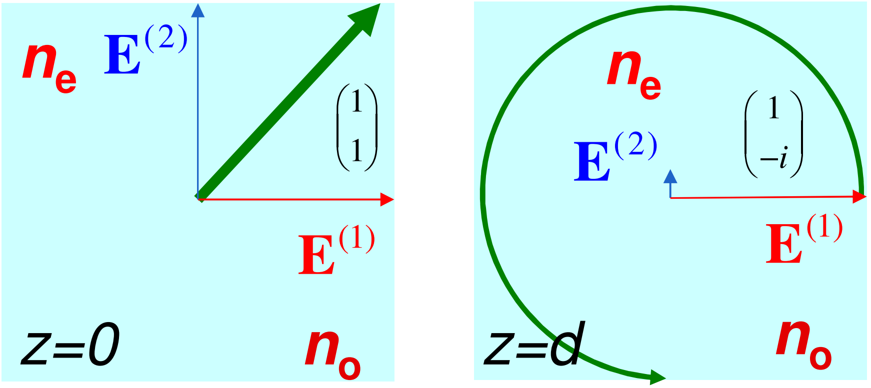
quarter wave plate

E_2 phase delay $\pi/2$

left circular polarization

E counter-clockwise in time

(left-handed helix in space)

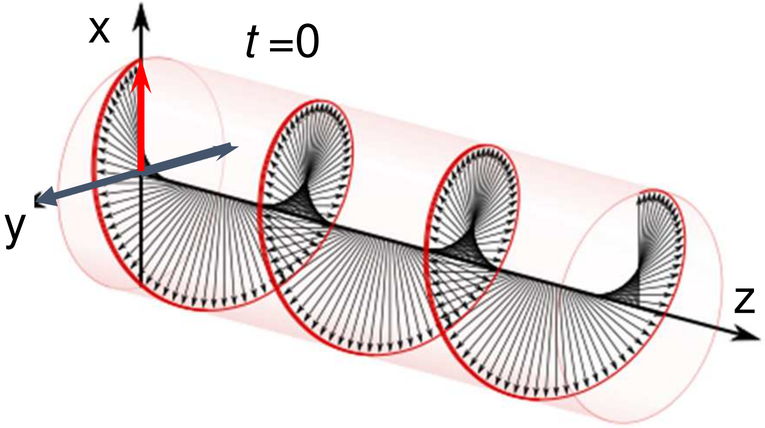


POLARIZATION STATES OF LIGHT

right-handed circularly polarized light (propagation along z)

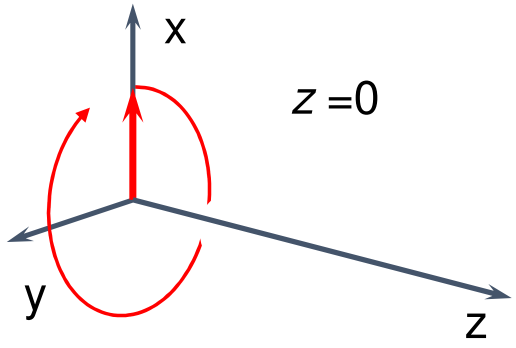
$$\mathbf{E} = \mathrm{Re}\left[\left(\mathbf{e}_x + i\mathbf{e}_y\right)e^{i(\omega t - kz)}\right]$$

$$\begin{pmatrix}1\\ i\end{pmatrix}$$



variation of E with position:
right handed helix

wikipedia



variation of E with time
(clockwise when seen
from the destination)

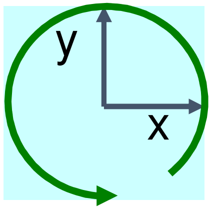
POLARIZATION STATES OF LIGHT (TOOLBOX 2)

$$\mathbf{E} = \mathrm{Re}\left[\left(J_x\mathbf{e}_x + J_y\mathbf{e}_y\right)e^{i(\omega t - kz)}\right]$$

Polarisation	**Jones vector**
Linearly polarised parallel to the *x*-axis	$\begin{bmatrix}1\\0\end{bmatrix}$
Linearly polarised parallel to the *y*-axis	$\begin{bmatrix}0\\1\end{bmatrix}$
Linearly polarised making an angle α with the *x*-axis	$\begin{bmatrix}\cos(\alpha)\\\sin(\alpha)\end{bmatrix}$
Circularly polarised (right)	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$
Circularly polarised (left)	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

LH pol

$$\mathrm{Re}\left[\left(\mathbf{e}_x - i\mathbf{e}_y\right)e^{i(\omega t - kz)}\right]$$



t =0: e_x

t =π/2ω: e_y

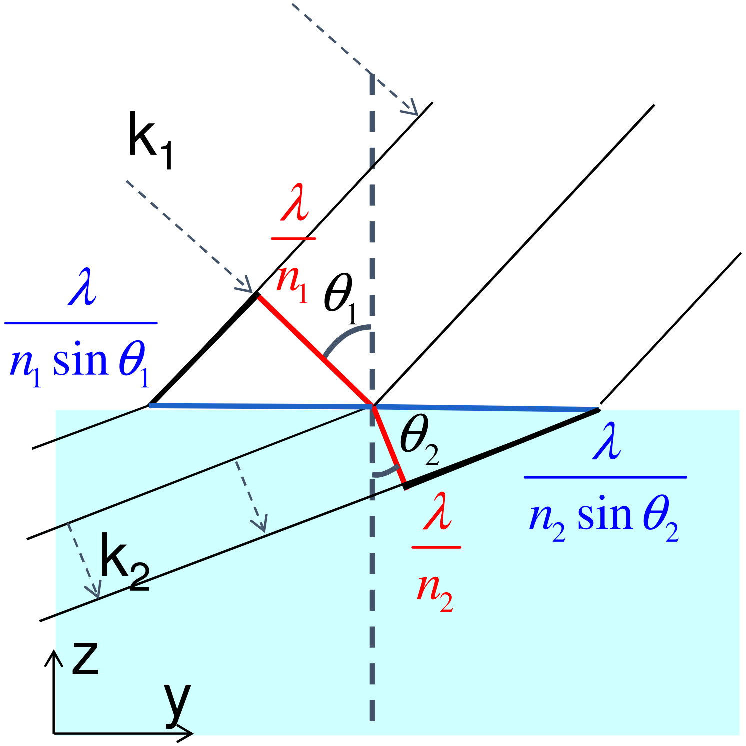
DOUBLE REFRACTION

wavelength in two media

distance between two wave fronts along the surface should be equal

$$\frac{\lambda}{n_1 \sin \theta_1} = \frac{\lambda}{n_2 \sin \theta_2}$$

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ Law of Snellius



$$\frac{\omega}{c} n_1 \sin \theta_1 = \frac{\omega}{c} n_2 \sin \theta_2 \longrightarrow k_1 \sin \theta_1 = k_2 \sin \theta_2$$

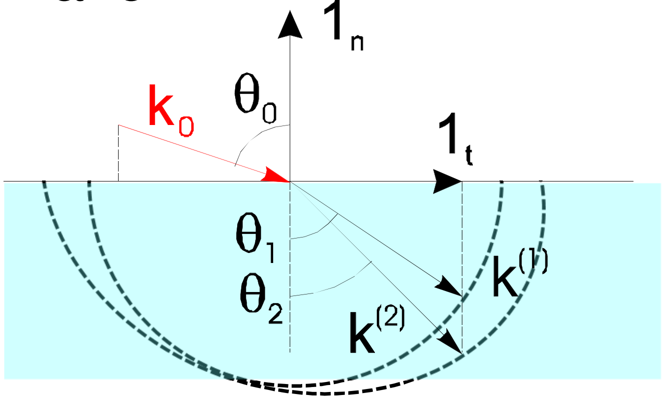
$$\boxed{k_{1y} = k_{2y}}$$

DOUBLE REFRACTION

Light incident from an isotropic medium into an anisotropic medium

$$k_0 = \frac{\omega}{c} n_0 = \frac{2\pi}{\lambda} n_0$$

k-vector represents periodicity of a wave



Tangential component k_t of

all *k*-vectors should be the same

Intersection with normal surface?

two solutions: bi-refringence

double refraction

$$n_0 \sin\theta_0 = n^{(1)} \sin\theta_1 = n^{(2)} \sin\theta_2$$

Kristiaan Neyts

DICHROISM

Linear dichroism

anisotropic absorption

$$\bar{\bar{\varepsilon}}_r = \begin{pmatrix} (n_1{}'+\mathrm{i}\,n_1{}'')^2 & 0 & 0 \\ 0 & (n_2{}'+\mathrm{i}\,n_2{}'')^2 & 0 \\ 0 & 0 & (n_3{}'+\mathrm{i}\,n_3{}'')^2 \end{pmatrix}$$

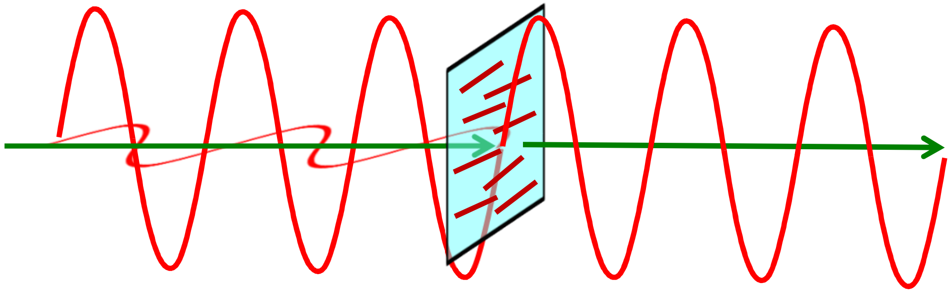
linear polarizer: *y* polarization is absorbed, *x*-pol is transmitted

$$\bar{\bar{\varepsilon}}_r = \begin{pmatrix} n_1{}'^2 & 0 & 0 \\ 0 & (n_2{}'+\mathrm{i}\,n_2{}'')^2 & 0 \\ 0 & 0 & n_3{}'^2 \end{pmatrix}$$

DICHROISM

Polaroid polarizer

oriented absorbing polymer molecules



Wire grid polarizer

parallel metal wires absorb the *E*-field component parallel to the wires