FACULTY OF ENGINEERING
AND ARCHITECTURE

ELECTRONICS AND INFORMATION SYSTEMS DEPARTMENT
LIQUID CRYSTALS AND PHOTONICS RESEARCH GROUP

LIQUID CRYSTALS AND LIGHT EMITTING MATERIALS FOR PHOTONIC APPLICATIONS

Kristiaan Neyts April 2018 Lecture series at WAT in Warsaw

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nb-photonics
Center for Nano- and Biophotonics
Ghent University

Liquid Crystals and Photonics

OVERVIEW

Electrical and optical properties of materials (6h)

Polarizability of dielectric materials
Light propagation
Refraction of light
Conductors and semiconductors
Light propagation in anisotropic media
Polarized light
Spontaneous emission

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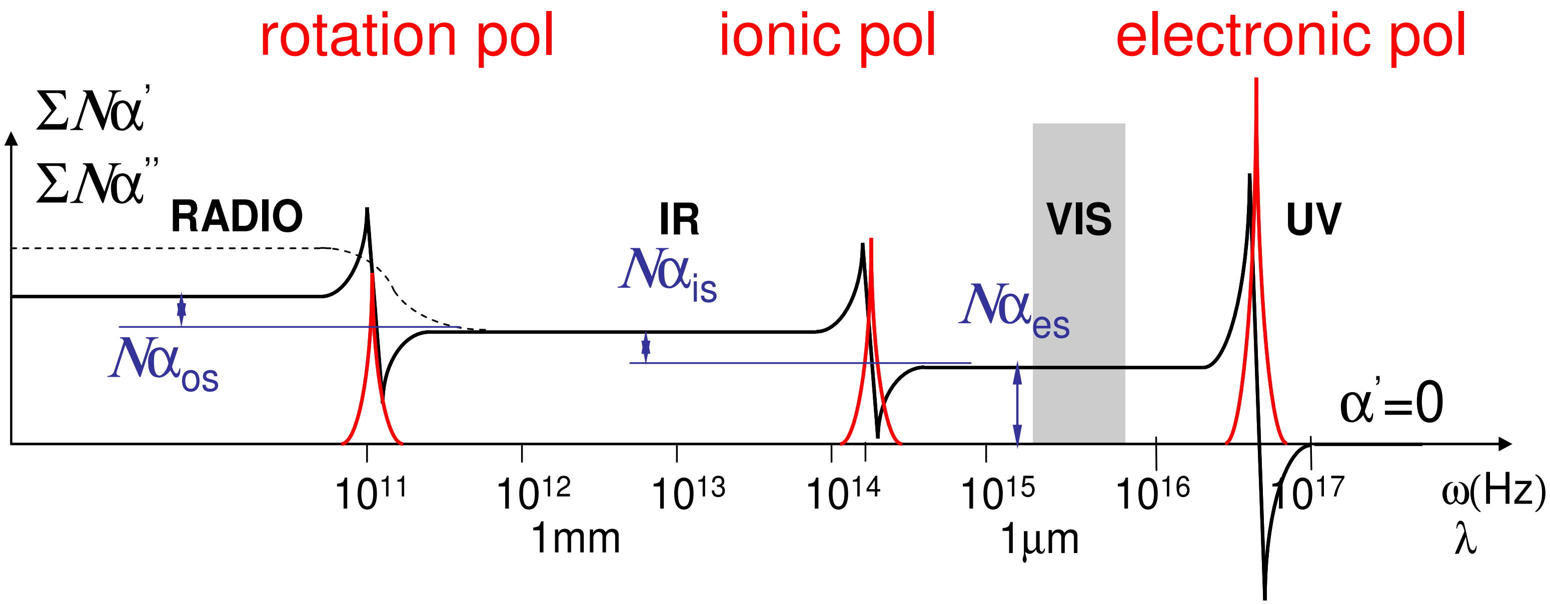
GENERAL DIELECTRIC BEHAVIOR

different resonance/relaxation processes

sum of all polarizabilities per unit volume:



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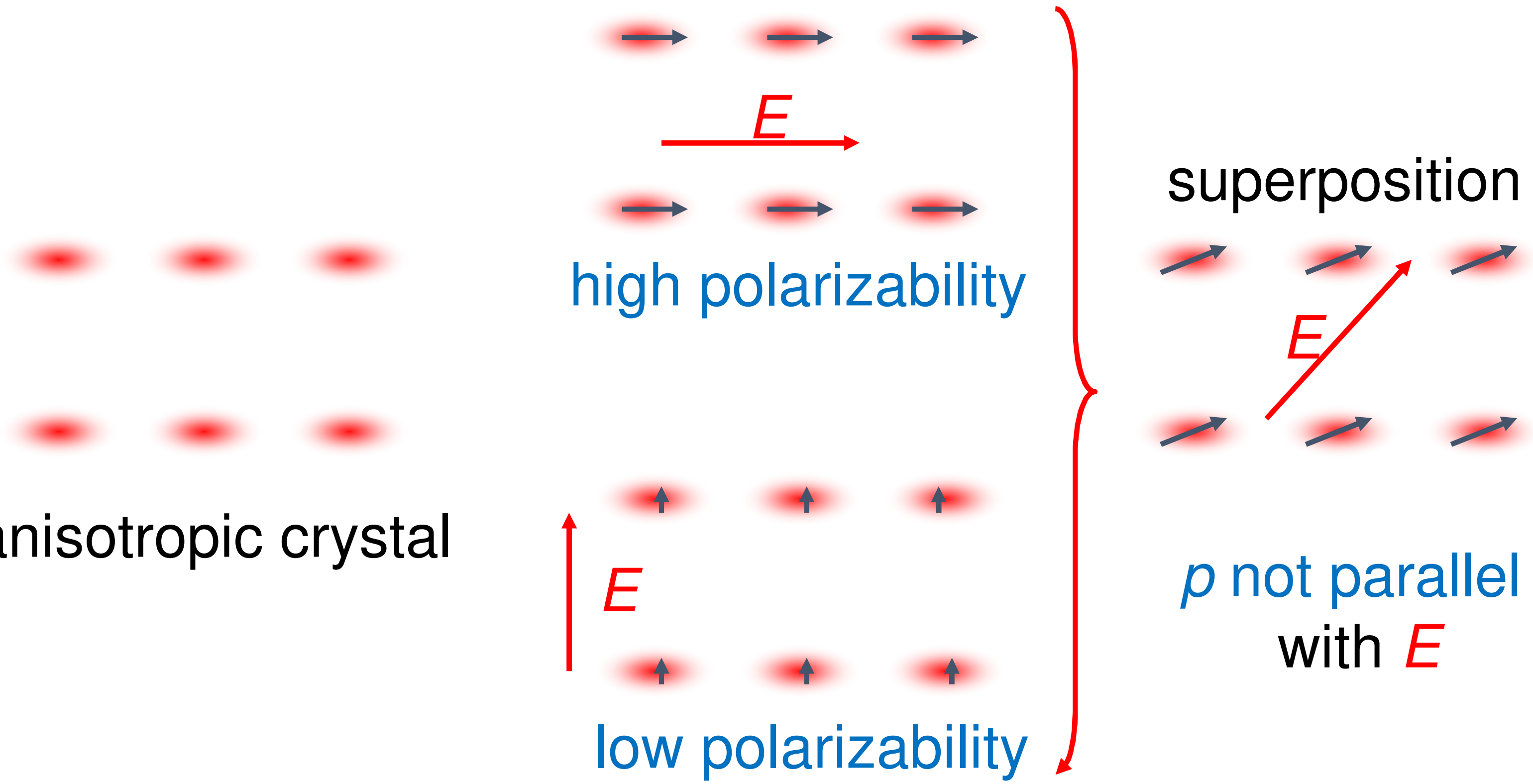
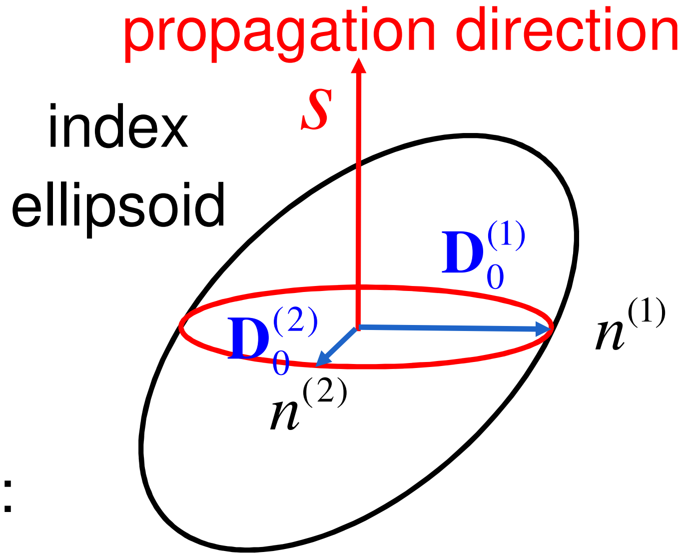
ANISOTROPIC POLARIZABILITY

Anisotropic polarizability in a crystalline structure



THE INDEX ELLIPSOID

$$\mathbf{D} = \mathbf{D}_0\, \mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n \mathbf{s} \cdot \mathbf{r}\right)}$$



Conclusion:

to find the wave solutions with *k* along *s*:

- use the index ellipsoid
- intersect with a plane perpendicular to *s*
- the result is an ellipse
- the principle axes of the ellipse give the eigenmodes for *D*
- the lengths of the intersections give the refractive indices

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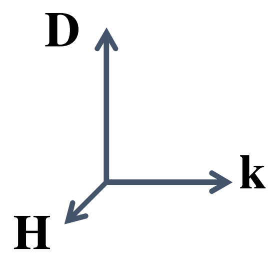
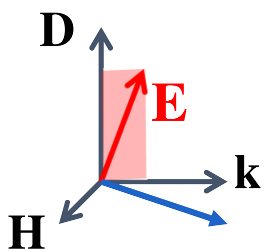
PROPERTIES OF EIGENMODES

$$\mathbf{H} \perp \mathbf{k}; \mathbf{D} \perp \mathbf{H}; \mathbf{D} \perp \mathbf{k}$$

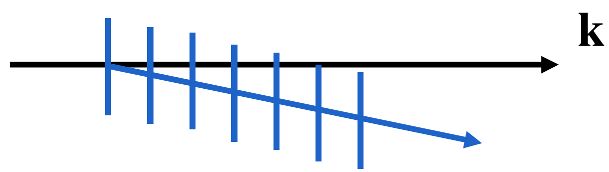


E is normally not parallel to *D*

E is normally not perpendicular to *k*



vector of pointing S



vector of Poynting S

OPTICAL CLASSIFICATION OF ANISOTROPIC MEDIA

Three optical classes

Biaxial crystals: $n_1 \neq n_2 \neq n_3$

2 optical axes $n_1 < n_2 < n_3$

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_1^2 & 0 & 0 \\ 0 & n_2^2 & 0 \\ 0 & 0 & n_3^2 \end{pmatrix}$$

Uniaxial crystals: $n_1 = n_2 = n_o$

1 optical axis $n_3 = n_e$

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$$

Isotropic materials: $n_1 = n_2 = n_3 = n$

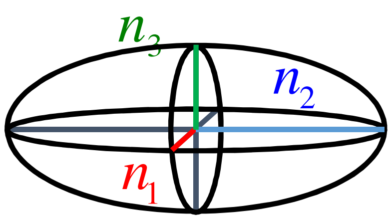
all directions optical axes

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix}$$

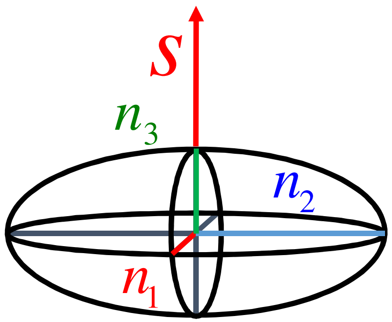
LIGHT PROPAGATION ALONG THE PRINCIPLE AXES

By rotation (coordinate transformation) this can be reduced to:

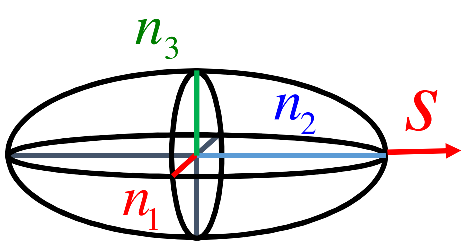
$$\frac{X^2}{n_1^2}+\frac{Y^2}{n_2^2}+\frac{Z^2}{n_3^2}=1$$



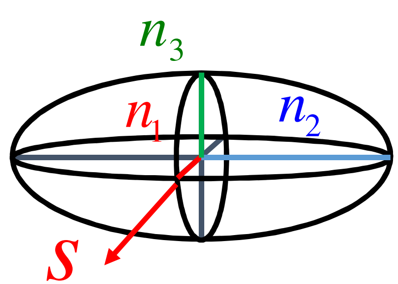
$$\bar{\bar{\eta}}=\begin{pmatrix} \frac{1}{n_1^2} & 0 & 0 \\ 0 & \frac{1}{n_2^2} & 0 \\ 0 & 0 & \frac{1}{n_3^2} \end{pmatrix}$$



two plane waves with n_1 and n_2



two plane waves with n_1 and n_3



two plane waves with n_2 and n_3

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LIGHT PROPAGATION IN UNIAXIAL CRYSTALS

dielectric constant

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$$

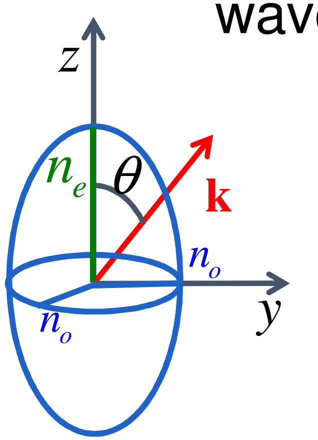
index ellipsoid

$$\frac{X^2 + Y^2}{n_o^2} + \frac{Z^2}{n_e^2} = 1$$

rotational symmetry around the *z*-axis

we can assume that the *k* vector is in the *yz* plane

wave vector makes an angle θ with the *z*-axis



$$\mathbf{k} = \frac{\omega}{c} n \left(\sin\theta \, \mathbf{e}_y + \cos\theta \, \mathbf{e}_z \right)$$

$$\begin{cases} k_x = 0 \\ k_y = \dfrac{\omega}{c} n \sin\theta \\ k_z = \dfrac{\omega}{c} n \cos\theta \end{cases}$$

LIGHT PROPAGATION IN UNIAXIAL CRYSTALS

Index ellipsoid: $$\frac{X^2+Y^2}{n_o^2}+\frac{Z^2}{n_e^2}=1$$

$$\mathbf{k}=\frac{\omega}{c}n\left(\sin\theta\,\mathbf{e}_y+\cos\theta\,\mathbf{e}_z\right)$$

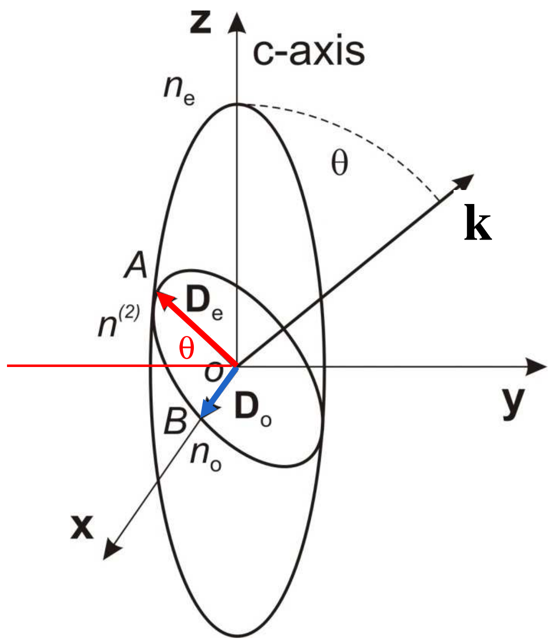
plane perpendicular to the *k*-vector

intersection with index ellipsoid is an ellipse

two eigenmodes:

- ordinary mode: D_o along x with n_o
- extra-ordinary mode: $n^{(2)}$ or n_{eff}

  D_e in the yz plane, $\perp$ s



$$\begin{cases} X=0 \\ Y=-n^{(2)}\cos\theta \\ Z=n^{(2)}\sin\theta \end{cases}$$

$$\frac{n^{(2)2}\cos^2\theta}{n_o^2}+\frac{n^{(2)2}\sin^2\theta}{n_e^2}=1$$

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POLARIZED LIGHT

Linearly polarized light along *x* (LP)

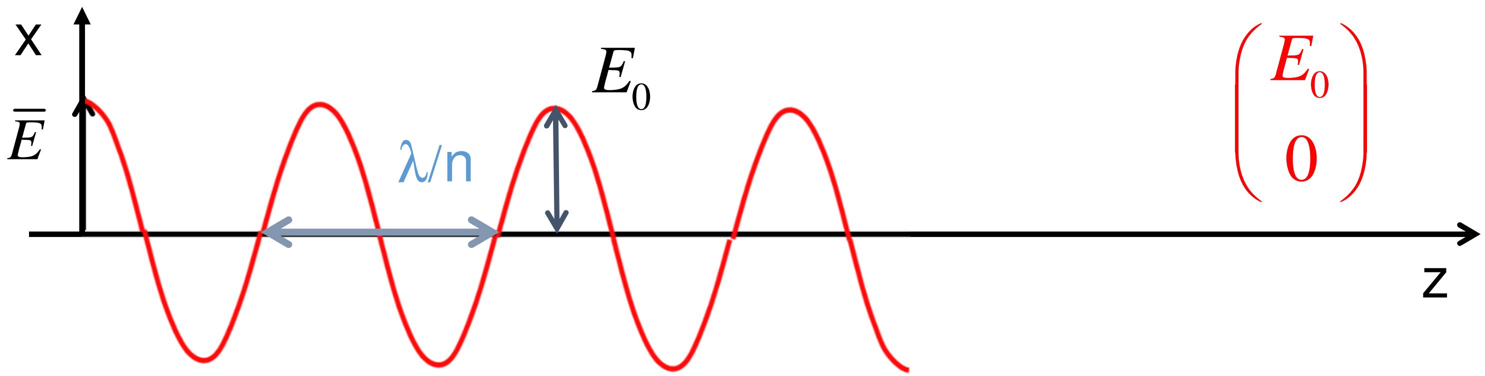
field, amplitude

$$\overline{E}(z,t) = E(z,t)\overline{1}_x = E_0\overline{1}_x \,\mathrm{Re}\left[e^{i(\omega t - kz)} \right]$$

$$= E_0\overline{1}_x \cos\left(\omega t - kz\right)$$

k: wave vector ω: pulsation

$$k = \frac{\omega}{c} n = \frac{2\pi}{\lambda} n; \qquad \omega = 2\pi f = \frac{2\pi}{T}$$



x and *y* component of the field in a matrix

$$\begin{pmatrix} E_0 \\ 0 \end{pmatrix}$$

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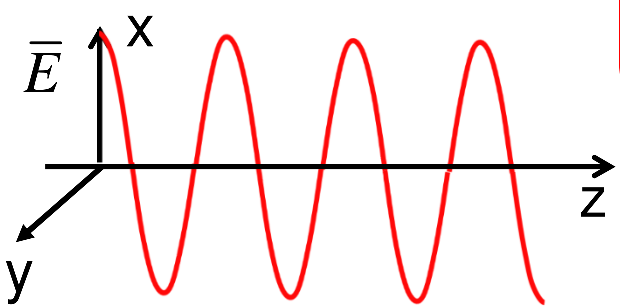
POLARIZED LIGHT

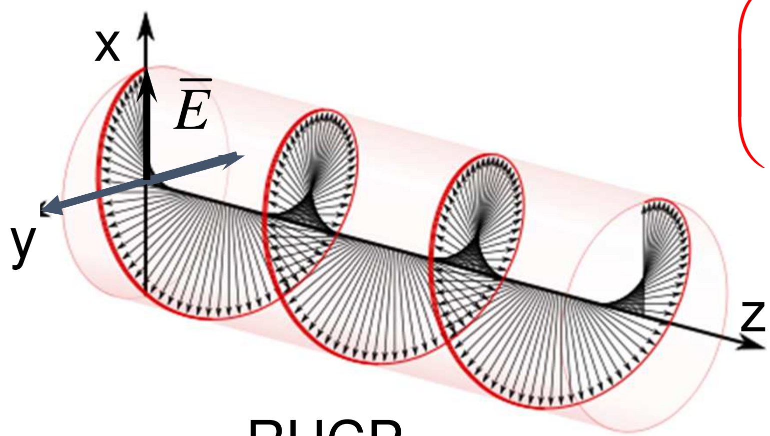
Light can have different polarization states

- linear polarization (LP: HLP, VLP or angle α)
- circular polarization (RHCP or LHCP)
- elliptical polarization
- unpolarized light (UNP)



$$\begin{pmatrix} E_0 \\ 0 \end{pmatrix}$$

LINEAR POLARZIATION (X)



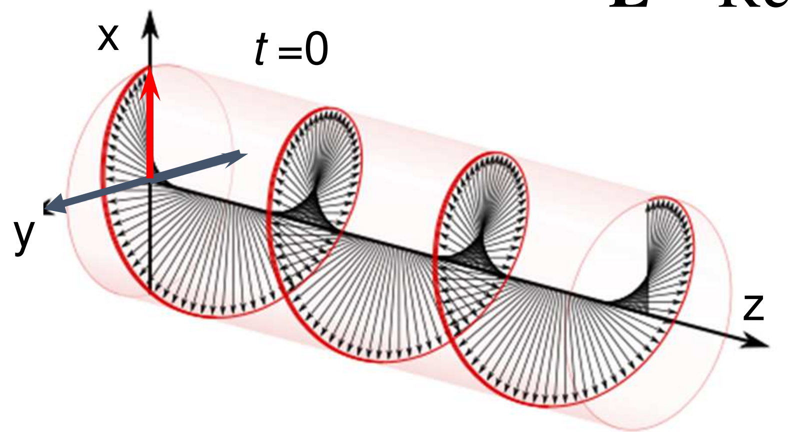
$$\begin{pmatrix} E_0 \\ iE_0 \end{pmatrix}$$

RHCP

POLARIZATION STATES OF LIGHT

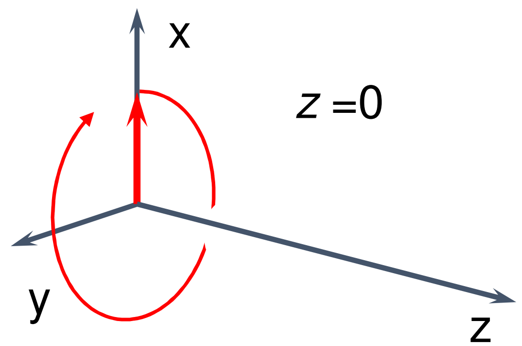
right-handed circularly polarized light (propagation along z)

$$\mathbf{E} = \mathrm{Re}\left[\left(\mathbf{e}_x + i\mathbf{e}_y\right)e^{i(\omega t - kz)}\right] \qquad \begin{pmatrix} 1 \\ i \end{pmatrix}$$



variation of *E* with position:
right handed helix

wikipedia



variation of *E* with time
(clockwise when seen
from the destination)

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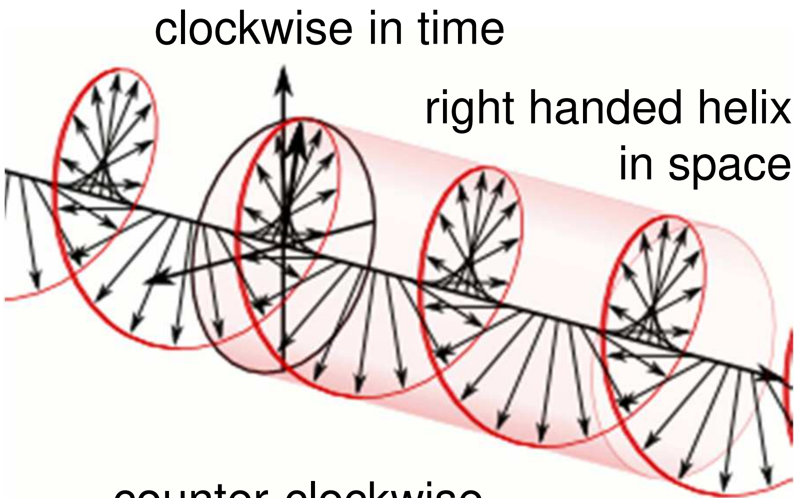
POLARIZED LIGHT

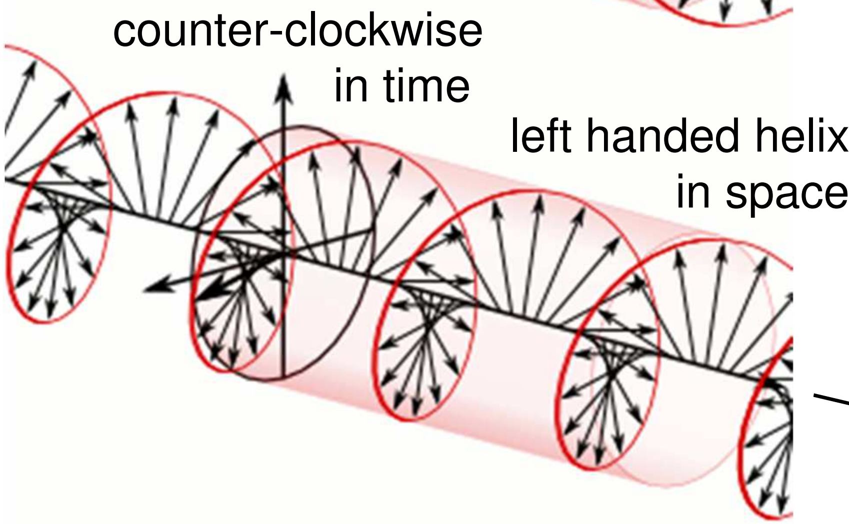
different conventions can be found in literature!

RH CP



$$\begin{pmatrix} E_0 \\ iE_0 \end{pmatrix}$$

LH CP



$$\begin{pmatrix} E_0 \\ -iE_0 \end{pmatrix}$$

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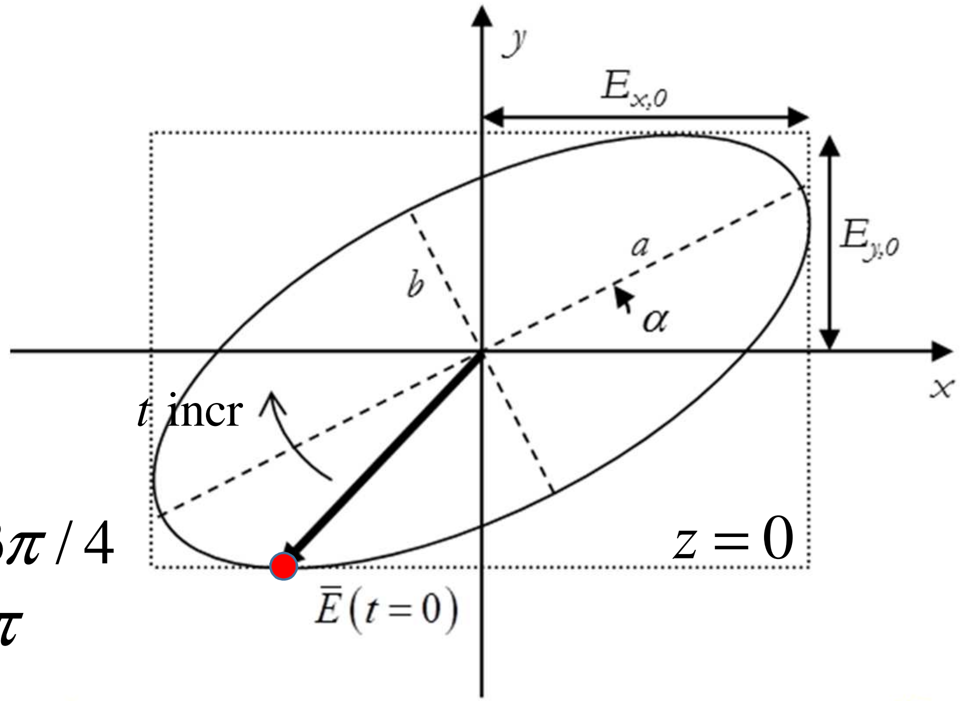
POLARIZED LIGHT

Polarized light in general: elliptical

$$\bar{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \mathrm{Re}\begin{pmatrix} E_{x,0} e^{i(\omega t - kz + \phi_x)} \\ E_{y,0} e^{i(\omega t - kz + \phi_y)} \end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix} E_{x,0} e^{i\phi_x} \\ E_{y,0} e^{i\phi_y} \end{pmatrix} e^{i(\omega t - kz)}\right]$$

Jones vector

$$\bar{J} = \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} E_{x,0} e^{i\phi_x} \\ E_{y,0} e^{i\phi_y} \end{pmatrix}$$



RH elliptical clockwise

$$\begin{cases} \phi_x = 3\pi/4 \\ \phi_y = \pi \end{cases}$$

POLARIZED LIGHT

$$\mathbf{E} = \mathrm{Re}\left[\left(J_x \mathbf{e}_x + J_y \mathbf{e}_y\right) e^{i(\omega t - kz)}\right]$$

Polarisation	Jones vector
Linearly polarised parallel to the *x*-axis	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Linearly polarised parallel to the *y*-axis	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Linearly polarised making an angle α with the *x*-axis	$\begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$
Circularly polarised (right)	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$
Circularly polarised (left)	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

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STOKES PARAMETERS

Definition for light propagating along z

$$S_0 = 1$$

Degree of polarization
we will use S_0=1

$$S_1 = \frac{|J_x|^2 - |J_y|^2}{|J_x|^2 + |J_y|^2}$$

Polarized along *x*
(versus *y*)

$$S_2 = \frac{1}{2}\frac{|J_x + J_y|^2 - |J_x - J_y|^2}{|J_x|^2 + |J_y|^2}$$

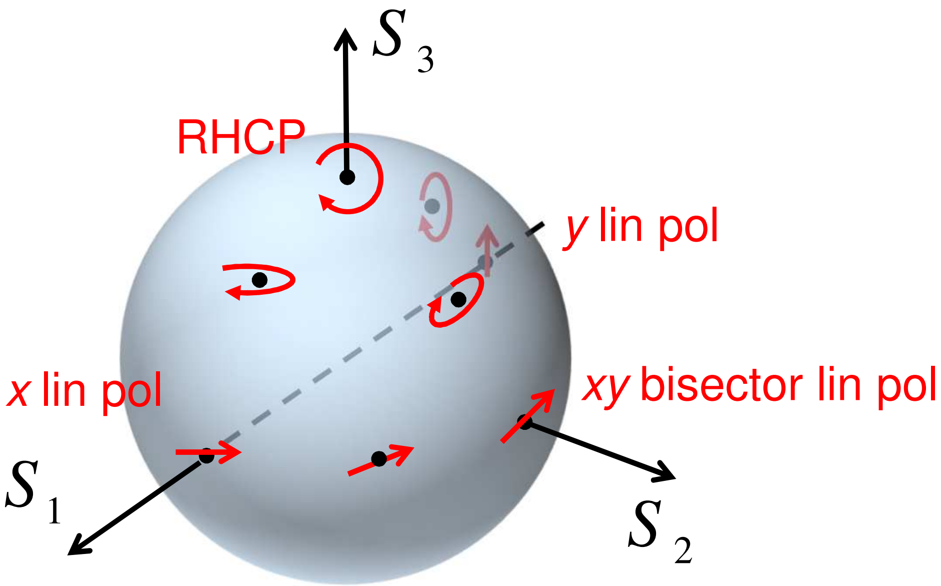
Polarized along bisector *xy*
(versus bisector –*xy*)

$$S_3 = \frac{1}{2}\frac{|J_x - iJ_y|^2 - |J_x + iJ_y|^2}{|J_x|^2 + |J_y|^2}$$

RH circularly polarized
(versus LH circularly pol.)

STOKES PARAMETERS

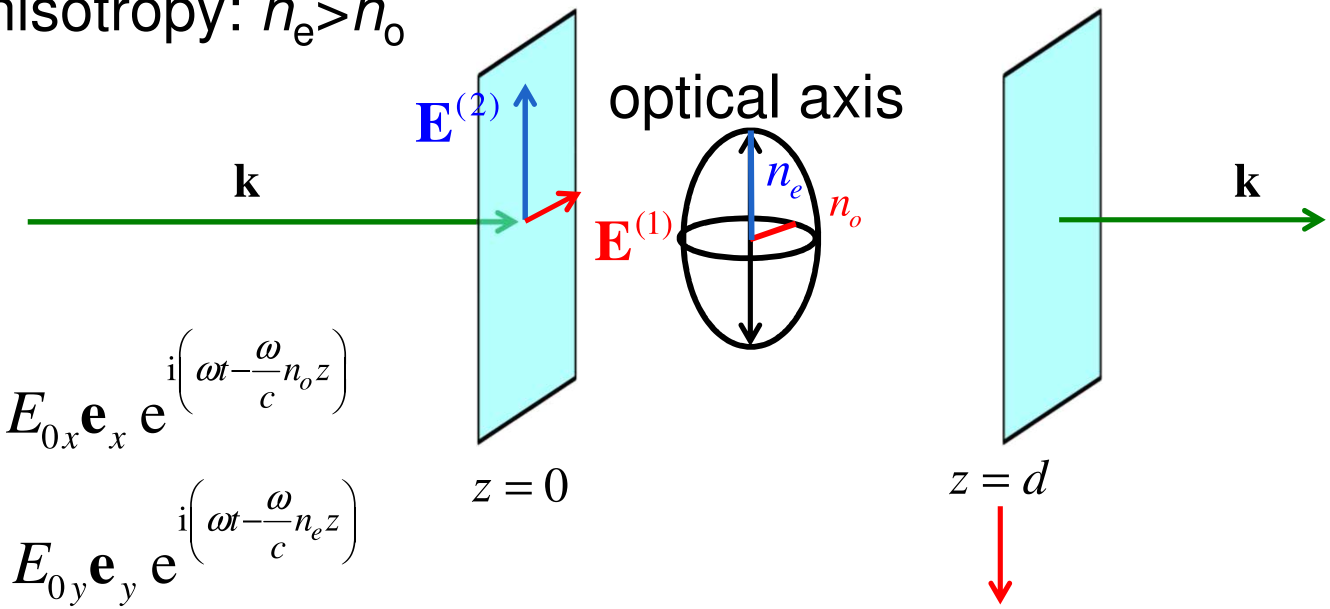
Stokes parameters
shown on the sphere of Poincaré



WAVELENGTH RETARDATION PLATES

Uniaxial material, optic axis in the plane (A-plate)

positive anisotropy: $n_e > n_o$



$$\mathbf{E}^{(1)} = E_{0x}\mathbf{e}_x\, \mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n_o z\right)}$$

$$\mathbf{E}^{(2)} = E_{0y}\mathbf{e}_y\, \mathrm{e}^{\mathrm{i}\left(\omega t - \frac{\omega}{c} n_e z\right)}$$

phase delay of $E^{(2)}$: $\varphi_1 - \varphi_2 = \frac{\omega}{c}(n_e - n_o)d$

optical path length difference: $(n_e - n_o)d$

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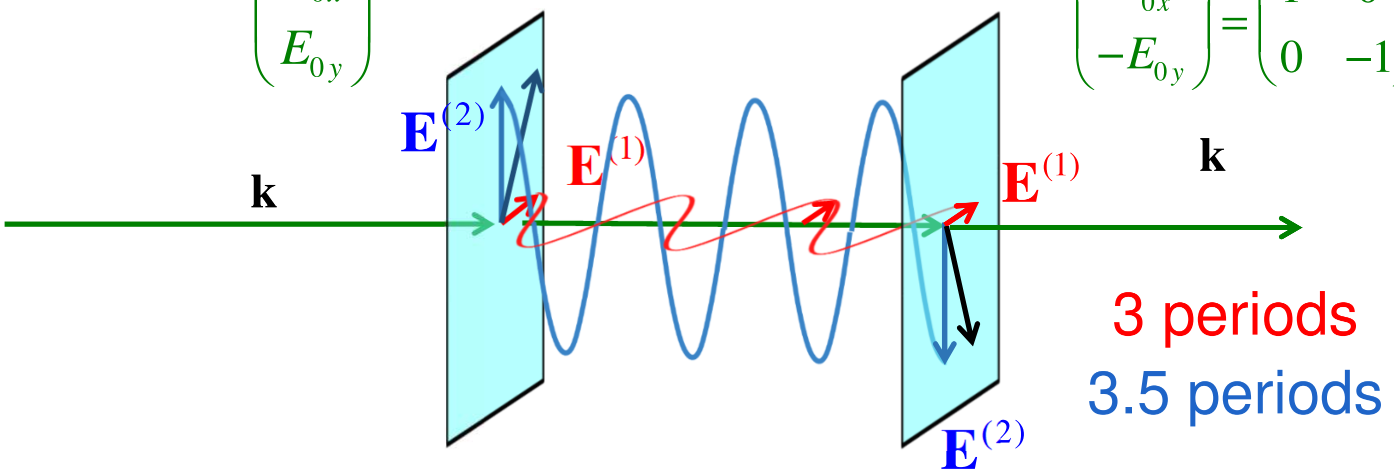
WAVELENGTH RETARDATION PLATES

Example: half-wave plate
the wave $\mathbf{E}_2$ is delayed by a phase π

$$\frac{\omega}{c}(n_e - n_o)d = \frac{2\pi}{\lambda}(n_e - n_o)d = \pi$$

$$\begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix}$$

$$\begin{pmatrix} E_{0x} \\ -E_{0y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix}$$



$$J = \begin{pmatrix} e^{-i\frac{\omega}{c}n_o d} & 0 \\ 0 & e^{-i\frac{\omega}{c}n_e} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\omega}{c}(n_e - n_o)d} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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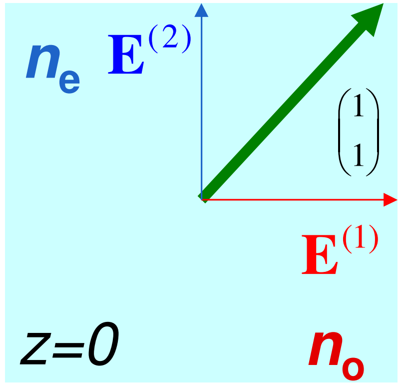
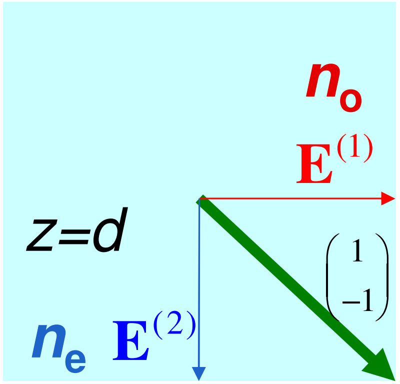
WAVELENGTH RETARDATION PLATES

z=0: linearly polarized at 45°

phase delay π

$$J=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$





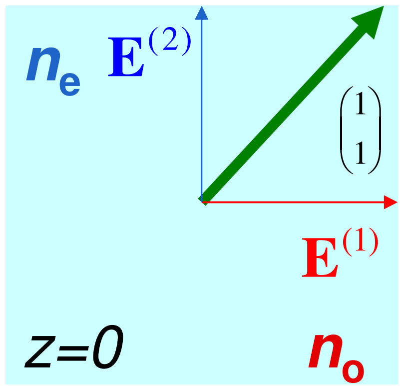
seen from the destination

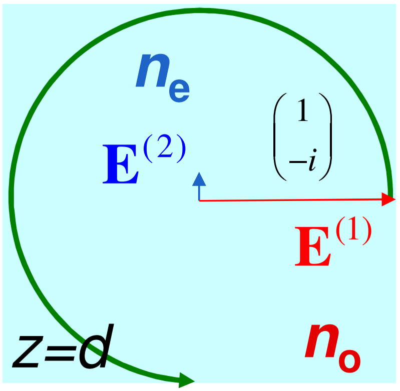
quarter wave plate

E_2 phase delay π/2

left circular polarization

$$J=\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}$$

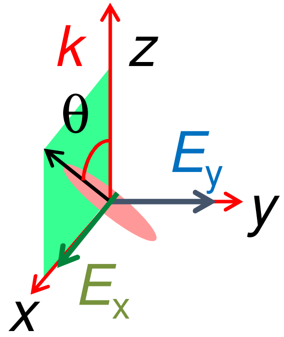




LIGHT IN ANISOTROPIC MATERIALS

propagation along the *z*-axis (distance *d*)

two polarizations with different speed



ordinary mode E_y : $n=n_o$

extra-ordinary mode E_x : $n=n_{eff}$

$$n_{eff}=\frac{1}{\sqrt{\frac{\cos^2\theta}{n_o^2}+\frac{\sin^2\theta}{n_e^2}}}$$

$$\begin{bmatrix}E_{x,out}\\E_{y,out}\end{bmatrix}=\begin{bmatrix}e^{-i\frac{2\pi n_{eff}d}{\lambda}} & 0\\0 & e^{-i\frac{2\pi n_o d}{\lambda}}\end{bmatrix}\cdot\begin{bmatrix}E_{x,in}\\E_{y,in}\end{bmatrix}=e^{-i\frac{2\pi(n_{eff}+n_0)d}{2\lambda}}\begin{bmatrix}e^{-i\Gamma/2} & 0\\0 & e^{i\Gamma/2}\end{bmatrix}\cdot\begin{bmatrix}E_{x,in}\\E_{y,in}\end{bmatrix}$$

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Jones matrix

$$\overline{\overline{J}}=\begin{bmatrix}e^{-i\Gamma/2} & 0\\0 & e^{i\Gamma/2}\end{bmatrix}$$

retardation

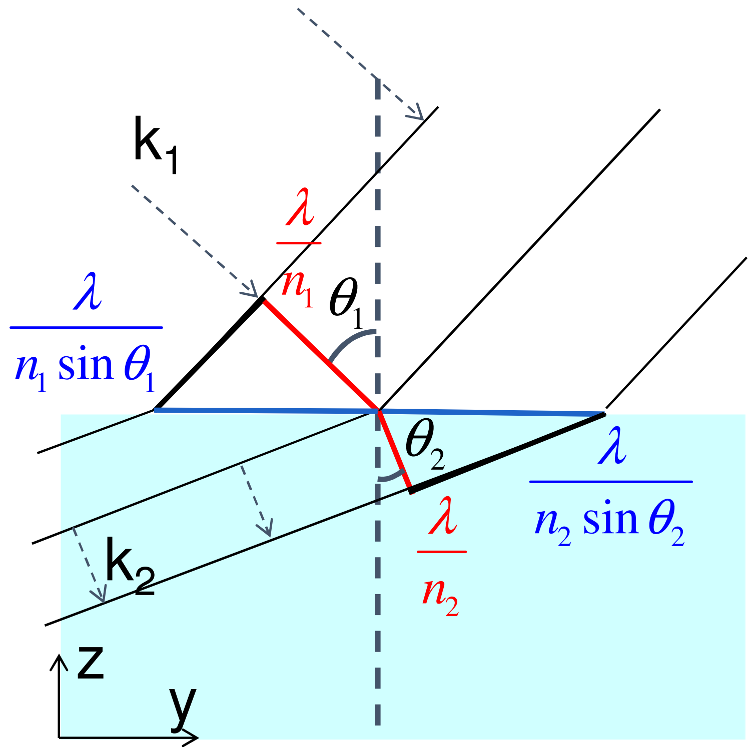
$$\Gamma=\frac{2\pi}{\lambda}\left(n_{eff}-n_o\right)d$$

REFRACTION

wavelengths in two media

distance between two wave fronts along the surface should be equal

$$\frac{\lambda}{n_1 \sin\theta_1} = \frac{\lambda}{n_2 \sin\theta_2}$$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$ Law of Snellius

$$\frac{\omega}{c} n_1 \sin\theta_1 = \frac{\omega}{c} n_2 \sin\theta_2 \longrightarrow k_1 \sin\theta_1 = k_2 \sin\theta_2$$

$$\boxed{k_{1y} = k_{2y}}$$

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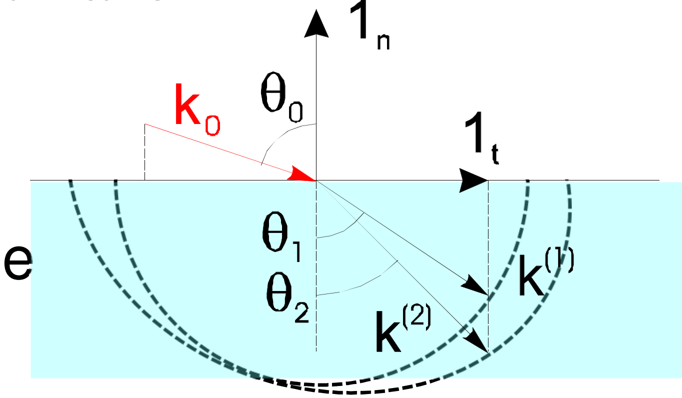
DOUBLE REFRACTION

Light incident from an isotropic medium
into an anisotropic medium
k-vector represents periodicity of a wave

$$k_0 = \frac{\omega}{c} n_0 = \frac{2\pi}{\lambda} n_0$$

Tangential component k_t
of *k*-vectors should be the same



two solutions: bi-refringence
double refraction

$$n_0 \sin\theta_0 = n^{(1)} \sin\theta_1 = n^{(2)} \sin\theta_2$$

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DICHROISM

Linear dichroism

anisotropic absorption

$$\bar{\bar{\varepsilon}}_r = \begin{pmatrix} (n_1' + \mathrm{i}\, n_1'')^2 & 0 & 0 \\ 0 & (n_2' + \mathrm{i}\, n_2'')^2 & 0 \\ 0 & 0 & (n_3' + \mathrm{i}\, n_3'')^2 \end{pmatrix}$$

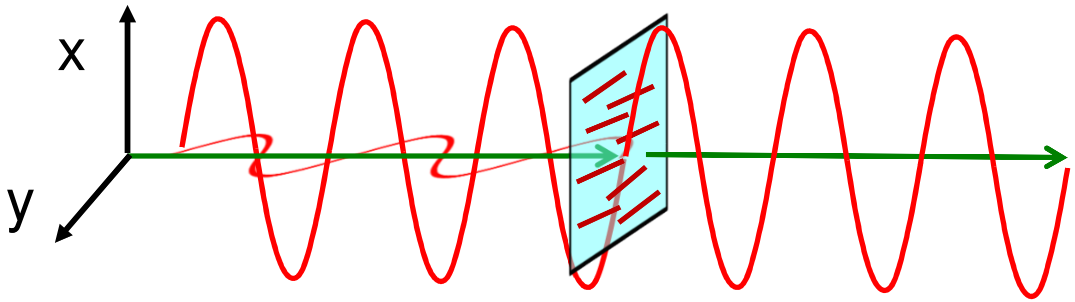
linear polarizer: *y* polarization is absorbed, *x*-pol is transmitted

$$\bar{\bar{\varepsilon}}_r = \begin{pmatrix} n_1'^2 & 0 & 0 \\ 0 & (n_2' + \mathrm{i}\, n_2'')^2 & 0 \\ 0 & 0 & n_3'^2 \end{pmatrix}$$

DICHROISM

Polaroid polarizer

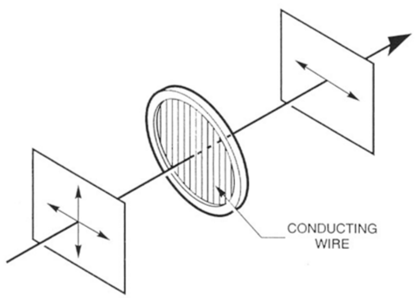
oriented absorbing polymer molecules



$$J = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Wire grid polarizer

parallel metal wires absorb the *E*-field component parallel to the wires



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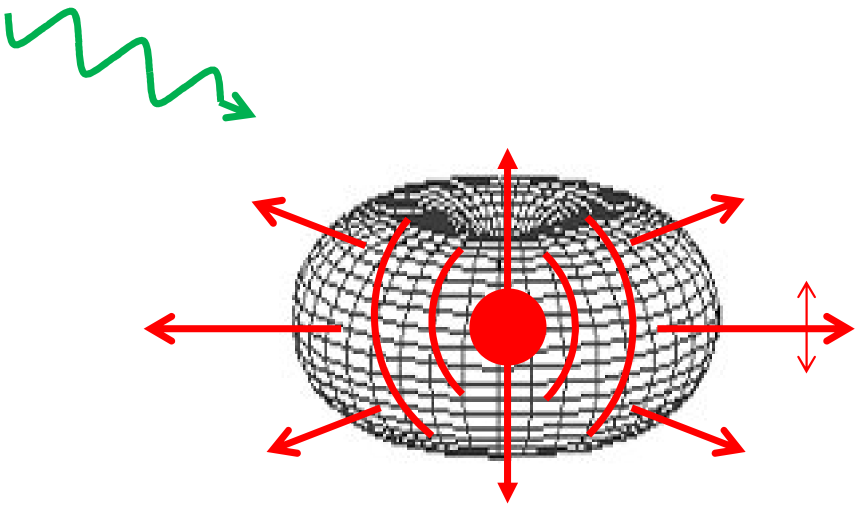
LIGHT EMITTING MATERIALS

excitation by absorption of a photon: mainly one polarization is absorbed



emission from a dipole transition, mainly one polarization

probability ~ electrical dipole antenna radiation (linear pol)
dipole may have different orientations

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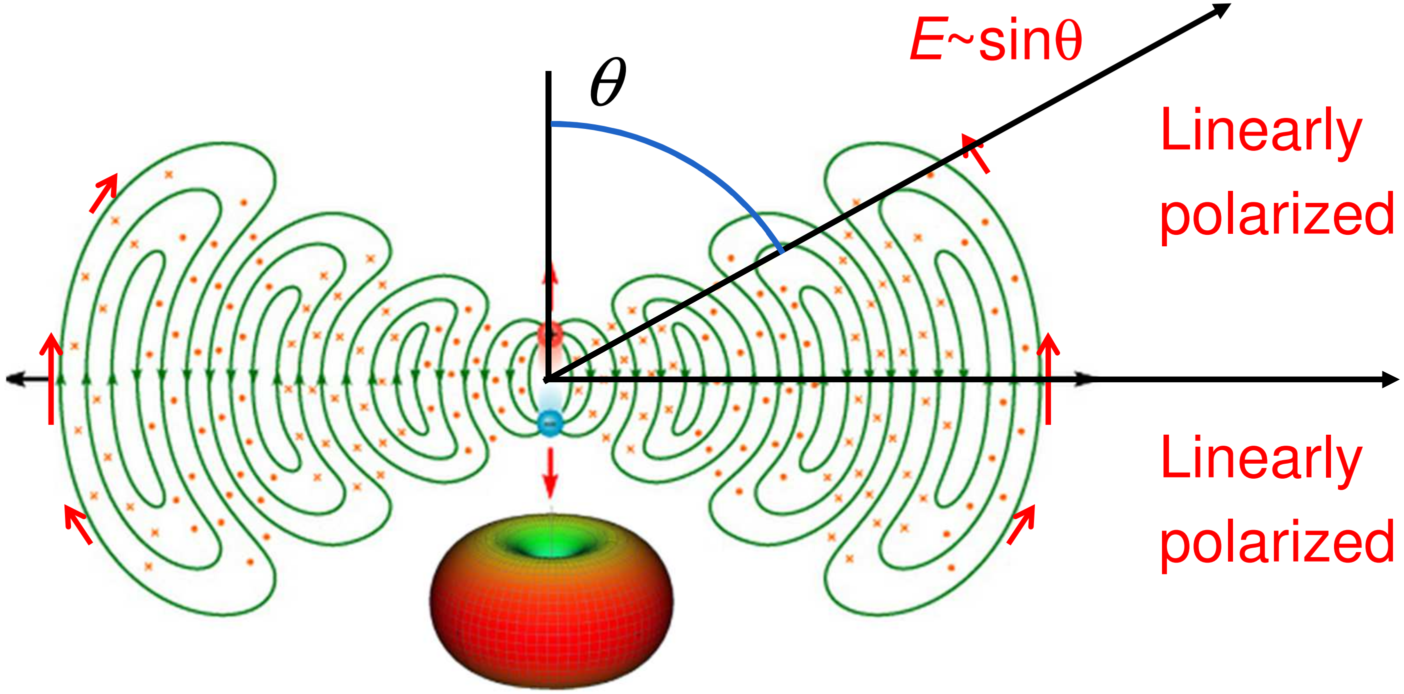
LIGHT EMITTING MATERIALS

Oscillating elementary dipole antenna *p*

$$P(\theta_e)\,d\Omega = \frac{\omega^4 |\mathbf{p}_e|^2}{32\pi^2 \varepsilon_e c^3} \sin^2\theta_e d\Omega$$



only p polarization (// plane of dipole and k, =TM)

DEPOLARIZATION FIELD IN A PARTICLE

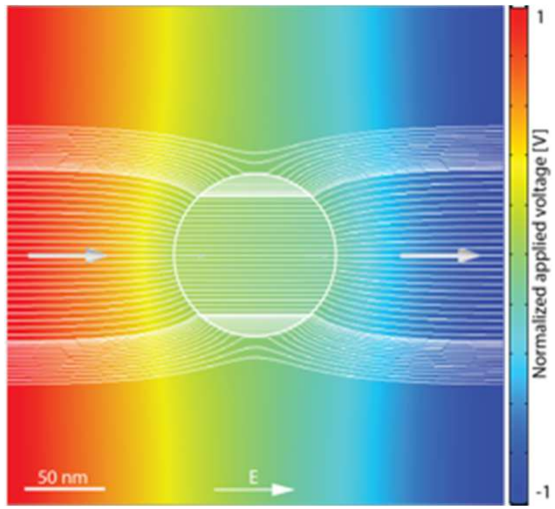
Particle with $\varepsilon_p > \varepsilon_m$ in a homogeneous electric field

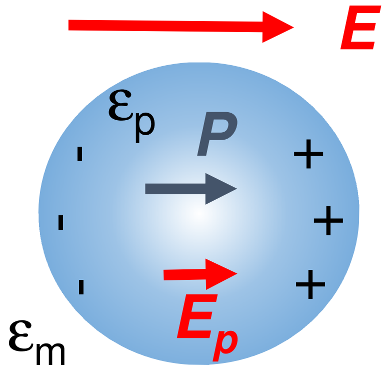
particle is polarized

charges generate depolarization field

lower field E_p inside the particle

field lines $\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}$





$$\mathbf{E}_p = \frac{3\varepsilon_m}{2\varepsilon_m+\varepsilon_p}\mathbf{E}_e < \mathbf{E}_e$$

Liquid Crystals and Photonics

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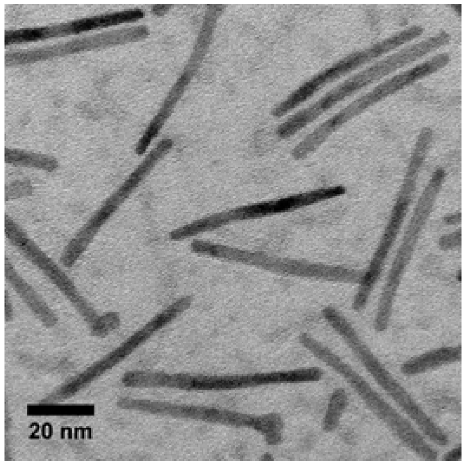
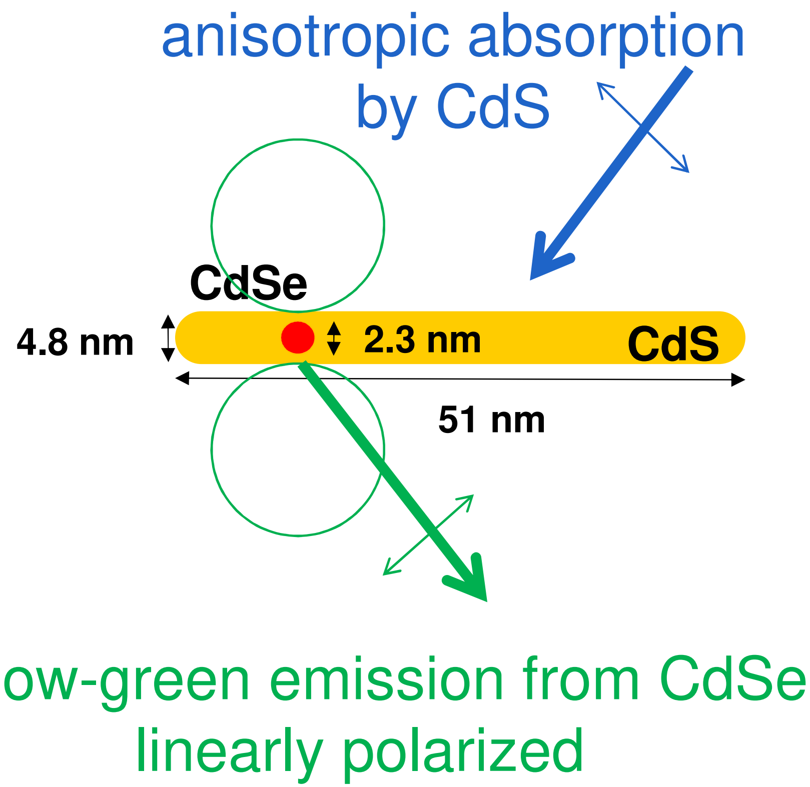
LIGHT EMITTING MATERIALS

TEM: CdSe/CdS nanorods (2 semiconductors)
field is reduced along x and y,
not along the z-axis





Liquid Crystals and Photonics

WAT

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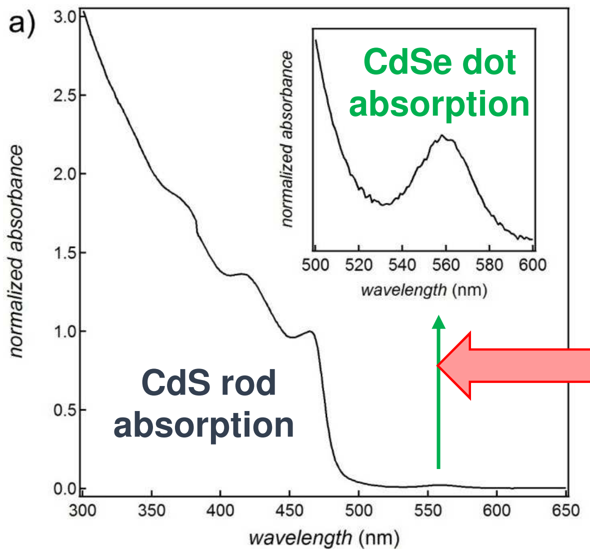
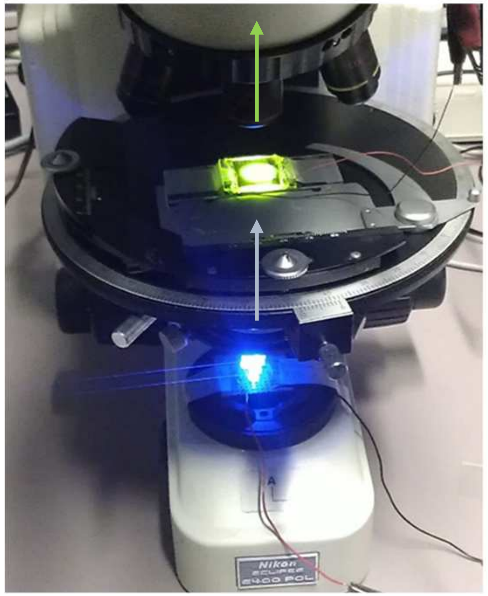
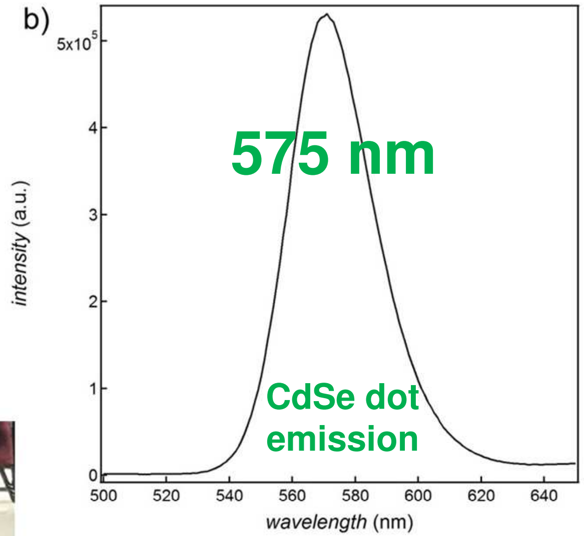
LIGHT EMITTING MATERIALS



QD shifted band gap

absorption spectrum





Emission spectrum

excitation by 365 nm

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