FACULTY OF ENGINEERING
AND ARCHITECTURE

ELECTRONICS AND INFORMATION SYSTEMS DEPARTMENT
LIQUID CRYSTALS AND PHOTONICS RESEARCH GROUP

LIQUID CRYSTALS AND LIGHT EMITTING MATERIALS FOR PHOTONIC APPLICATIONS

Kristiaan Neyts April 2018 Lecture series at WAT in Warsaw

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nb-photonics
Center for Nano- and Biophotonics
Ghent University

Liquid Crystals and Photonics

OVERVIEW

Liquid crystal properties (10h)

Properties of nematic liquid crystals
Nematic order parameter
Polarization and dielectric constant
Elastic energy
Surface alignment
Electrical energy
Jones matrix method
Variable phase retarder
VAN mode
IPS mode
TN mode

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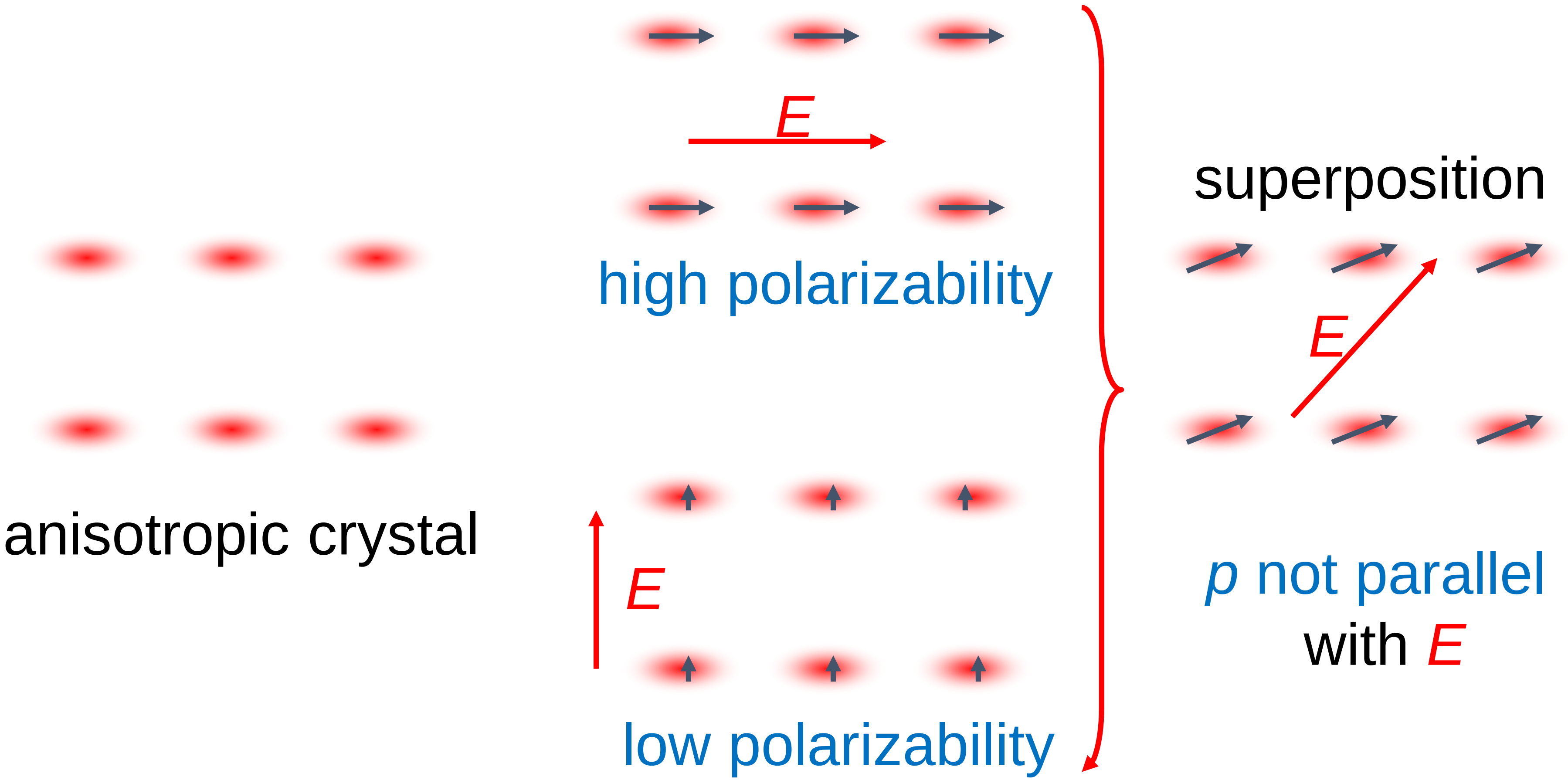
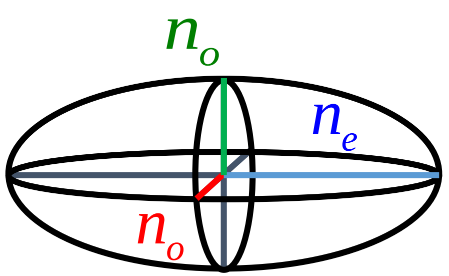
ANISOTROPIC POLARIZABILITY

Anisotropic polarizability in a crystalline structure

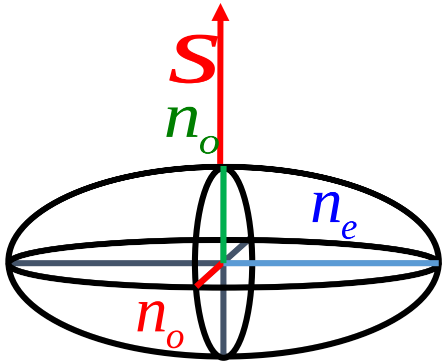


LIGHT PROPAGATION

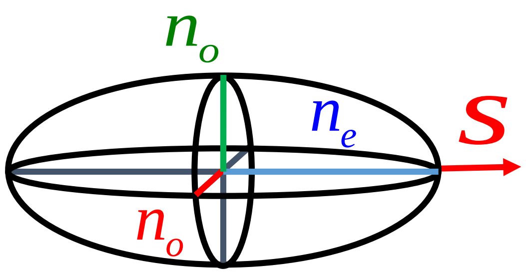
$$\frac{X^2}{n_o^2}+\frac{Y^2}{n_o^2}+\frac{Z^2}{n_e^2}=1$$



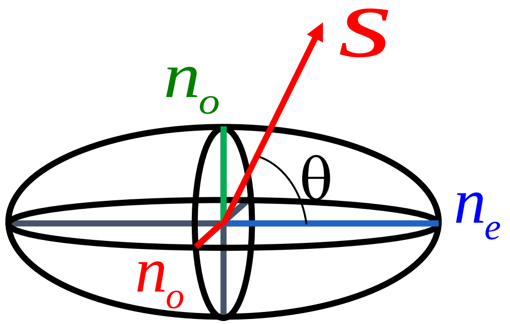
$$\bar{\bar{\eta}}=\begin{pmatrix}\frac{1}{n_o^2} & 0 & 0\\ 0 & \frac{1}{n_o^2} & 0\\ 0 & 0 & \frac{1}{n_e^2}\end{pmatrix}$$



two plane waves with n_o and n_e



two plane waves with n_o and n_o



two plane waves with n_o and

$$\frac{1}{\sqrt{\frac{\cos^2\theta}{n_o^2}+\frac{\sin^2\theta}{n_e^2}}}$$

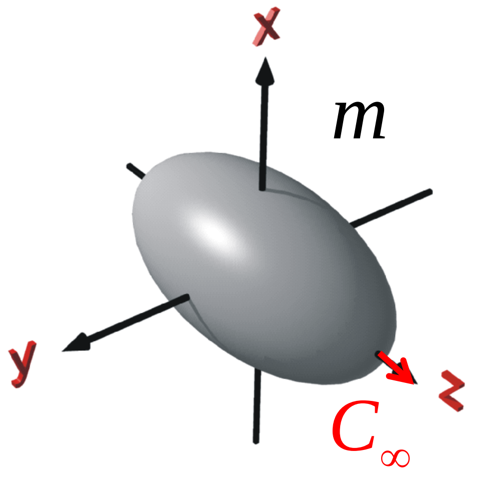
HOMOGENEOUS NEMATIC LIQUID CRYSTAL

Symmetry elements for nematic LC

- full rotation symmetry axis C_∞ (*z*-axis)
- mirror plane *m* (*xy* plane)
- C_2 axes in the plane *xy*
- mirror planes containing the *z*-axis

tensors of rank 2

have uniaxial symmetry



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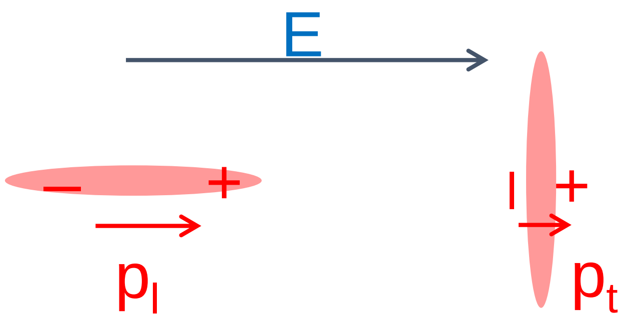
LIQUID CRYSTAL POLARIZABILITY

Anisotropic polarization:

$$p = \alpha \varepsilon_0 E_{local}$$



polarization due to
molecular polarizability α
induced dipoles

$$p_l = \alpha_l \varepsilon_0 E_{local,l} \qquad p_t = \alpha_t \varepsilon_0 E_{local,t}$$

$$\Delta\alpha = \alpha_l - \alpha_t > 0$$

$$\alpha_{avg} = \frac{1}{3}\alpha_l + \frac{2}{3}\alpha_t$$

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MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

Polarizability tensor

α_l **long axis**

α_t **transverse**

for a molecule, related to its axes

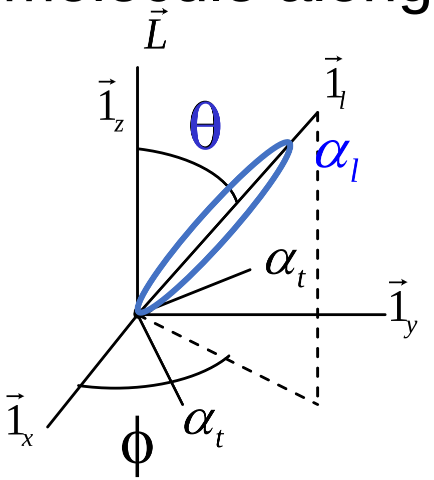
α_t is average of two transverse α's

average of three α's: $\alpha_{avg} = \dfrac{\alpha_l + 2\alpha_t}{3}$

$$\bar{\bar{\alpha}}_{mol} = \begin{pmatrix} \alpha_t & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_l \end{pmatrix}$$

for a molecule, related to the axes of nematic phase

molecule along $\vec{1}_l$ has inclination θ and azimuth ϕ



$$\vec{1}_l = \begin{pmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{pmatrix}$$

$\vec{1}_l \neq \vec{L}$ due to thermal motion

MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

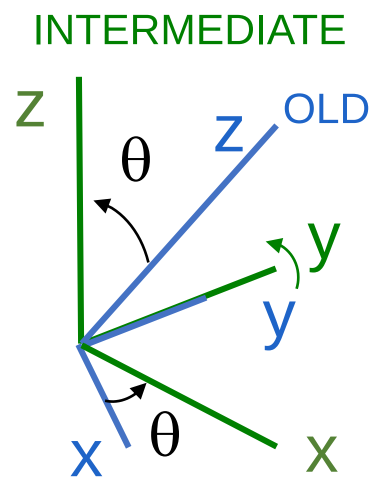
Coordinate transformations

OLD to INTERMEDIATE

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

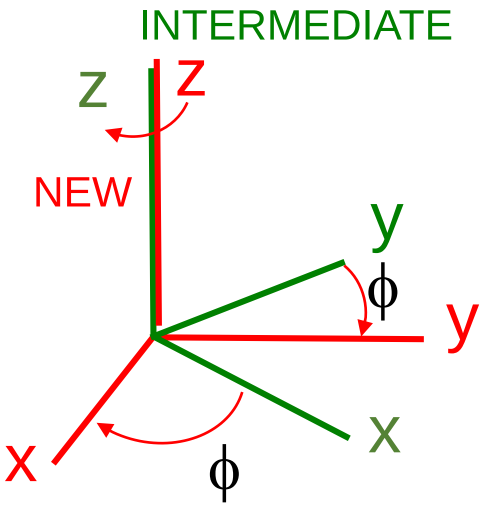
INTERMEDIATE to NEW

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



columns are *xyz* coordinates of old unit vectors

$$\begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$



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TENSOR TRANSFORMATION FORMULA

Transformation of a tensor?

transformation of *p* (old coordinates)

to a new system of axes *p'* (new coordinates)

$$p_k' = R_{ki} p_i$$

$$p_i = \varepsilon_0 \alpha_{ij} E_j$$

$$p_k' = R_{ki} p_i$$

$$E_l' = R_{lj} E_j \Rightarrow E_j = R_{lj} E_l'$$

$$p_k' = \varepsilon_0 R_{ki} \alpha_{ij} R_{lj} E_l'$$

$$p_k' = \varepsilon_0 \alpha_{kl}' E_l'$$

$$\alpha_{kl}' = R_{ki} R_{lj} \alpha_{ij}$$

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TENSOR TRANSFORMATION FORMULA

Transformation of a tensor?

$$\alpha_{kl}' = R_{ki} R_{lj} \alpha_{ij}$$

transformation of α (old coordinates)

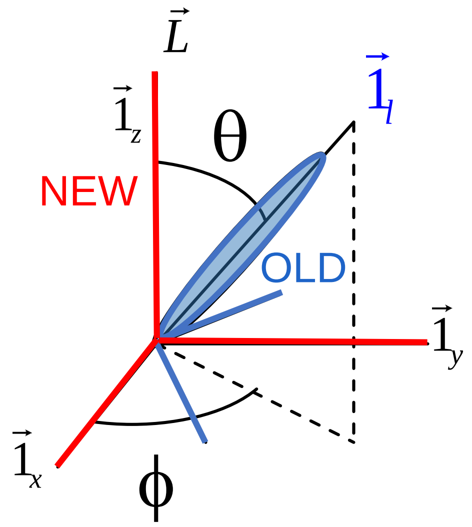
to a new system of axes α' (new coordinates)

$$\bar{\bar{\alpha}}_{mol} = \begin{pmatrix} \alpha_t & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_l \end{pmatrix}$$

$$\bar{\bar{\alpha}}_{mol}' = \begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \alpha_t & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_l \end{pmatrix} \begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}^{-1}$$

MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

Coordinate transformation: OLD to NEW



$$R=\begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\bar{\bar{\alpha}}_{mol}=\begin{pmatrix} \alpha_t & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_l \end{pmatrix}=\alpha_t\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}+\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_l-\alpha_t \end{pmatrix}$$

$$\alpha_{mol,mn}=\alpha_t\delta_{mn}+(\alpha_l-\alpha_t)\delta_{m3}\delta_{n3}$$

δ_{ij}: 1when $i=j$; 0 when $i\neq j$

transform →

$$\alpha_{ij}=R_{im}R_{jn}\alpha_{mol,mn}=R_{im}R_{jn}\alpha_t\delta_{mn}+(\alpha_l-\alpha_t)R_{im}R_{jn}\delta_{m3}\delta_{n3}$$
$$=R_{im}R_{jm}\alpha_t+(\alpha_l-\alpha_t)R_{i3}R_{j3}$$
$$=\alpha_t\delta_{ij}+(\alpha_l-\alpha_t)l_il_j$$

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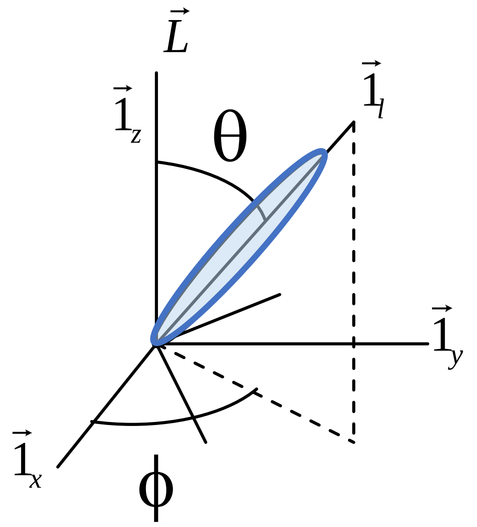
MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

Value of α in *xyz* ?

$$\alpha_{ij} = \alpha_t \delta_{ij} + (\alpha_l - \alpha_t) l_i l_j$$

$$\bar{\bar{\alpha}} = \alpha_t \bar{\bar{\delta}} + (\alpha_l - \alpha_t) \vec{l}\vec{l}$$

diade product: $\vec{l}\vec{l} = \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} \begin{pmatrix} l_x & l_y & l_z \end{pmatrix}$



$$\bar{\bar{\alpha}} = \begin{pmatrix} \alpha_t + (\alpha_l - \alpha_t)\cos^2\phi \sin^2\theta & (\alpha_l - \alpha_t)\sin\phi\cos\phi\sin^2\theta & (\alpha_l - \alpha_t)\cos\phi\sin\theta\cos\theta \\ (\alpha_l - \alpha_t)\sin\phi\cos\phi\sin^2\theta & \alpha_t + (\alpha_l - \alpha_t)\sin^2\phi\sin^2\theta & (\alpha_l - \alpha_t)\sin\phi\sin\theta\cos\theta \\ (\alpha_l - \alpha_t)\cos\phi\sin\theta\cos\theta & (\alpha_l - \alpha_t)\sin\phi\sin\theta\cos\theta & \alpha_t + (\alpha_l - \alpha_t)\cos^2\theta \end{pmatrix}$$

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$\langle \bar{\bar{\alpha}} \rangle$? average over orientations

ϕ random between 0 and 2π
$$\begin{cases} \langle \cos\phi \rangle = \langle \sin\phi \rangle = \langle \cos\phi \sin\phi \rangle = 0 \\ \langle \cos^2\phi \rangle = \frac{1}{2} \end{cases}$$

MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

average polarizability tensor

$$\langle \bar{\bar{\alpha}} \rangle = \begin{pmatrix} \alpha_t + \frac{1}{2}(\alpha_l - \alpha_t)\langle \sin^2\theta \rangle & 0 & 0 \\ 0 & \alpha_t + \frac{1}{2}(\alpha_l - \alpha_t)\langle \sin^2\theta \rangle & 0 \\ 0 & 0 & \alpha_t + (\alpha_l - \alpha_t)\langle \cos^2\theta \rangle \end{pmatrix}$$

definition for nematic phase: $\perp$ and // with director $\vec{L}$

$$\langle \bar{\bar{\alpha}} \rangle = \begin{pmatrix} \alpha_\perp & 0 & 0 \\ 0 & \alpha_\perp & 0 \\ 0 & 0 & \alpha_{//} \end{pmatrix} \Rightarrow \begin{cases} \alpha_\perp = \alpha_t + \frac{1}{2}(\alpha_l - \alpha_t)\langle \sin^2\theta \rangle \\ \alpha_{//} = \alpha_t + (\alpha_l - \alpha_t)\langle \cos^2\theta \rangle \end{cases}$$

for a molecule:

$$\alpha_{avg} = \frac{\alpha_l + 2\alpha_t}{3}$$

check that:

$$\frac{\alpha_{//} + 2\alpha_\perp}{3} = \alpha_{avg}$$

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MICROSCOPIC THEORY FOR UNIAXIAL DIELECTRICS

Relation with the order parameter

$$S = \tfrac{1}{2}\left\langle 3\cos^2\theta - 1\right\rangle$$

$$\left\langle \cos^2\theta \right\rangle = \frac{1+2S}{3}$$

$$\left\langle \sin^2\theta \right\rangle = \frac{2-2S}{3}$$

$$\alpha_{//} = \alpha_t + (\alpha_l - \alpha_t)\left\langle \cos^2\theta \right\rangle = \ldots = \alpha_{avg} + (\alpha_l - \alpha_t)\tfrac{2}{3}S$$

$$\alpha_{\perp} = \alpha_t + \tfrac{1}{2}(\alpha_l - \alpha_t)\left\langle \sin^2\theta \right\rangle = \ldots = \alpha_{avg} - (\alpha_l - \alpha_t)\tfrac{1}{3}S$$

check that: $\alpha_{//} - \alpha_{\perp} = (\alpha_l - \alpha_t)S$

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$$\langle \bar{\bar{\alpha}} \rangle = \begin{pmatrix} \alpha_\perp & 0 & 0 \\ 0 & \alpha_\perp & 0 \\ 0 & 0 & \alpha_{//} \end{pmatrix} = \begin{pmatrix} \alpha_{avg} - (\alpha_l - \alpha_t)\frac{1}{3}S & 0 & 0 \\ 0 & \alpha_{avg} - (\alpha_l - \alpha_t)\frac{1}{3}S & 0 \\ 0 & 0 & \alpha_{avg} + (\alpha_l - \alpha_t)\frac{2}{3}S \end{pmatrix}$$

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Susceptibility tensor?

$$\bar{\bar{\chi}} \approx N \langle \bar{\bar{\alpha}} \rangle$$

$$\chi_{//} \approx N\alpha_{//}$$

$$\chi_\perp \approx N\alpha_\perp$$

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \left(1 + \bar{\bar{\chi}}\right)$$

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DIELECTRIC TENSOR

Dielectric tensor? from *D*-field

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \left(1 + \bar{\bar{\chi}}\right) \vec{E} = \bar{\bar{\varepsilon}} \vec{E}$$

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{pmatrix}$$

$$\varepsilon_{//} = \varepsilon_0 \left(1 + \chi_{//}\right)$$

$$\varepsilon_\perp = \varepsilon_0 \left(1 + \chi_\perp\right)$$

$$\varepsilon_{avg} = \frac{\varepsilon_{//} + 2\varepsilon_\perp}{3} = \varepsilon_0 \left(1 + \chi_{avg}\right)$$

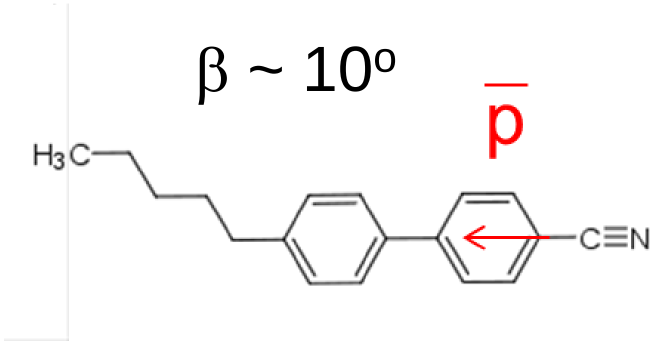
$$\Rightarrow \quad \frac{\varepsilon_{//} - \varepsilon_\perp}{\varepsilon_{avg} - \varepsilon_0} = \frac{\chi_{//} - \chi_\perp}{\chi_{avg}}$$

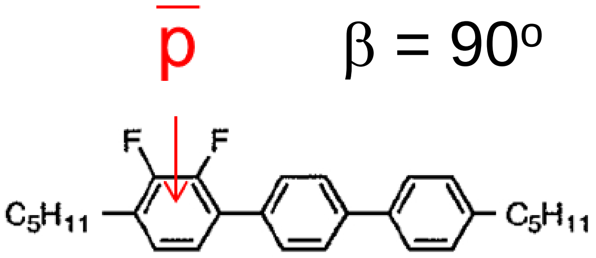
Conclusion

$$\frac{\varepsilon_{//} - \varepsilon_\perp}{\varepsilon_{avg} - \varepsilon_0} = \frac{\chi_{//} - \chi_\perp}{\chi_{avg}} = \frac{\alpha_{//} - \alpha_\perp}{\alpha_{avg}} = \frac{\alpha_l - \alpha_t}{\alpha_{avg}} S$$

LIQUID CRYSTAL POLARIZABILITY

molecules with permanent dipole moment

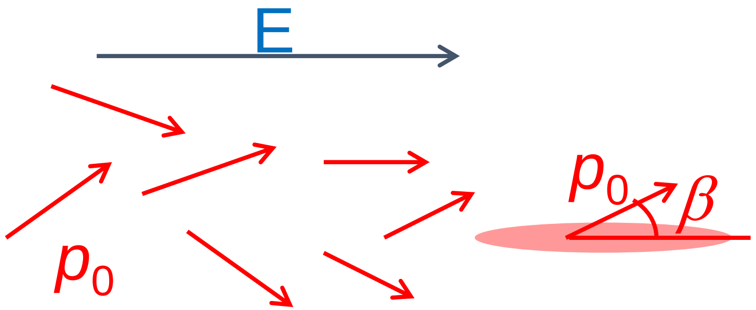




orientation polarization

permanent dipoles

(low frequencies)



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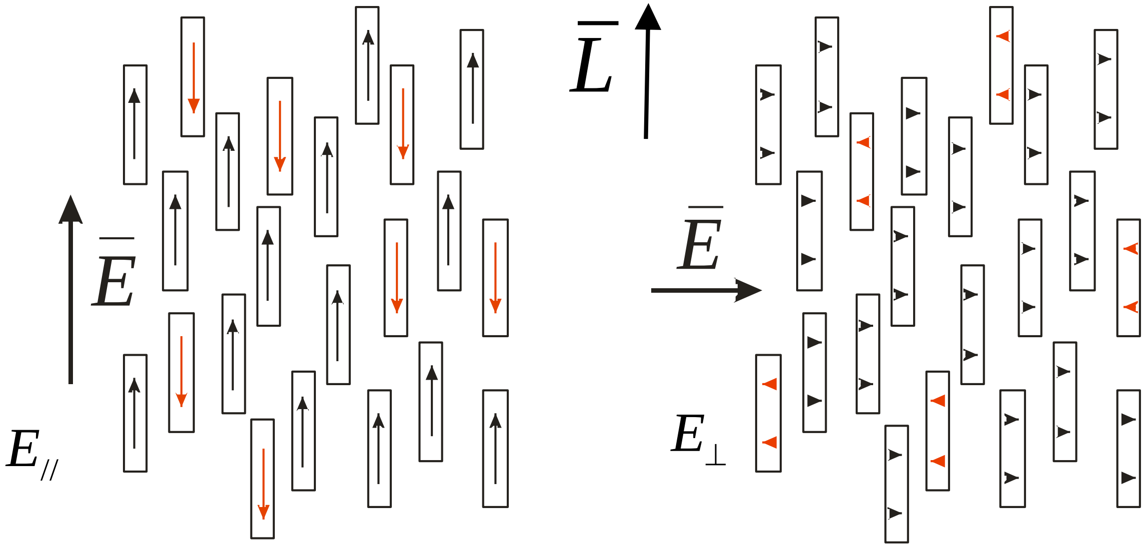
ELECTRICAL PROPERTIES

More dipoles parallel to the electric field (Boltzmann, Maier)



shown for S=1
(perfect order)

$\beta = 0°$ $\beta = 90°$

h: cavity field factor
F: reaction field factor

$$p_{ori,//} = \frac{F^2 p_0^2}{3kT}\left[1+2S\right]hE_{//} \qquad \left[1-\left(1-3\cos^2\beta\right)S\right] \qquad p_{ori,//} = \frac{F^2 p_0^2}{3kT}\left[1-S\right]hE_{//}$$

// long axis

$$p_{ori,\perp} = \frac{F^2 p_0^2}{3kT}\left[1-S\right]hE_{\perp} \qquad \left[1+\frac{1}{2}\left(1-3\cos^2\beta\right)S\right] \qquad p_{ori,\perp} = \frac{F^2 p_0^2}{3kT}\left[1+\frac{1}{2}S\right]hE_{\perp}$$

⊥ long axis

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ELECTRICAL PROPERTIES

Anisotropic (uniaxial) dielectric :

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

h: cavity field factor
F: reaction field factor

polarization due to
molecular polarizability α
induced dipoles

$$\begin{cases} p_{ele,//} = \varepsilon_0 F \left(\alpha_{avg} + \frac{2}{3} \Delta\alpha S \right) hE_{//} \\ p_{ele,\perp} = \varepsilon_0 F \left(\alpha_{avg} - \frac{1}{3} \Delta\alpha S \right) hE_{\perp} \end{cases}$$

orientation polarization
permanent dipoles
(low frequencies)

$$\begin{cases} p_{ori,//} = \frac{F^2 p_0^2}{3kT} \left[1 - \left(1 - 3\cos^2\beta \right) S \right] hE_{//} \\ p_{ori,\perp} = \frac{F^2 p_0^2}{3kT} \left[1 + \frac{1}{2} \left(1 - 3\cos^2\beta \right) S \right] hE_{\perp} \end{cases}$$

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DIELECTRIC PROPERTIES

Anisotropic (uniaxial) dielectric :

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\varepsilon_{//} = \varepsilon_0 + \varepsilon_0 NFh \left(\alpha_{avg} + \frac{2}{3}\Delta\alpha S + \frac{Fp_0^2}{3kT}\left[1 - \left(1 - 3\cos^2\beta\right)S\right] \right)$$

$$\varepsilon_\perp = \varepsilon_0 + \varepsilon_0 NFh \left(\alpha_{avg} - \frac{1}{3}\Delta\alpha S + \frac{Fp_0^2}{3kT}\left[1 + \frac{1}{2}\left(1 - 3\cos^2\beta\right)S\right] \right)$$

only at low frequencies <100kHz
orientation polarization is slow

$$\Delta\varepsilon = \varepsilon_{//} - \varepsilon_\perp$$

$\Delta\varepsilon < 0$ at low frequency if $\beta = 90^o$

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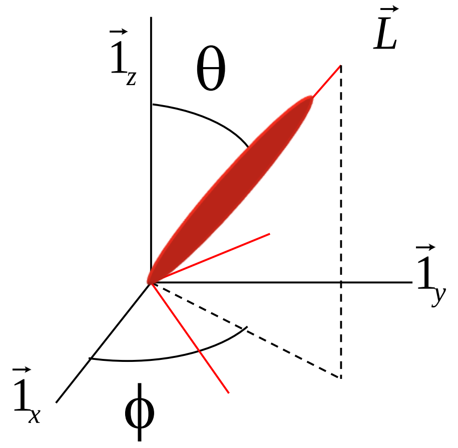
DIELECTRIC TENSOR

Dielectric tensor in laboratory axes?

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{pmatrix} \qquad \vec{L} = \begin{pmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{pmatrix}$$



$$R = \begin{pmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\varepsilon_{\alpha\beta} = R_{\alpha k} R_{\beta l} \varepsilon_{kl}$$

$$= \varepsilon_\perp \delta_{\alpha\beta} + \left(\varepsilon_{//} - \varepsilon_\perp\right) R_{\alpha 3} R_{\beta 3}$$

$$= \varepsilon_\perp \delta_{\alpha\beta} + \left(\varepsilon_{//} - \varepsilon_\perp\right) L_\alpha L_\beta$$

$$\varepsilon_{kl} = \varepsilon_\perp \delta_{kl} + \left(\varepsilon_{//} - \varepsilon_\perp\right) \delta_{k3} \delta_{l3}$$

DIELECTRIC TENSOR

Dielectric tensor

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp + (\varepsilon_{//} - \varepsilon_\perp)\cos^2\phi\sin^2\theta & (\varepsilon_{//} - \varepsilon_\perp)\sin\phi\cos\phi\sin^2\theta & (\varepsilon_{//} - \varepsilon_\perp)\cos\phi\sin\theta\cos\theta \\ (\varepsilon_{//} - \varepsilon_\perp)\sin\phi\cos\phi\sin^2\theta & \varepsilon_\perp + (\varepsilon_{//} - \varepsilon_\perp)\sin^2\phi\sin^2\theta & (\varepsilon_{//} - \varepsilon_\perp)\sin\phi\sin\theta\cos\theta \\ (\varepsilon_{//} - \varepsilon_\perp)\cos\phi\sin\theta\cos\theta & (\varepsilon_{//} - \varepsilon_\perp)\sin\phi\sin\theta\cos\theta & \varepsilon_\perp + (\varepsilon_{//} - \varepsilon_\perp)\cos^2\theta \end{pmatrix}$$

equivalent with:

$$\varepsilon_{\alpha\beta} = \varepsilon_\perp \delta_{\alpha\beta} + (\varepsilon_{//} - \varepsilon_\perp) L_\alpha L_\beta$$

$$\bar{\bar{\varepsilon}} = \varepsilon_\perp \bar{\bar{\delta}} + (\varepsilon_{//} - \varepsilon_\perp)\vec{L}\vec{L}$$

$$= \varepsilon_\perp \bar{\bar{\delta}} + \Delta\varepsilon\,\vec{L}\vec{L}$$

$$\vec{L} = \begin{pmatrix} \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{pmatrix}$$

diade product $\vec{L}\vec{L} = \begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} \begin{pmatrix} L_x & L_y & L_z \end{pmatrix}$

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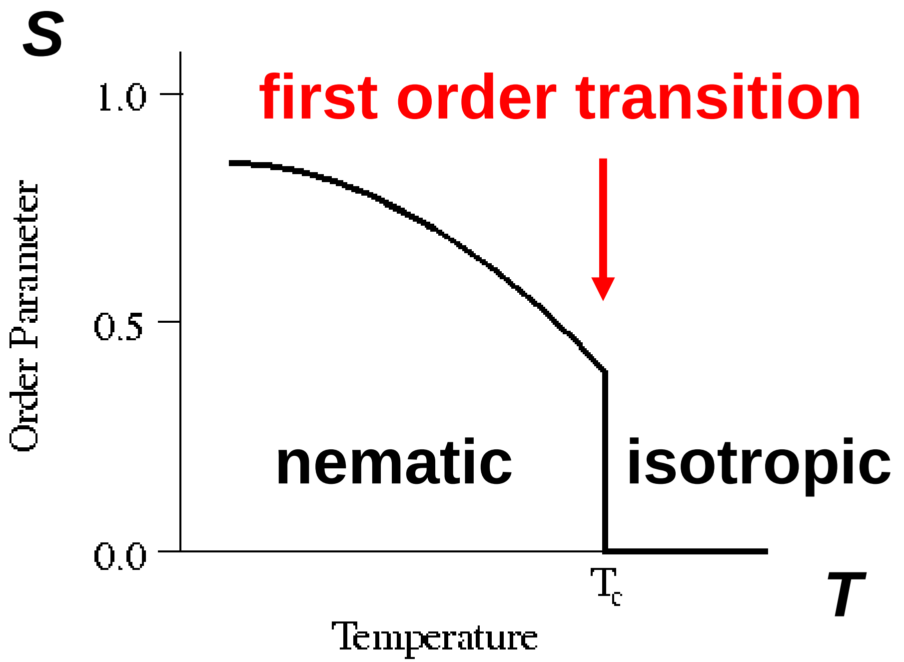
INFLUENCE OF TEMPERATURE

Transition from nematic to isotropic

is related with a jump in the order parameter

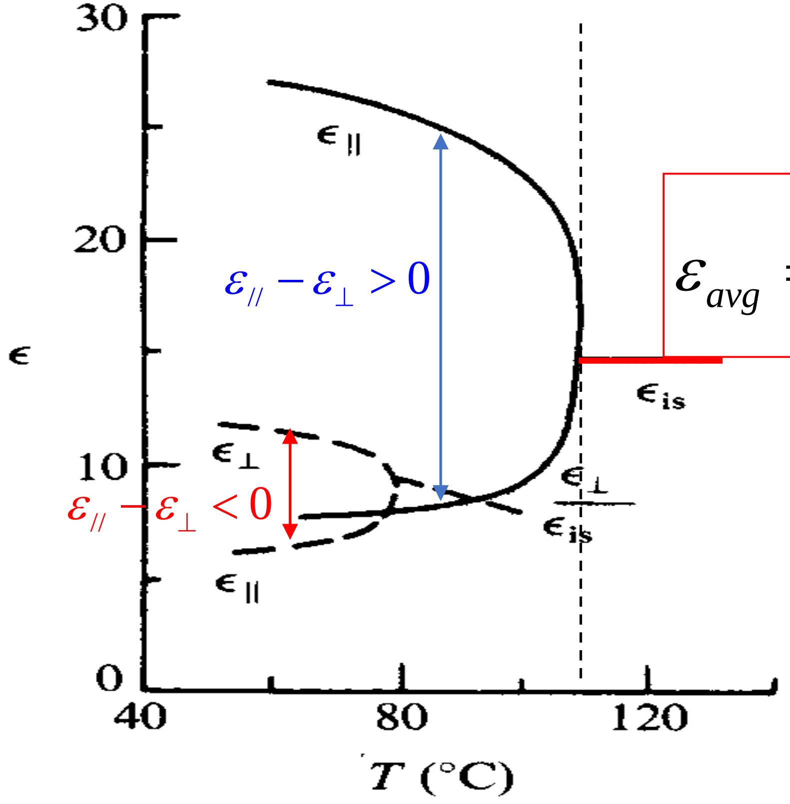
this is called a first order transition

(with change in enthalpy, heat is needed to cross T_C)



DIELECTRIC TENSOR

nematic isotropic



$$\varepsilon_{avg} = \frac{\varepsilon_{//} + 2\varepsilon_{\perp}}{3}$$

T ↗ { **S** ↘ ; $\varepsilon_{//}$ ↘ }

low frequencies

$$\begin{cases} 5\varepsilon_0 < \varepsilon_{//}, \varepsilon_{\perp} < 80\varepsilon_0 \\ -5\varepsilon_0 < \varepsilon_{//} - \varepsilon_{\perp} < 20\varepsilon_0 \end{cases}$$

optical frequencies

$$\begin{cases} 2\varepsilon_0 < \varepsilon_{//}, \varepsilon_{\perp} < 4\varepsilon_0 \\ 0 < \varepsilon_{//} - \varepsilon_{\perp} < 1.5\,\varepsilon_0 \end{cases}$$

REFRACTIVE INDICES

Ordinary and extra-ordinary indices:

$$n_e = \sqrt{\frac{\varepsilon_{//}}{\varepsilon_0}}; \quad n_o = \sqrt{\frac{\varepsilon_{\perp}}{\varepsilon_0}}$$

optical frequencies:
$$\begin{cases} 1.4 < n_e, n_o < 2 \\ 0 < n_e - n_o < 0.4 \end{cases}$$

For light propagation with *k*-vector at an angle θ from *L*

$$n^{(2)} = \left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2} \right)^{-\frac{1}{2}}$$

Liquid Crystals and Photonics

WAT

GHENT UNIVERSITY

TENSOR OF MAGNETIC PERMEABILITY

Magnetic permeability tensor $\vec{M} = \mu_0 \overline{\overline{\chi}}_m \vec{H}$

$$\overline{\overline{\chi}}_m = \begin{pmatrix} \chi_{m\perp} & 0 & 0 \\ 0 & \chi_{m\perp} & 0 \\ 0 & 0 & \chi_{m\parallel} \end{pmatrix}$$

diamagnetism: $\chi<0$ $\chi_{m\perp}, \chi_{m//} < 0$

easier magnetization for the phenyl ring, perpendicular to director

$$|\chi_{m\perp}| > |\chi_{m//}| \Rightarrow \Delta\chi_m = \chi_{m//} - \chi_{m\perp} > 0$$

relation with order parameter: $S = \dfrac{\chi_{//} - \chi_{\perp}}{(\chi_{//} - \chi_{\perp})_{\max}}$

(H_{local}~H)

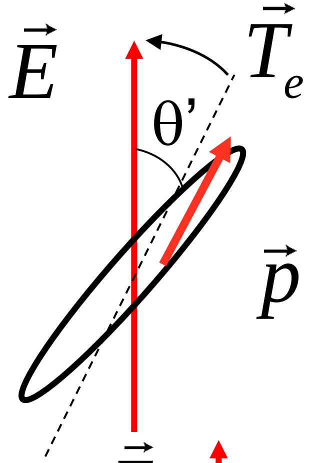
INFLUENCE OF ELECTRIC FIELD

Torque by electric field on molecules: $\vec{T}_e = \vec{p} \times \vec{E}_{local}$

permanent dipole moment

p aligns with *E*, due to torque T_e

$$\left|\vec{T}_e\right| = pE_{local} \sin\theta'$$



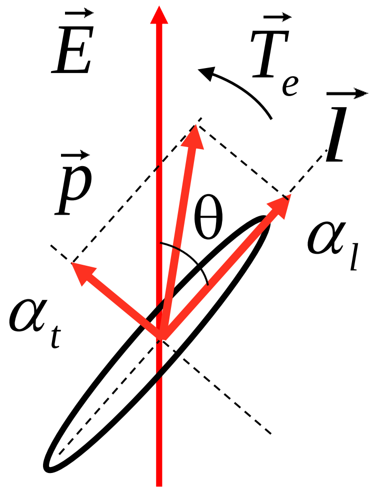
induced dipole moment $\vec{p} = \varepsilon_0 \bar{\bar{\alpha}} \vec{E}_{local}$

$$\left|\vec{T}_e\right| = \left|\varepsilon_0 \bar{\bar{\alpha}} \vec{E}_{local} \times \vec{E}_{local}\right|$$

$$= \left|\left(\varepsilon_0 \alpha_t \bar{\bar{\delta}} \vec{E}_{local} + \varepsilon_0(\alpha_l - \alpha_t)\left(\vec{l}\,\vec{l}\right)\vec{E}_{local}\right) \times \vec{E}_{local}\right|$$

$$= \left|\left(\varepsilon_0(\alpha_l - \alpha_t)\cos\theta E_{local}\vec{l}\right) \times \vec{E}_{local}\right|$$

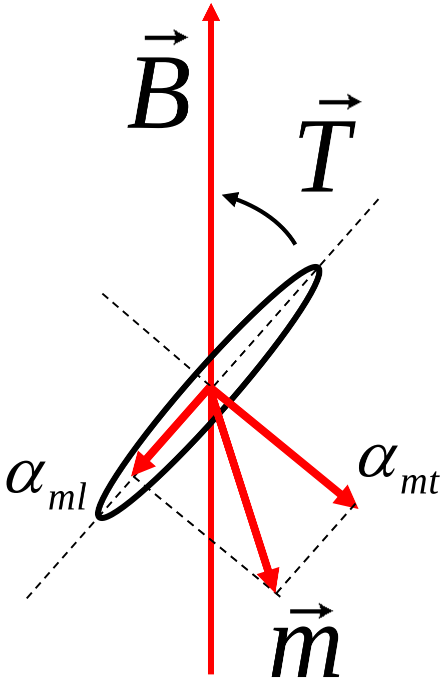
$$= \varepsilon_0(\alpha_l - \alpha_t)\cos\theta \sin\theta E_{local}^2$$



for $\alpha_l > \alpha_t$

INFLUENCE OF MAGNETIC FIELD

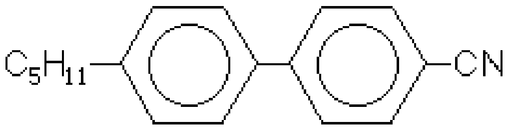
Torque by magnetic field on molecules



diamagnetism

(stronger for phenyl rings)

$$\alpha_{mt} \quad < \quad \alpha_{ml} \quad < 0$$



Torque $\vec{T}_m = \vec{m} \times \vec{B}$

aligns long axis with *B*

PRINCIPLE OF VIRTUAL WORK

Conservation of energy, in the presence of a source

$$VdQ + Id\Phi = dW_{electric} + dW_{magnetic} + dW_{elastic} + dA$$

$VdQ + Id\Phi$: Electric / magnetic energy supplied by sources

$dW_{electric} + dW_{magnetic} + dW_{elastic}$: energy stored in the system

dA: dissipated energy (always>0)
ex: viscous rotation in LC

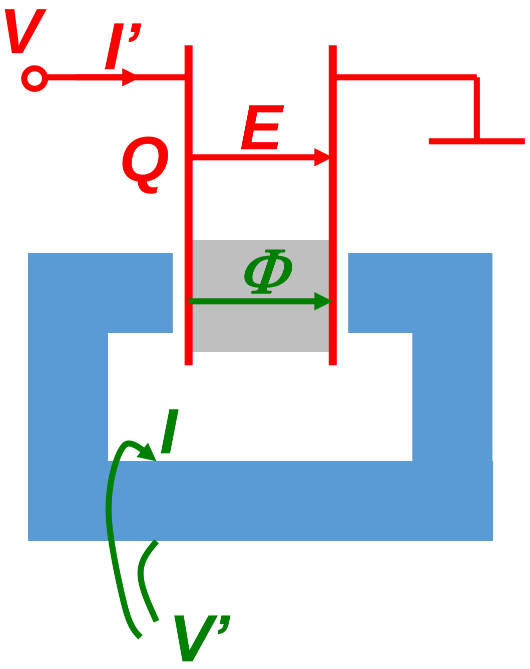
"Stable director distribution is obtained
if every virtual variation leads to $dA < 0$ (not allowed)"

Without sources, the stored energy should be minimal

PRINCIPLE OF VIRTUAL WORK

Using sources that can supply energy to the LC

electric: source with cst *V* or magnetic: source with cst *I*



Electric

energy

$$VdQ$$

current

$$I' = \frac{dQ}{dt}$$

Magnetic

energy

$$Id\Phi$$

voltage

$$V' = \frac{d\Phi}{dt}$$

PRINCIPLE OF VIRTUAL WORK

Internal energy in the system

$$\begin{cases} W_{electric} = \int_{vol} \frac{1}{2}\vec{D}\cdot\vec{E}\,dV = \frac{1}{2}V\cdot Q \\ W_{magnetic} = \int_{vol} \frac{1}{2}\vec{B}\cdot\vec{H}\,dV = \frac{1}{2}I\cdot\Phi \end{cases}$$

cst V over a capacitor with charge Q

cst I over a gap with flux Φ

virtual work

$$VdQ + Id\Phi = d\left(\tfrac{1}{2}V\cdot Q\right) + d\left(\tfrac{1}{2}I\cdot\Phi\right) + dW_{elastic} + dA$$

$$0 = -\tfrac{1}{2}d\left(VQ\right) - \tfrac{1}{2}d\left(I\Phi\right) + dW_{elastic} + dA$$

$$0 = d\int_{vol}\left(-\tfrac{1}{2}\vec{D}\cdot\vec{E}\right)dV + d\int_{vol}\left(-\tfrac{1}{2}\vec{B}\cdot\vec{H}\right)dV + dW_{elastic} + dA$$

$$0 = d\int_{vol}\left(f_{electric} + f_{magnetic} + f_{elastic}\right)dV + dA$$

dissipation>0

energy density

integral should be minimal for stability