

LIQUID CRYSTALS AND LIGHT EMITTING MATERIALS FOR PHOTONIC APPLICATIONS

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Lecture series at WAT in Warsaw

OVERVIEW

Liquid crystal properties (10h)

Properties of nematic liquid crystals

Nematic order parameter

Polarization and dielectric constant

Elastic energy

Surface alignment

Electrical energy

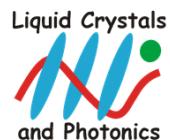
Freederickz threshold

VAN mode

Variable phase retarder

IPS mode

TN mode



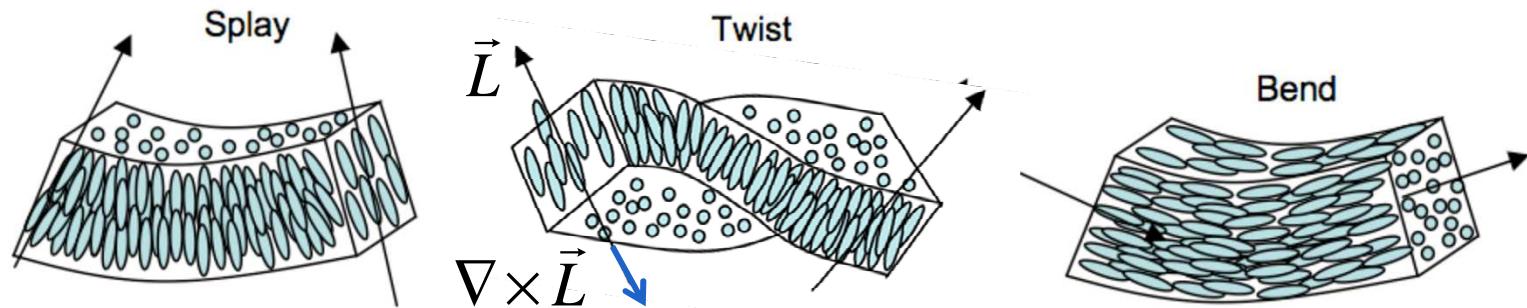
ELASTIC ENERGY

Elastic energy for **chiral** liquid crystal

lowest energy when: $\vec{L} \cdot (\nabla \times \vec{L}) = -q_0$

right handed CLC: $q_0 > 0$

$$f_{elastic} = \frac{1}{2} \left[K_{11} (\nabla \cdot \vec{L})^2 + K_{22} \left(\vec{L} \cdot (\nabla \times \vec{L}) + q_0 \right)^2 + K_{33} \left(\vec{L} \times (\nabla \times \vec{L}) \right)^2 \right]$$



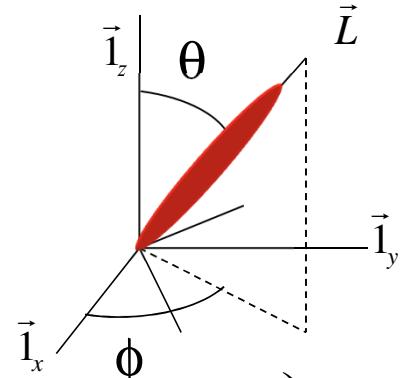
right handed twist
 $\vec{L} \cdot (\nabla \times \vec{L}) < 0$

ELASTIC ENERGY

In a **1D geometry** (L depending only on z)

$$\vec{L} = \begin{pmatrix} \cos \phi(z) \sin \theta(z) \\ \sin \phi(z) \sin \theta(z) \\ \cos \theta(z) \end{pmatrix}$$

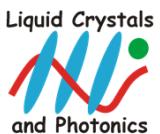
only $\frac{\partial}{\partial z}$



$$f_{elastic} = \frac{1}{2} \left(K_{11} (\nabla \cdot \vec{L})^2 + K_{22} (\vec{L} \cdot (\nabla \times \vec{L}))^2 + K_{33} (\vec{L} \times (\nabla \times \vec{L}))^2 \right)$$

after calculation: ...

$$f_{elastic} = \frac{1}{2} \left(K_{11} \left(\sin \theta \frac{\partial \theta}{\partial z} \right)^2 + K_{22} \left(\sin^2 \theta \frac{\partial \phi}{\partial z} \right)^2 + K_{33} \left(\left(\cos \theta \frac{\partial \theta}{\partial z} \right)^2 + \left(\cos \theta \sin \theta \frac{\partial \phi}{\partial z} \right)^2 \right) \right)$$



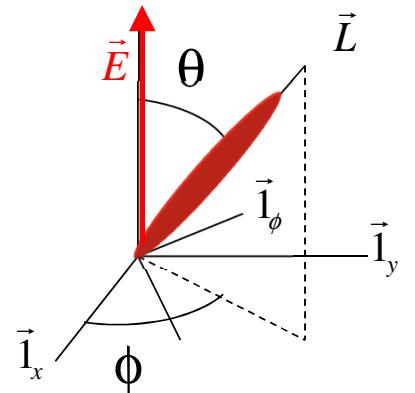
ELECTRIC ENERGY

Electric energy density (with source)

$$f_{electric} = -\frac{1}{2} \vec{D} \cdot \vec{E} = -\frac{1}{2} (\bar{\epsilon} \vec{E}) \vec{E} = -\frac{1}{2} \epsilon_{\perp} E^2 - \frac{1}{2} \Delta \epsilon (\vec{L} \cdot \vec{E})^2$$

$\bar{\epsilon} \vec{E} = \epsilon_{\perp} \vec{E} + \Delta \epsilon (\vec{L} \vec{L}) \vec{E} = \epsilon_{\perp} \vec{E} + \Delta \epsilon \vec{L} (\vec{L} \cdot \vec{E})$

$$\left\{ \begin{array}{l} \vec{L} \cdot \vec{E} = E \cos \theta \\ \vec{L} \times \vec{E} = -E \sin \theta \vec{l}_{\phi} \end{array} \right.$$



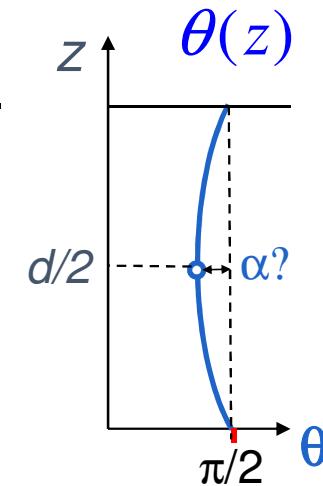
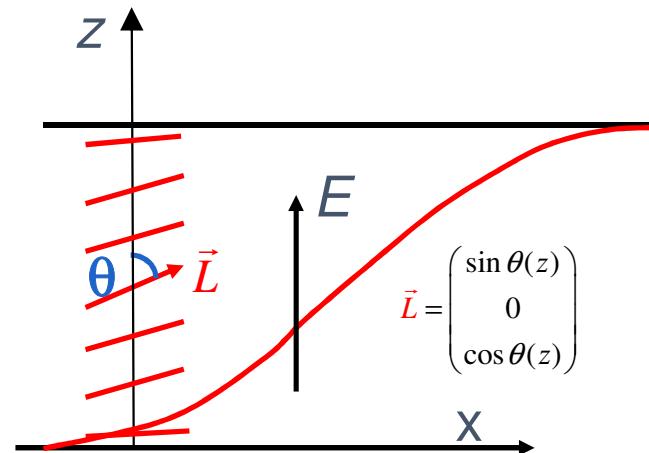
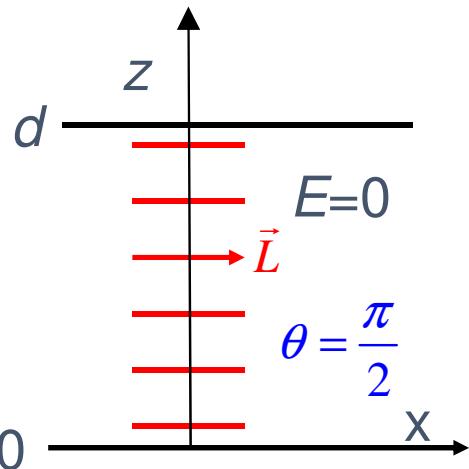
given field E (for example V/d)

$\Delta \epsilon > 0$: lowest energy when L and E are parallel

$\Delta \epsilon < 0$: lowest energy when L and E are perpendicular

EXAMPLE: FRÉEDERICKSZ TRANSITION

LC between substrates with E -field, $\phi = 0$: $\theta(z)$?



assume small deviation $\alpha \ll 1$ due to E -field
with given z -dependency:

$$\theta(z) = \frac{\pi}{2} - \alpha \sin \frac{\pi z}{d}$$

$$\begin{cases} \cos \theta(z) = \cos \left(\frac{\pi}{2} - \alpha \sin \frac{\pi z}{d} \right) = \sin \left(\alpha \sin \frac{\pi z}{d} \right) \approx \alpha \sin \frac{\pi z}{d} \ll 1 \\ \sin \theta(z) \approx 1 \end{cases}$$

EXAMPLE: FRÉEDERICKSZ TRANSITION

calculation of elastic energy

$$\theta(z) = \frac{\pi}{2} - \alpha \sin \frac{\pi z}{d}$$

$$f_{elastic} = \frac{1}{2} K_{11} \left(\sin \theta \frac{\partial \theta}{\partial z} \right)^2 + \frac{1}{2} K_{33} \left(\cos \theta \frac{\partial \theta}{\partial z} \right)^2$$

$\cos \theta \ll 1$

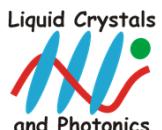
$$\rightarrow F_{elastic} \approx \int_0^d \frac{1}{2} K_{11} \left(-\alpha \frac{\pi}{d} \cdot \cos \frac{\pi z}{d} \right)^2 dz = \frac{1}{4} K_{11} \alpha^2 \frac{\pi^2}{d}$$

calculation of electric energy

field approximately homogeneous: $E=V/d$

$$f_{electric} = -\frac{1}{2} \epsilon_{\perp} E^2 - \frac{1}{2} \Delta \epsilon (\vec{E} \cdot \vec{L})^2 = -\frac{1}{2} \epsilon_{\perp} \frac{V^2}{d^2} - \frac{1}{2} \Delta \epsilon \left(\frac{V}{d} \cos \theta \right)^2$$

$$\rightarrow F_{electric} \approx \int_0^d \left[-\frac{1}{2} \epsilon_{\perp} \left(\frac{V}{d} \right)^2 - \frac{1}{2} \Delta \epsilon \left(\frac{V}{d} \cdot \alpha \sin \frac{\pi z}{d} \right)^2 \right] dz = -\frac{1}{2} \epsilon_{\perp} \frac{V^2}{d} - \frac{1}{4} \Delta \epsilon \cdot \alpha^2 \frac{V^2}{d}$$



EXAMPLE: FRÉEDERICKSZ TRANSITION

calculation of the total energy

$$\theta(z) = \frac{\pi}{2} - \alpha \sin \frac{\pi z}{d}$$

$$F_{elastic} \approx \frac{1}{4} K_{11} \alpha^2 \frac{\pi^2}{d}$$

$$F_{electric} \approx -\frac{1}{2} \epsilon_{\perp} \frac{V^2}{d} - \frac{1}{4} \Delta \epsilon \cdot \alpha^2 \frac{V^2}{d}$$

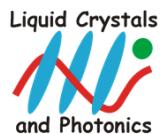


Minimum of the total energy, for which α ?

$$\frac{\partial}{\partial \alpha} (F_{elastic} + F_{electric}) \approx \frac{1}{2} K_{11} \alpha \frac{\pi^2}{d} - \frac{1}{2} \Delta \epsilon \cdot \alpha \frac{V_{th}^2}{d} = 0$$



$$V_{th} = \sqrt{\frac{K_{11}}{\Delta \epsilon}} \cdot \pi$$



Fréedericksz threshold

VARIABLE PHASE RETARDER

low voltage: no switching

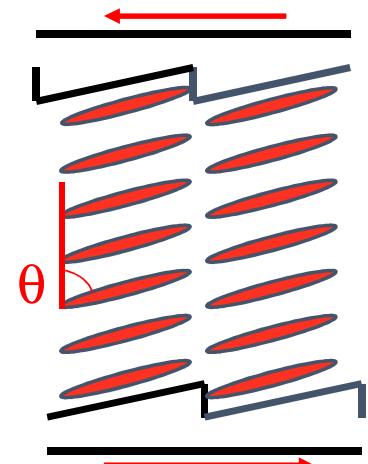
y polarized light: n_o

x polarized light: n_e

$$\Delta n = n_e - n_o$$

$$\Gamma = \frac{2\pi \Delta n d}{\lambda}$$

$$V=0$$



high voltage:

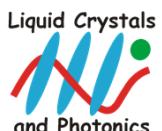
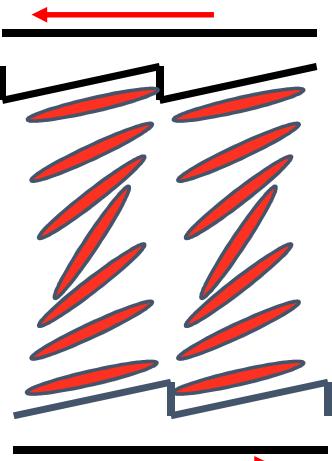
y polarized light: n_o

x polarized light: $n_{eff}(z)$

$$n_{avg} = \frac{1}{d} \int_0^d \left(\frac{\cos^2 \theta(z)}{n_o^2} + \frac{\sin^2 \theta(z)}{n_e^2} \right)^{-\frac{1}{2}} dz$$

$$\Delta n_{avg} = n_{eff,avg} - n_o$$

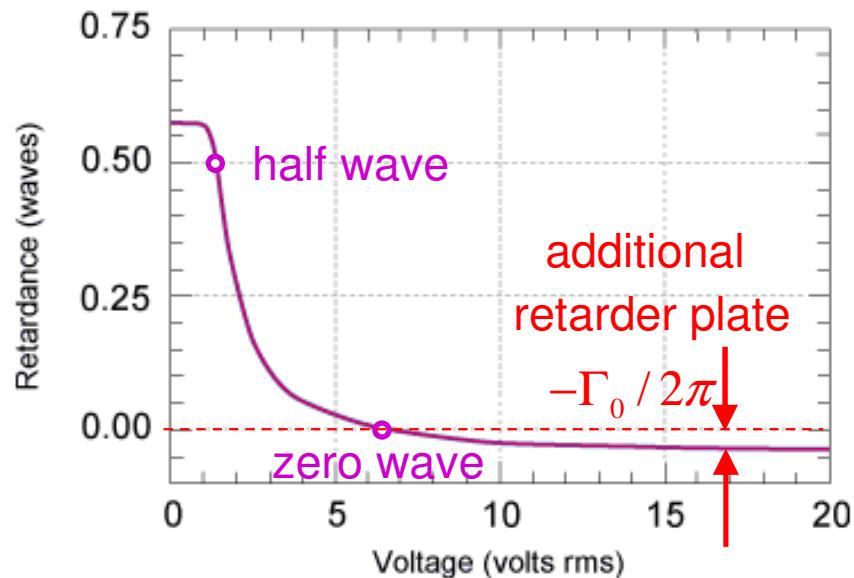
$$V > V_{th} = \sqrt{\frac{K_{11}}{\Delta \epsilon}} \cdot \pi$$



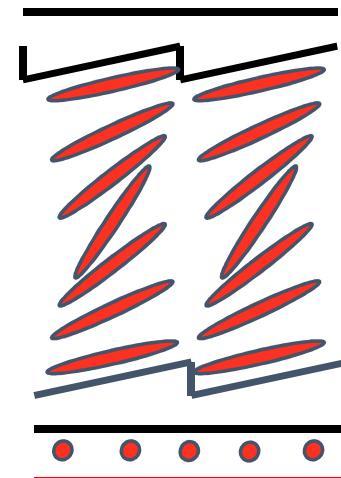
VARIABLE PHASE RETARDER

total retardation: LC retardation + plate

$$\frac{\Gamma_{tot}}{2\pi} = \frac{\Delta n_{avg} d}{\lambda} - \frac{\Gamma_0}{2\pi}$$

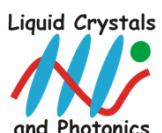


$$V > V_{th} = \sqrt{\frac{K_{11}}{\Delta\epsilon}} \cdot \pi$$



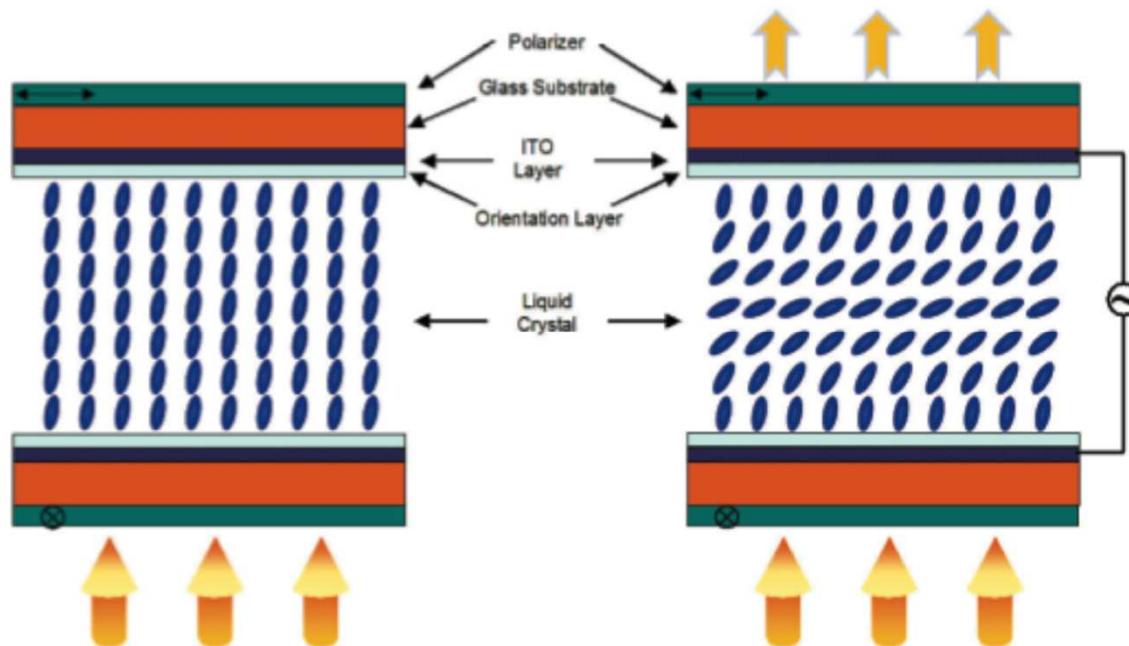
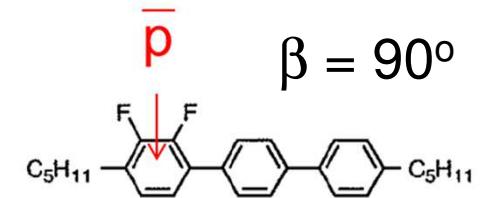
additional
retarder plate
slow axis
perpendicular

$$-\frac{\Gamma_0}{2\pi}$$



VERTICALLY ALIGNED NETMATIC (VAN)

Initially vertically aligned, and $\Delta\epsilon < 0$
with particular alignment material

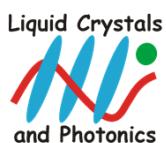
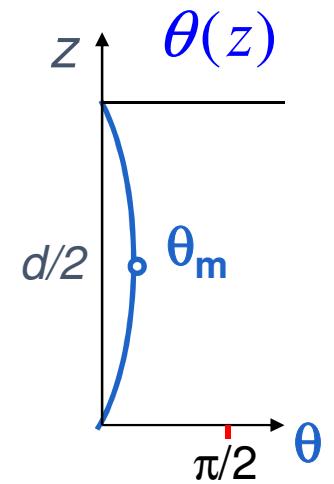
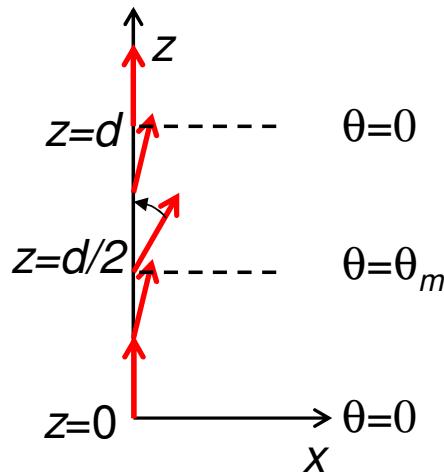


VERTICALLY ALIGNED NEMATIC (VAN)

Small variation of the angle θ
suggestion:

$$\theta = \theta_m \sin\left(\frac{\pi z}{d}\right)$$

$$\begin{aligned} \varphi &= 0 \\ \theta_m &\ll 1 \\ \sin \theta &\ll 1 \\ \cos \theta &\approx 1 \end{aligned}$$



$$f_{elastic} = \frac{1}{2} \left(K_{11} \left(\sin \theta \frac{\partial \theta}{\partial z} \right)^2 + K_{22} \left(\sin^2 \theta \frac{\partial \phi}{\partial z} - q_0 \right)^2 + K_{33} \left(\left(\cos \theta \frac{\partial \theta}{\partial z} \right)^2 + \left(\cos \theta \sin \theta \frac{\partial \phi}{\partial z} \right)^2 \right) \right)$$

$$\begin{aligned} F_{elastic} &= \frac{1}{2} K_{33} A \int_0^d \left(\frac{\partial \theta}{\partial z} \right)^2 dz = \frac{1}{2} K_{33} A \int_0^d \left(\theta_m \frac{\pi}{d} \cos \left(\frac{\pi z}{d} \right) \right)^2 dz \\ &= \frac{1}{2} K_{33} A d \left(\theta_m \frac{\pi}{d} \right)^2 \frac{1}{2} = \frac{1}{4} K_{33} A \theta_m^2 \frac{\pi^2}{d} \end{aligned}$$

VERTICALLY ALIGNED NEMATIC (VAN)

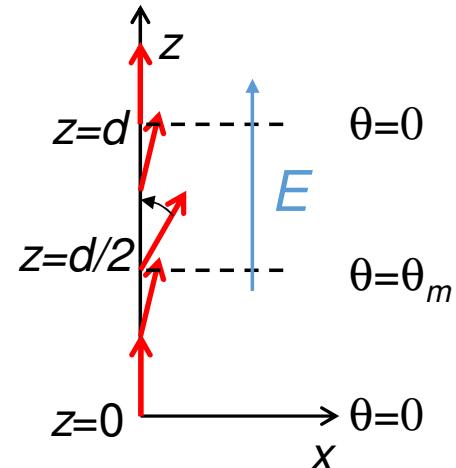
Variation of the angle θ

$$\theta = \theta_m \sin\left(\frac{\pi z}{d}\right)$$

$$\begin{aligned} \varphi &= 0 \\ \theta_m &\ll 1 \\ \sin \theta &\approx \theta \ll 1 \end{aligned}$$

$$\Delta\epsilon < 0$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

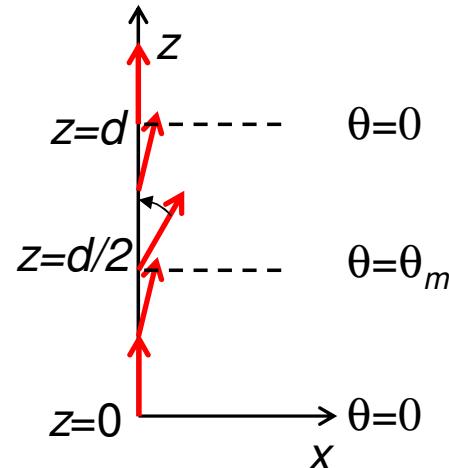


$$\begin{aligned} F_{electric} &= -\frac{1}{2} A \int_0^d (\epsilon_{\perp} + \Delta\epsilon \cos^2 \theta) E^2 dz = -\frac{1}{2} A \int_0^d \left(\epsilon_{\perp} + \Delta\epsilon \left[1 - \left(\theta_m \cos\left(\frac{\pi z}{d}\right) \right)^2 \right] \right) \frac{V^2}{d^2} dz \\ &= -\frac{1}{2} A \int_0^d \left(\epsilon_{||} - \Delta\epsilon \left(\theta_m \cos\left(\frac{\pi z}{d}\right) \right)^2 \right) \frac{V^2}{d^2} dz = -\frac{1}{2} A \epsilon_{||} \frac{V^2}{d^2} d + \frac{1}{2} A \Delta\epsilon \theta_m^2 \frac{V^2}{d^2} \left(d \frac{1}{2} \right) \\ &= -\frac{1}{2} A \epsilon_{||} \frac{V^2}{d} - \frac{1}{4} A |\Delta\epsilon| \theta_m^2 \frac{V^2}{d} \end{aligned}$$

VERTICALLY ALIGNED NEMATIC (VAN)

Does $\theta_m = 0$
minimize the total energy?

$$\theta = \theta_m \sin\left(\frac{\pi z}{d}\right)$$



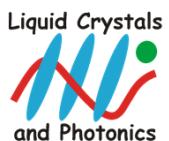
$$F_{total} = F_{elastic} + F_{electric} = \frac{1}{4} K_{33} A \theta_m^2 \frac{\pi^2}{d} - \frac{1}{2} A \epsilon_{||} \frac{V^2}{d} - \frac{1}{4} A |\Delta \epsilon| \theta_m^2 \frac{V^2}{d}$$

$$\frac{\partial F_{total}}{\partial \theta_m} = \frac{1}{4} K_{33} A 2\theta_m \frac{\pi^2}{d} - \frac{1}{4} A |\Delta \epsilon| 2\theta_m \frac{V^2}{d} = \frac{1}{2} A \theta_m \frac{1}{d} \left(K_{33} \pi^2 - |\Delta \epsilon| V^2 \right)$$

threshold voltage
makes this zero

$$V_{TH} = \pi \sqrt{\frac{K_{33}}{|\Delta \epsilon|}}$$

$$V < V_{TH} \rightarrow \theta = 0$$

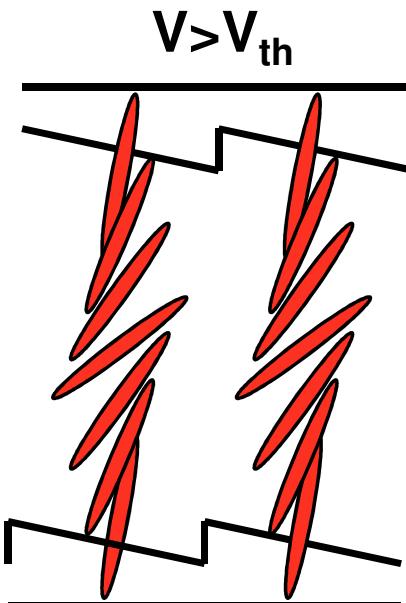
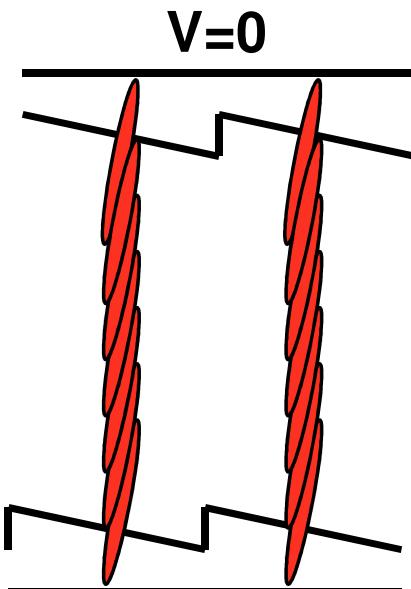


VERTICALLY ALIGNED NEMATIC (VAN)

1D structure, material with $\Delta\epsilon < 0$, $\Delta n > 0$

homeotropic alignment

- [for $V = 0$: $\theta \approx 0$ $\Gamma \approx 0$]
- [for $V > V_{th}$: molecules rotate: θ , Γ , Δn increase]



$$V_{th} = \sqrt{\frac{K_{33}}{|\Delta\epsilon|}} \cdot \pi$$

bend and splay

pretilt determines switching

LIGHT IN ANISOTROPIC MATERIALS

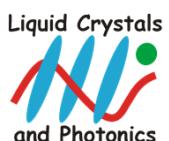
propagation along the z -axis (distance d)

two polarizations with different speed

ordinary mode E_y : $n=n_o$

extra-ordinary mode E_x : $n=n_{\text{eff}}$

$$n_{\text{eff}} = \frac{1}{\sqrt{\frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}}}$$

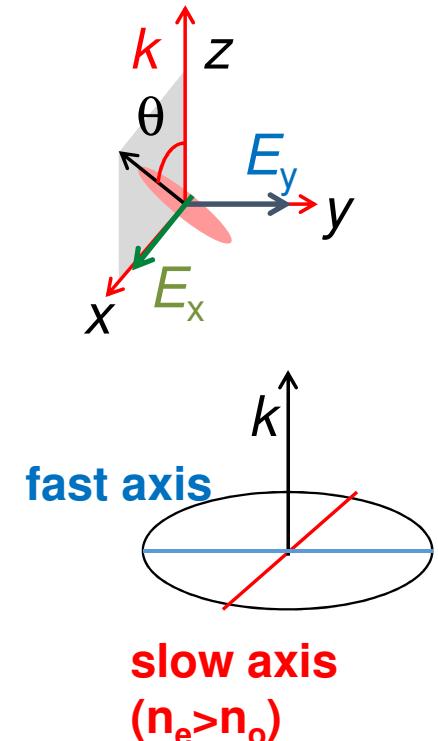


Jones matrix

$$\bar{J} = \begin{bmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{bmatrix}$$

retardation

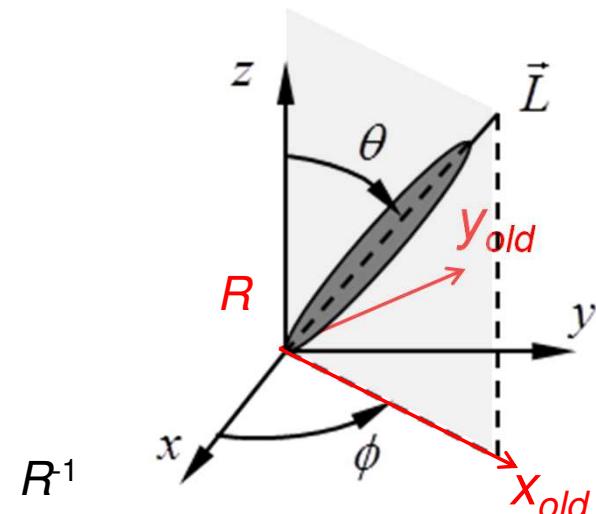
$$\Gamma = \frac{2\pi}{\lambda} (n_{\text{eff}} - n_o) d$$



LIGHT IN ANISOTROPIC MATERIALS

Transformation of the Jones matrix for a general system of axes:

$$\bar{\bar{J}}' = R \bar{\bar{J}}_{old} R^{-1}$$



$$\begin{aligned}
 R &= \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \\
 &= \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi \cdot e^{-i\Gamma/2} & \sin \varphi \cdot e^{-i\Gamma/2} \\ -\sin \varphi \cdot e^{i\Gamma/2} & \cos \varphi \cdot e^{-i\Gamma/2} \end{bmatrix} \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix} \\
 &= \begin{bmatrix} \cos^2 \varphi \cdot e^{-i\Gamma/2} + \sin^2 \varphi \cdot e^{i\Gamma/2} & \sin \varphi \cos \varphi \cdot (e^{-i\Gamma/2} - e^{i\Gamma/2}) \\ \sin \varphi \cos \varphi \cdot (e^{-i\Gamma/2} - e^{i\Gamma/2}) & \sin^2 \varphi \cdot e^{-i\Gamma/2} + \cos^2 \varphi \cdot e^{i\Gamma/2} \end{bmatrix} \cdot \begin{bmatrix} E_{x,in} \\ E_{y,in} \end{bmatrix}
 \end{aligned}$$

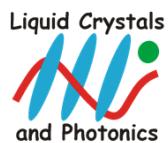
LIGHT IN ANISOTROPIC MATERIALS

$$\bar{\bar{J}} = \begin{bmatrix} \cos^2 \varphi \cdot e^{-i\Gamma/2} + \sin^2 \varphi \cdot e^{i\Gamma/2} & \sin \varphi \cos \varphi \cdot (e^{-i\Gamma/2} - e^{i\Gamma/2}) \\ \sin \varphi \cos \varphi \cdot (e^{-i\Gamma/2} - e^{i\Gamma/2}) & \sin^2 \varphi \cdot e^{-i\Gamma/2} + \cos^2 \varphi \cdot e^{i\Gamma/2} \end{bmatrix}$$

$$\begin{aligned} J_{11} &= \cos^2 \varphi \cdot e^{-i\Gamma/2} + \sin^2 \varphi \cdot e^{i\Gamma/2} \\ &= \cos^2 \varphi \cdot (\cos(\Gamma/2) - i \sin(\Gamma/2)) + \sin^2 \varphi \cdot (\cos(\Gamma/2) + i \sin(\Gamma/2)) \\ &= (\cos^2 \varphi + \sin^2 \varphi) \cos(\Gamma/2) + (-i \cos^2 \varphi + i \sin^2 \varphi) \sin(\Gamma/2) \\ &= \cos(\Gamma/2) - i \cos 2\varphi \sin(\Gamma/2) \end{aligned}$$

$$\begin{aligned} J_{12} &= \sin \varphi \cos \varphi \cdot (e^{-i\Gamma/2} - e^{i\Gamma/2}) \\ &= \sin \varphi \cos \varphi \cdot (-2i \sin(\Gamma/2)) \\ &= -i \sin 2\varphi \cdot \sin(\Gamma/2) \end{aligned}$$

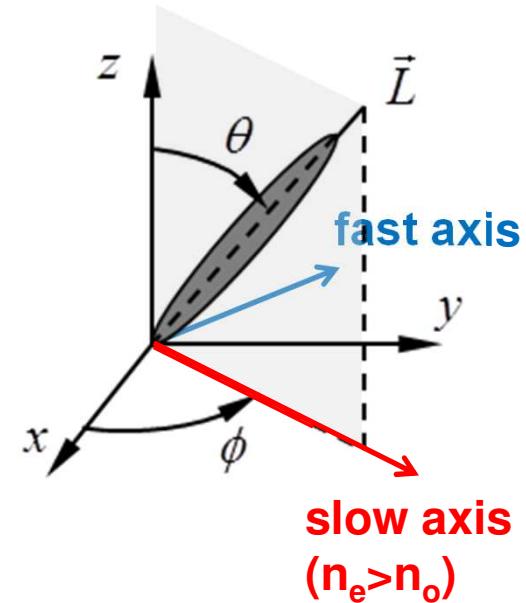
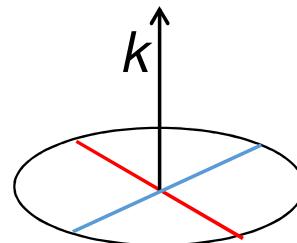
$$\bar{\bar{J}} = \begin{bmatrix} \cos(\Gamma/2) - i \cos 2\varphi \sin(\Gamma/2) & -i \sin 2\varphi \cdot \sin(\Gamma/2) \\ -i \sin 2\varphi \cdot \sin(\Gamma/2) & \cos(\Gamma/2) + i \cos 2\varphi \sin(\Gamma/2) \end{bmatrix}$$



LIGHT IN ANISOTROPIC MATERIALS

Jones matrix for a retardation plate

{
retardation Γ
azimuth ϕ



slow axis: with the highest refractive index

fast axis: with the lowest refractive index

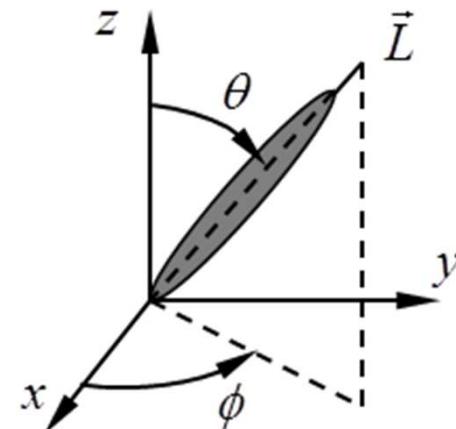
$$\bar{J} = \begin{bmatrix} \cos(\Gamma/2) - i \cos 2\phi \sin(\Gamma/2) & -i \sin 2\phi \cdot \sin(\Gamma/2) \\ -i \sin 2\phi \cdot \sin(\Gamma/2) & \cos(\Gamma/2) + i \cos 2\phi \sin(\Gamma/2) \end{bmatrix}$$

RETARDATION PLATES

homogeneous anisotropic layer
with given parameters

full wave plate $\Gamma=2\pi$

$$\bar{\bar{J}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{independent of } \phi$$



half wave plate $\Gamma=\pi$

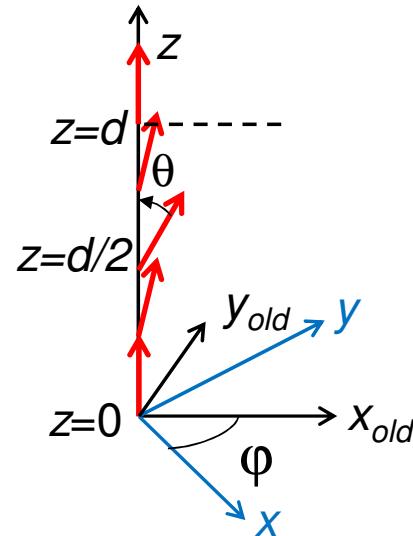
$$\bar{\bar{J}} = \begin{bmatrix} -i \cos 2\phi & -i \sin 2\phi \\ -i \sin 2\phi & i \cos 2\phi \end{bmatrix} \sim \begin{bmatrix} -\cos 2\phi & -\sin 2\phi \\ -\sin 2\phi & \cos 2\phi \end{bmatrix}$$

VERTICALLY ALIGNED NEMATIC (VAN)

Light transmission?

$$\bar{\bar{J}} = \begin{bmatrix} e^{-i\Gamma_{VA}/2} & 0 \\ 0 & e^{i\Gamma_{VA}/2} \end{bmatrix}$$

for inclination in the $\phi=0$ plane



General case, in the plane ϕ :

$$\bar{\bar{J}}' = R \bar{\bar{J}}_{old} R^{-1}$$

$$\bar{\bar{J}}_{VA} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} e^{-i\Gamma_{VA}/2} & 0 \\ 0 & e^{i\Gamma_{VA}/2} \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}$$

$$\Gamma_{VA} = \int_0^d \frac{2\pi}{\lambda} (n_{eff}(\theta(z)) - n_o) dz = \frac{2\pi \Delta n_{avg} d}{\lambda}$$

$$n_{eff} = \frac{1}{\sqrt{\frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}}}$$

VERTICALLY ALIGNED NEMATIC (VAN)

With polarizer and analyzer
along *x* and *y* respectively:

$$\begin{pmatrix} 0 \\ E_{out} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \overline{\overline{J}}_{VA} \begin{pmatrix} E_{in} \end{pmatrix}$$

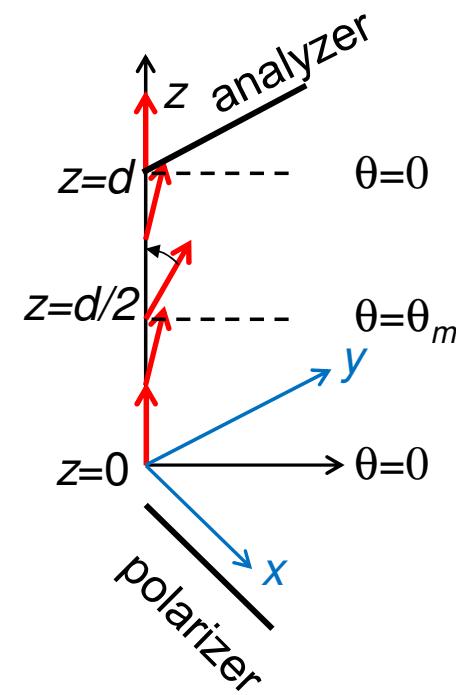
$$= \begin{pmatrix} 0 & 0 \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} e^{-i\Gamma_{VA}/2} & 0 \\ 0 & e^{i\Gamma_{VA}/2} \end{pmatrix} \begin{pmatrix} \cos \phi E_{in} \\ -\sin \phi E_{in} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ \sin \phi \cdot e^{-i\Gamma_{VA}/2} & \cos \phi e^{i\Gamma_{VA}/2} \end{pmatrix} \begin{pmatrix} \cos \phi E_{in} \\ -\sin \phi E_{in} \end{pmatrix}$$

$$E_{out} = (-i) \sin 2\phi \sin \left(\frac{\Gamma_{VA}}{2} \right) E_{in}$$

$$T = \frac{1}{2} \sin^2 2\phi \sin^2 \left(\frac{\pi \Delta n_{avg} d}{\lambda} \right)$$

for unpolarized incident light
(choose $\phi=45^\circ$; $\Gamma_{VA}=\pi$)

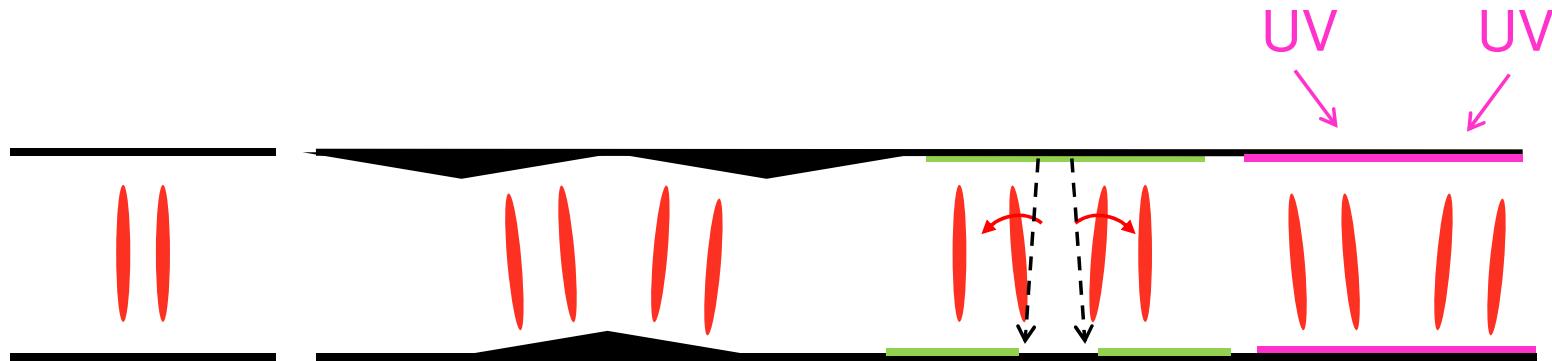


VERTICALLY ALIGNED NEMATIC (VAN)

Reproducible reorientation

how to define the azimuth ϕ of reorientation?

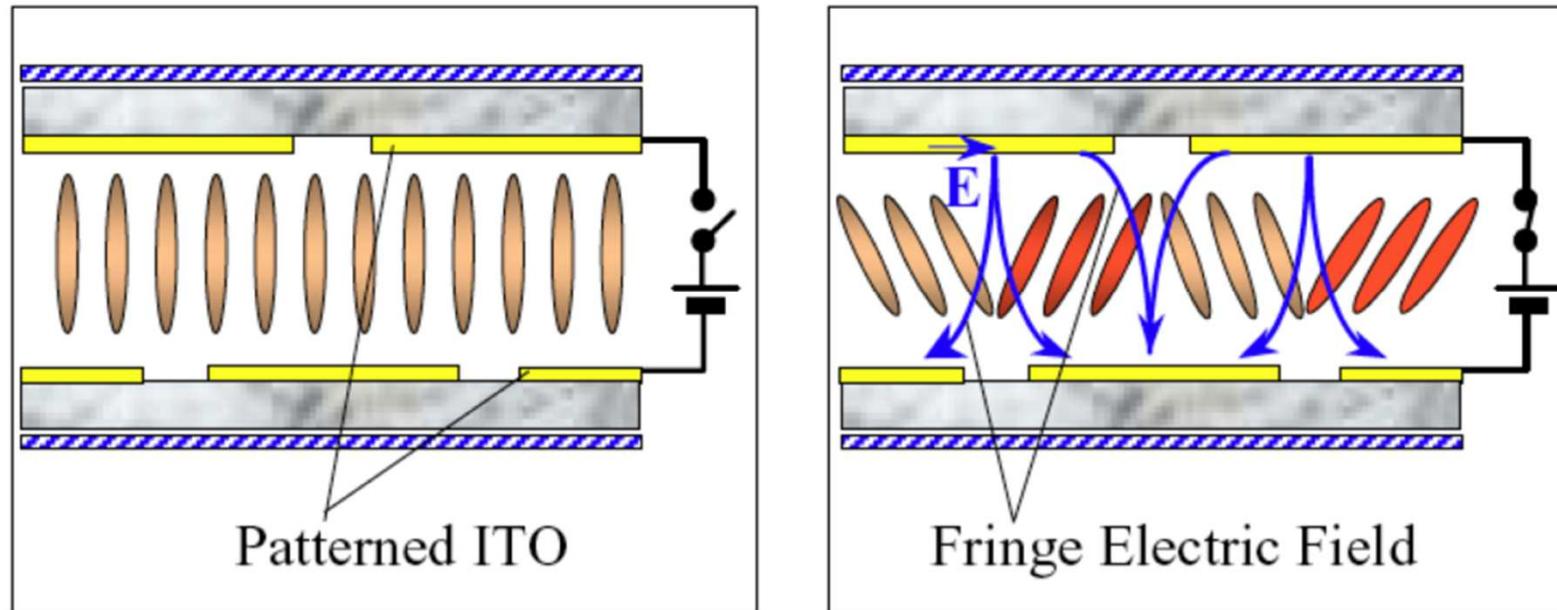
3 methods to determine ϕ



VERTICALLY ALIGNED NEMATIC (VAN)

patterned ITO electrodes (within a pixel)

lateral component of the electric field “fringe field”
determines the switching

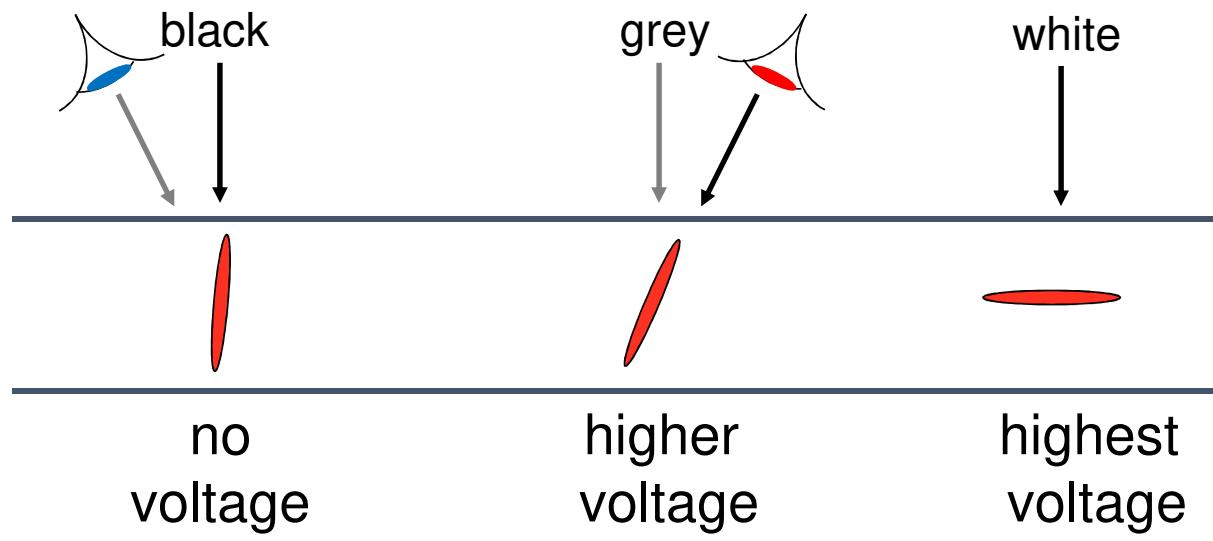


VERTICALLY ALIGNED NEMATIC (VAN)

for a direction that is not vertical
the transmission can have a different value...

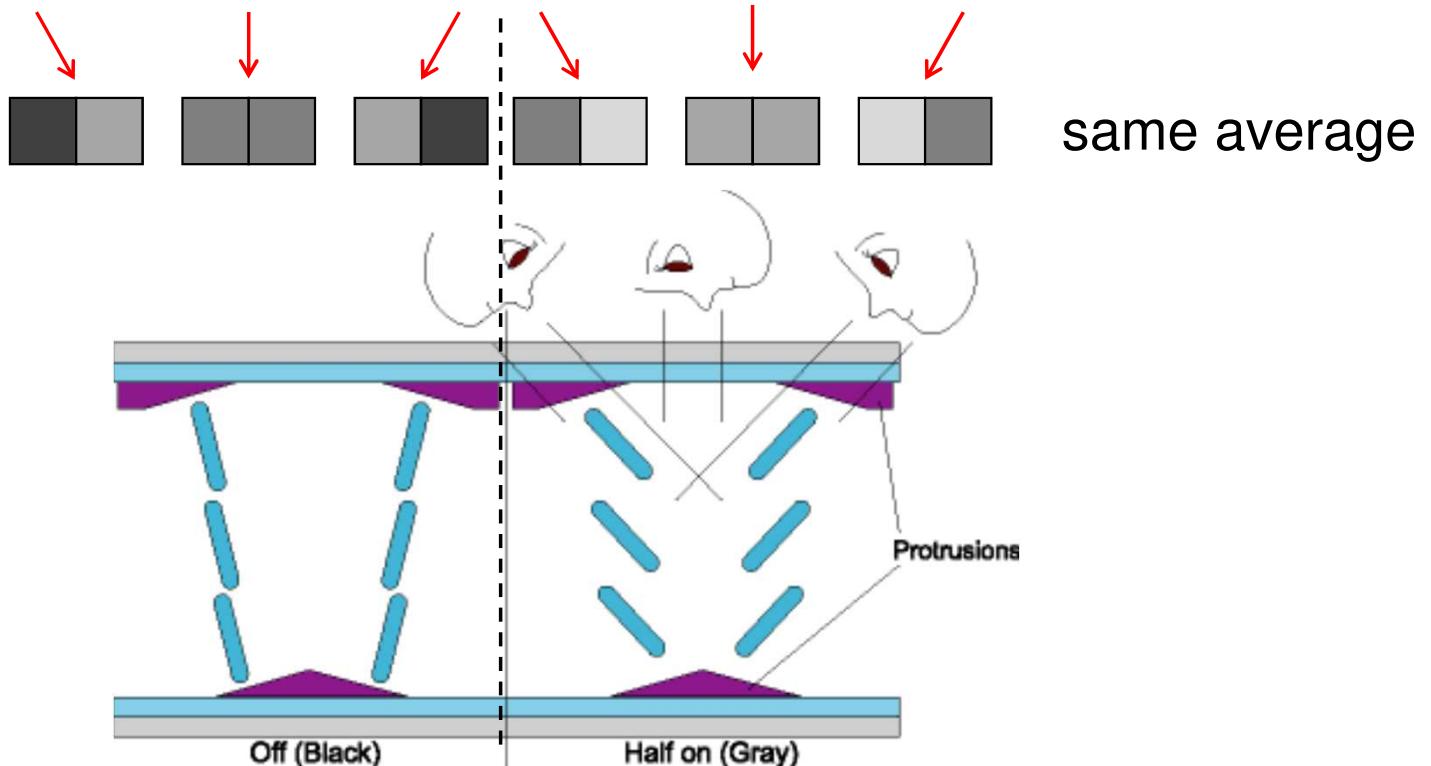
black? no... grey!
bad black state,
low contrast

grey? no...black!
grey scale
inversion



VERTICALLY ALIGNED NEMATIC (VAN)

Solving the viewing angle problem
by introducing different domains with different azimuth

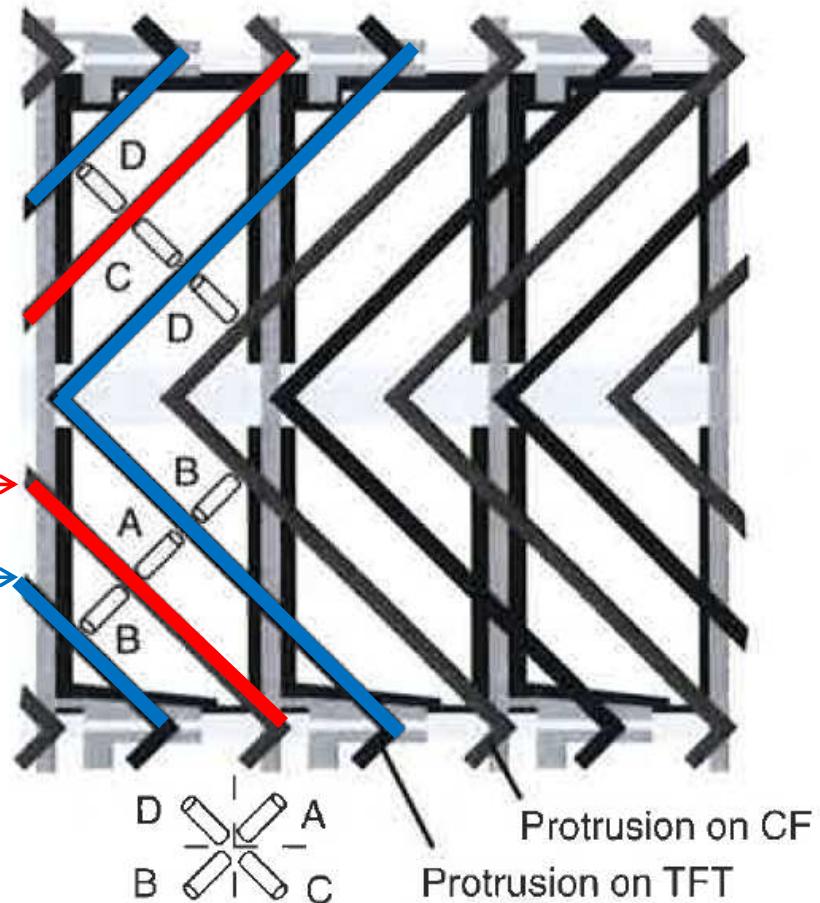


VERTICALLY ALIGNED NEMATIC (VAN)

Patterned VA
with 4 different
azimuthal angles

bottom substrate
protrusions on the color filters →

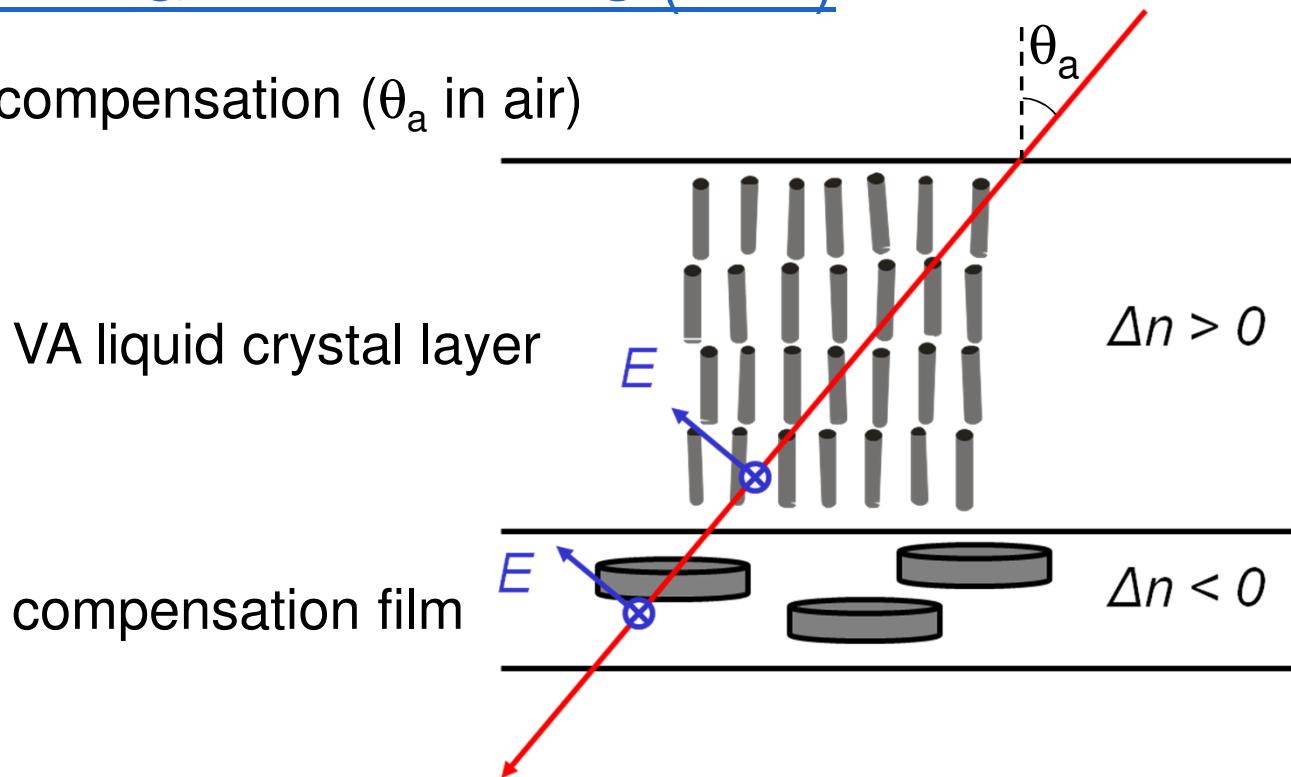
top substrate →
protrusion on TFT's



LC inclination azimuth
in each domain

VERTICALLY ALIGNED NEMATIC (VAN)

Viewing angle compensation (θ_a in air)



LIQUID CRYSTAL SWITCHING

Reorientation due to the torque

