

# LIQUID CRYSTALS AND LIGHT EMITTING MATERIALS FOR PHOTONIC APPLICATIONS

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# OVERVIEW

## Liquid crystal properties (10h)

Properties of nematic liquid crystals

Nematic order parameter

Polarization and dielectric constant

Elastic energy

Surface alignment

Electrical energy

Freederickz treshold

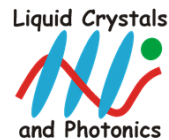
VAN mode

Variable phase retarder

IPS mode

TN mode

Polarization microscopy



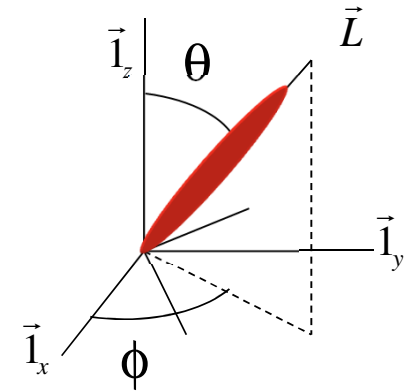
# ELASTIC ENERGY

In a **1D geometry** ( $L$  depending only on  $z$ )

$$f_{elastic} = \frac{1}{2} \left( K_{11} (\nabla \cdot \vec{L})^2 + K_{22} (\vec{L} \cdot (\nabla \times \vec{L}))^2 + K_{33} (\vec{L} \times (\nabla \times \vec{L}))^2 \right)$$

only  $\frac{\partial}{\partial z}$

$$\vec{L} = \begin{pmatrix} \cos \phi(z) \sin \theta(z) \\ \sin \phi(z) \sin \theta(z) \\ \cos \theta(z) \end{pmatrix}$$



$$f_{elastic} = \frac{1}{2} \left( K_{11} \left( \sin \theta \frac{\partial \theta}{\partial z} \right)^2 + K_{22} \left( \sin^2 \theta \frac{\partial \phi}{\partial z} \right)^2 + K_{33} \left( \left( \cos \theta \frac{\partial \theta}{\partial z} \right)^2 + \left( \cos \theta \sin \theta \frac{\partial \phi}{\partial z} \right)^2 \right) \right)$$

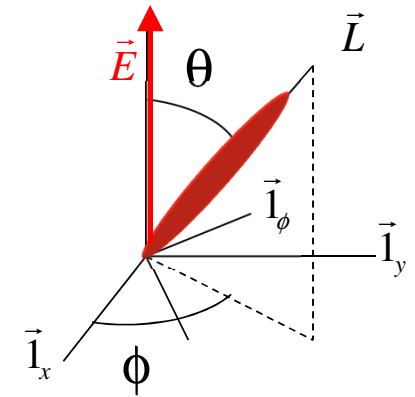
# ELECTRIC ENERGY

Electric energy density (with source)

$$f_{electric} = -\frac{1}{2} \vec{D} \cdot \vec{E} = -\frac{1}{2} (\overline{\overline{\boldsymbol{\epsilon}}} \vec{E}) \cdot \vec{E} = -\frac{1}{2} \epsilon_{\perp} E^2 - \frac{1}{2} \Delta \epsilon (\vec{L} \cdot \vec{E})^2$$

$$\overline{\overline{\boldsymbol{\epsilon}}} \vec{E} = \epsilon_{\perp} \vec{E} + \Delta \epsilon (\vec{L} \vec{L}) \vec{E} = \epsilon_{\perp} \vec{E} + \Delta \epsilon \vec{L} (\vec{L} \cdot \vec{E})$$

$$\left\{ \begin{array}{l} \vec{L} \cdot \vec{E} = E \cos \theta \\ \vec{L} \times \vec{E} = -E \sin \theta \vec{1}_{\phi} \end{array} \right.$$



given field E (for example V/d)

$\Delta \epsilon > 0$ : lowest energy when L and E are parallel

$\Delta \epsilon < 0$ : lowest energy when L and E are perpendicular

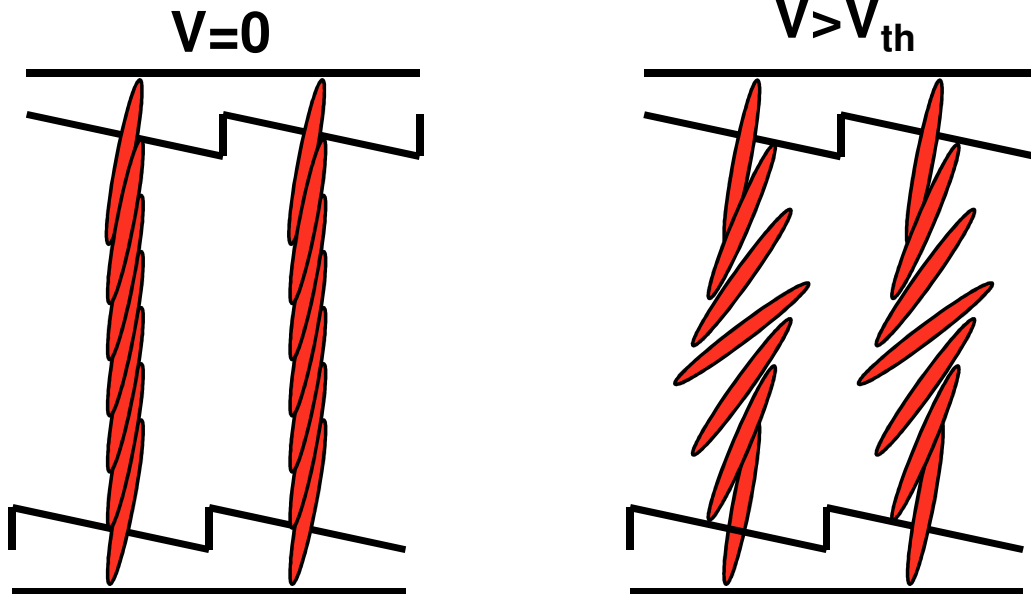
# VERTICALLY ALIGNED NEMATIC (VAN)

1D structure, material with  $\Delta\epsilon < 0$ ,  $\Delta n > 0$

homeotropic alignment

for  $V = 0$ :  $\theta \approx 0$   $\Gamma \approx 0$

for  $V > V_{th}$ : molecules rotate:  $\theta$ ,  $\Gamma$ ,  $\Delta n$  increase



$$V_{th} = \sqrt{\frac{K_{33}}{|\Delta\epsilon|}} \cdot \pi$$

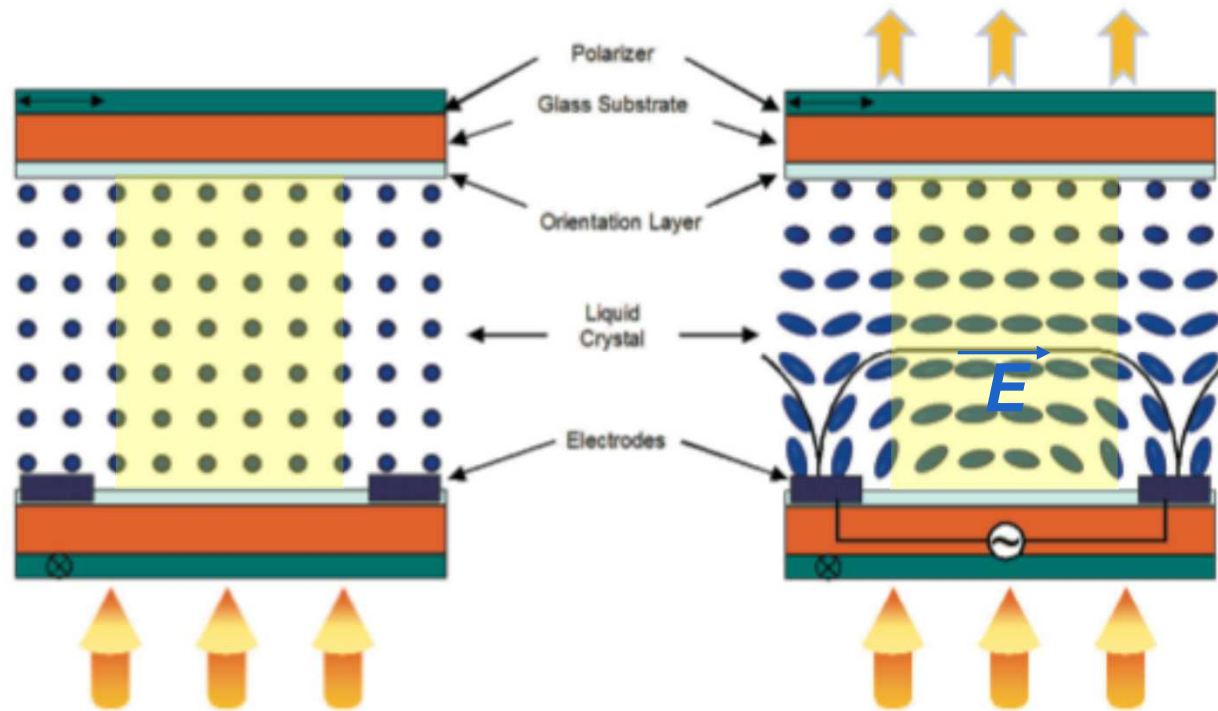
bend and splay

pretilt determines  
switching

# IN PLANE SWITCHING (IPS) MODE

initially: director  
homogeneous

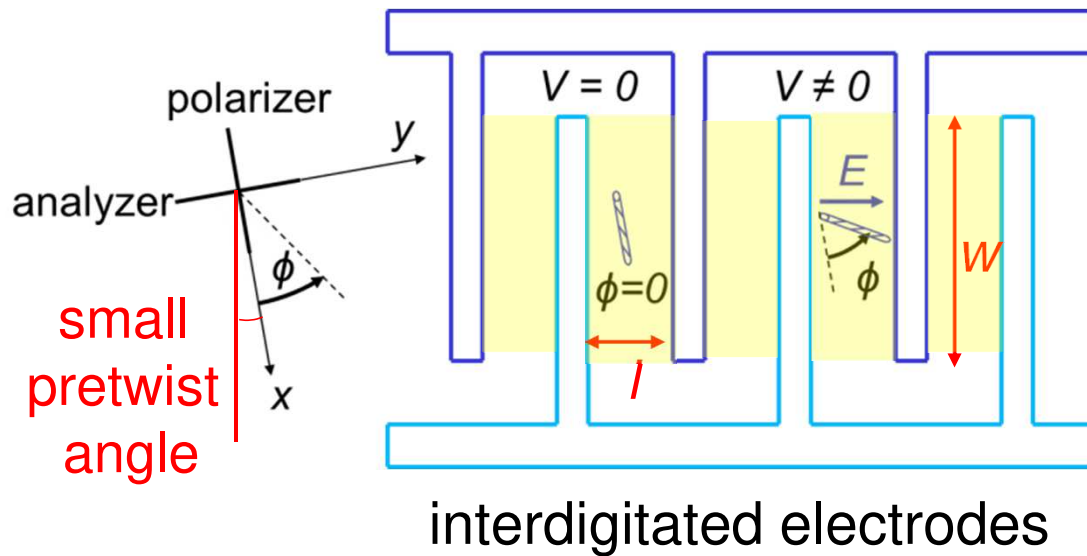
with voltage: director  
reorients in plane



# IN PLANE SWITCHING (IPS) MODE

initially: director  
homogeneous  
 $\phi=0$  when  $V=0$

with voltage: director  
reorients in plane ( $\Delta\varepsilon>0$ )  
 $\phi>0$ , increases with  $V$   
 $E$ -field:  $\phi_E = 80^\circ$



# IN PLANE SWITCHING (IPS) MODE

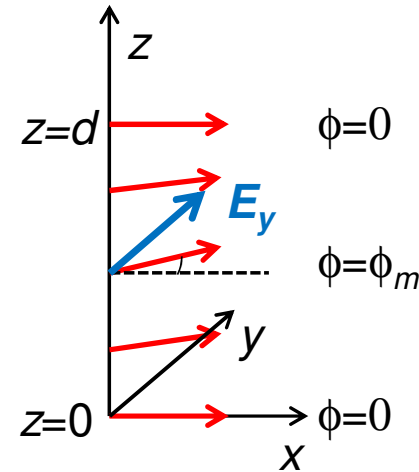
Variation of the **azimuth angle  $\phi$**   
only twist, neglect pretwist, **field  $E_y$**

$$\phi = \phi_m \sin\left(\frac{\pi z}{d}\right)$$

$$\theta = \pi/2$$

$$\phi_m \ll 1$$

$$\sin \phi \approx \phi_m \sin\left(\frac{\pi z}{d}\right)$$



$$f_{elastic} = \frac{1}{2} \left( K_{11} \left( \sin \theta \frac{\partial \theta}{\partial z} \right)^2 + K_{22} \left( \sin^2 \theta \frac{\partial \phi}{\partial z} - q_0 \right)^2 + K_{33} \left( \left( \cos \theta \frac{\partial \theta}{\partial z} \right)^2 + \left( \cos \theta \sin \theta \frac{\partial \phi}{\partial z} \right)^2 \right) \right)$$

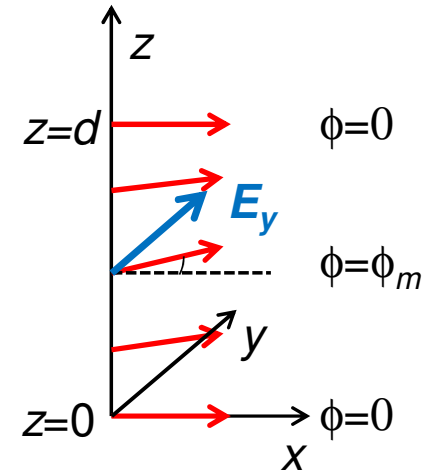
$$F_{elastic} = \frac{1}{2} K_{22} w l \int_0^d \left( \frac{\partial \phi}{\partial z} \right)^2 dz = \frac{1}{2} K_{22} w l \int_0^d \left( \phi_m \frac{\pi}{d} \cos \left( \frac{\pi z}{d} \right) \right)^2 dz = \frac{1}{4} K_{22} w l \left( \phi_m \frac{\pi}{d} \right)^2 d$$



# IN PLANE SWITCHING (IPS) MODE

Variation of the azimuth angle  $\phi$   
only twist, neglect pretwist, field  $E_y$

$$\phi = \phi_m \sin\left(\frac{\pi z}{d}\right) \quad \begin{array}{l} \theta = \pi/2 \\ \phi_m \ll 1 \\ \sin \phi \approx \phi_m \sin\left(\frac{\pi z}{d}\right) \end{array}$$



$$f_{electric} = -\frac{1}{2} \epsilon_{\perp} E^2 - \frac{1}{2} \Delta \epsilon (\bar{E} \cdot \bar{L})^2$$

$$\begin{aligned} F_{electric} &= -\frac{1}{2} w l \int_0^d \left( \epsilon_{\perp} + \Delta \epsilon \sin^2 \phi \right) E^2 dz = -\frac{1}{2} w l \int_0^d \left( \epsilon_{\perp} + \Delta \epsilon \left( \phi_m \sin\left(\frac{\pi z}{d}\right) \right)^2 \right) \frac{V^2}{l^2} dz \\ &= -\frac{1}{2} w l \epsilon_{\perp} \frac{V^2}{l^2} d - \frac{1}{4} w l \cdot \Delta \epsilon \cdot \phi_m^2 \frac{V^2}{l^2} d \end{aligned}$$

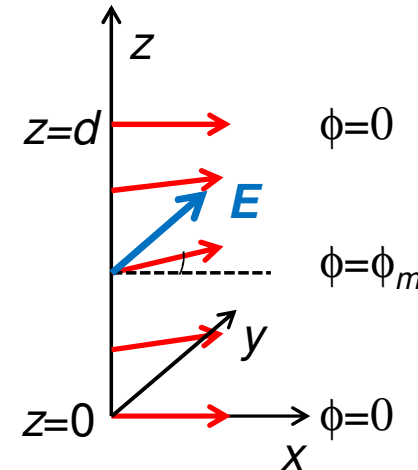
# IN PLANE SWITCHING (IPS) MODE

Variation of the azimuth angle  $\phi$   
only twist, field  $E_y$

$$\phi = \phi_m \sin\left(\frac{\pi z}{d}\right) \quad \theta = \pi/2$$

$$\phi_m \ll 1$$

$$\sin \phi \approx \phi_m \sin\left(\frac{\pi z}{d}\right)$$



$$F_{total} = F_{elastic} + F_{electric} = \frac{1}{4} K_{22} \omega l \left( \phi_m \frac{\pi}{d} \right)^2 d - \frac{1}{2} \omega l \epsilon_{\perp} \frac{V^2}{l^2} d - \frac{1}{4} \omega l \cdot \Delta \epsilon \cdot \phi_m^2 \frac{V^2}{l^2} d$$

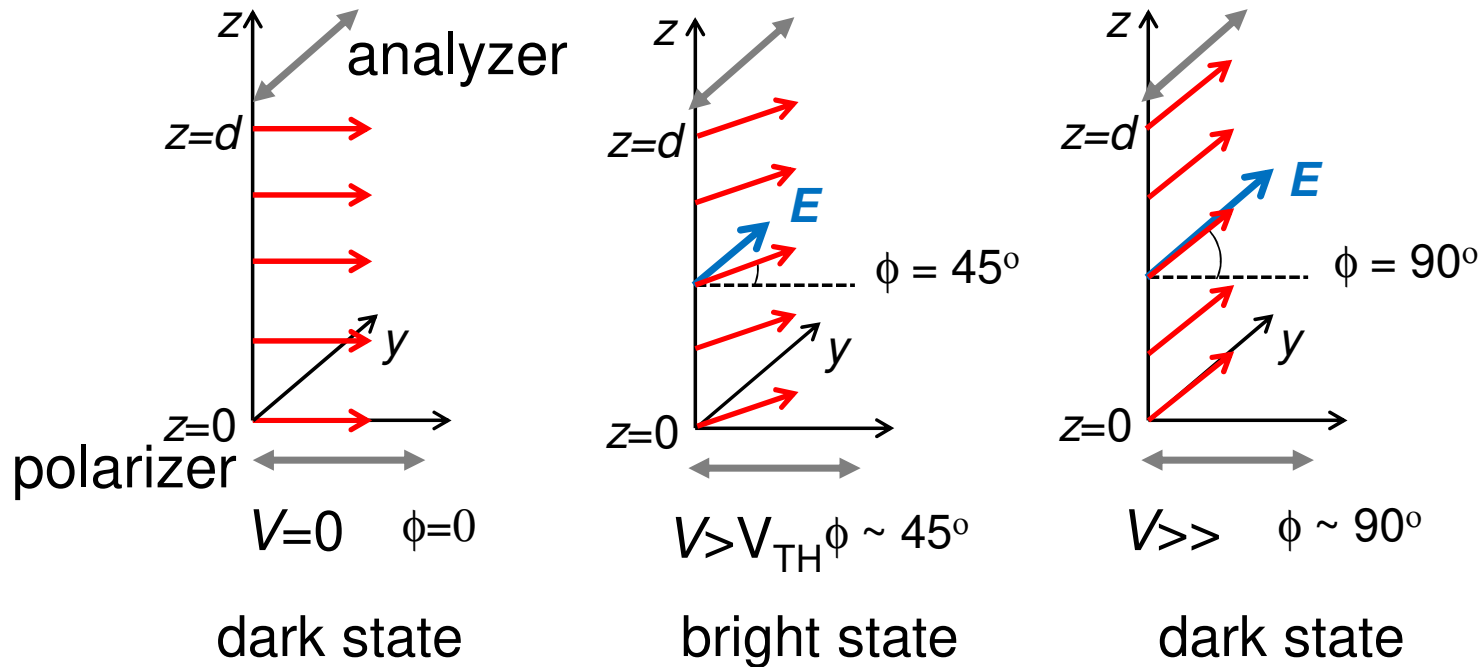
$$\frac{\partial F_{total}}{\partial \phi_m} = \frac{1}{4} K_{22} \omega l 2 \phi_m \frac{\pi^2}{d} - \frac{1}{4} \omega l \cdot \Delta \epsilon \cdot 2 \phi_m \frac{V^2}{l^2} d = \frac{1}{2} \omega l \phi_m \left( \underline{K_{22} \frac{\pi^2}{d} - \Delta \epsilon \frac{V^2}{l^2} d} \right)$$

threshold voltage  
makes this zero

$$V_{TH} = \pi \frac{l}{d} \sqrt{\frac{K_{22}}{\Delta \epsilon}} \quad V < V_{TH} \rightarrow \phi = 0$$

# IN PLANE SWITCHING (IPS) MODE

Transmission? ~ layer with homogeneous  $\phi$  (V)



$$T = \frac{1}{2} \sin^2 2\phi \sin^2 \left( \frac{\pi (n_e - n_o) d}{\lambda} \right)$$

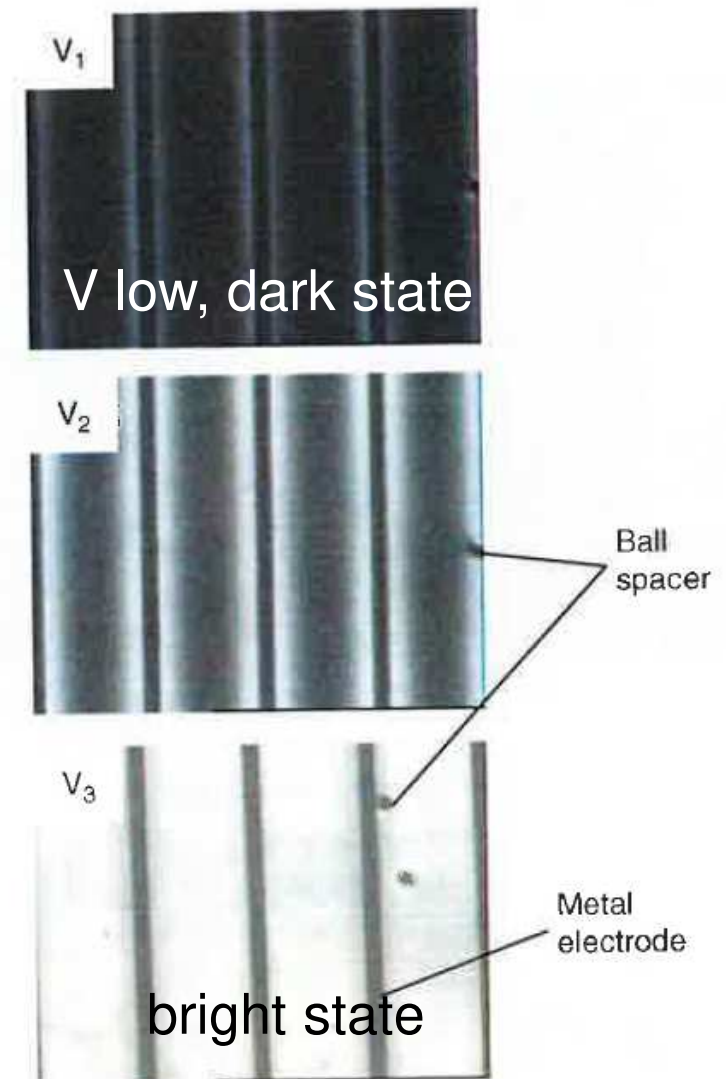
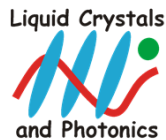
$\Gamma/2$

# IN PLANE SWITCHING (IPS) MODE

Images of IPS transmission

switching occurs first  
near the electrodes  
(highest field)

areas above the electrodes  
do not switch and remain dark  
Al can be used instead of ITO  
(lower brightness)



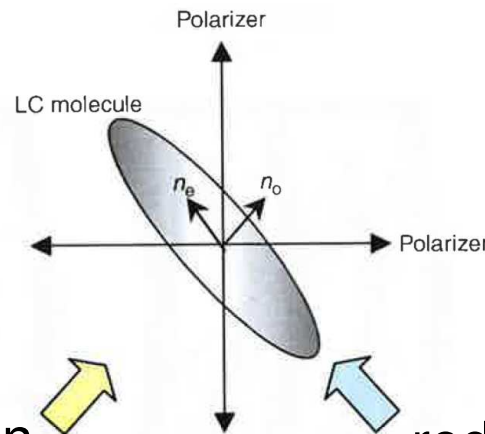
# IN PLANE SWITCHING (IPS) MODE

viewing angle dependency (off-normal) of the bright state?

(bright state,  $\varphi=45^\circ$  is optimized for green)

small dependency ( $<VA$ ), because the director is planar, retardation depends on the azimuth of observation

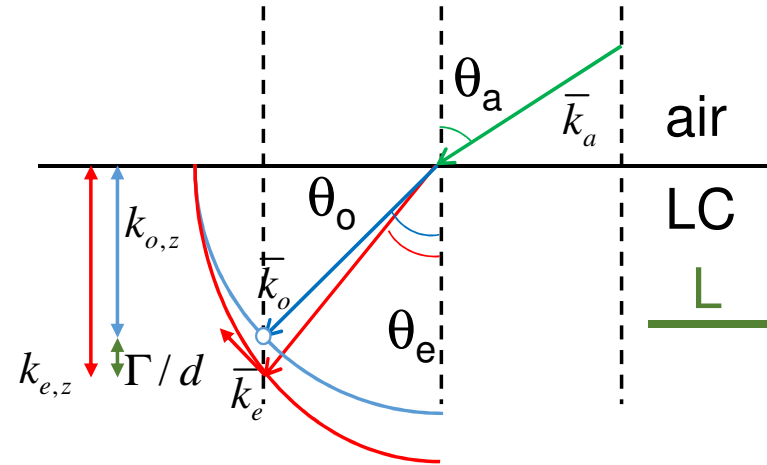
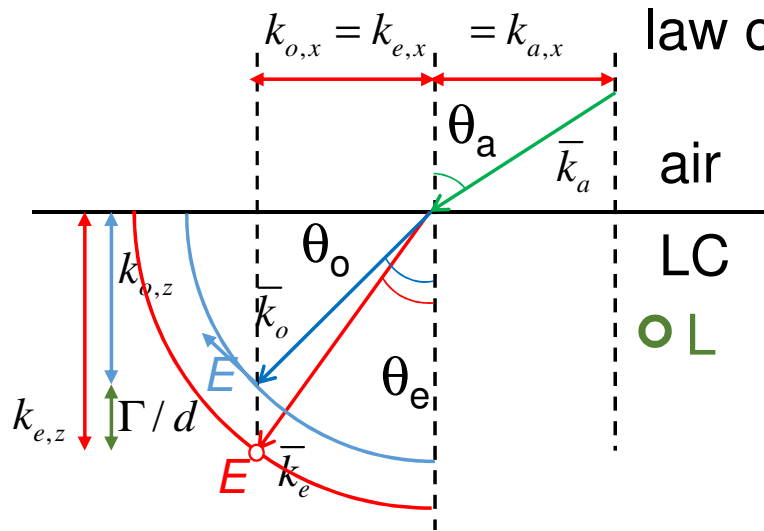
vertical:  
retardation  $\Gamma = \pi$   
for green light



increased retardation  
(see next slide)  
 $\Gamma = \pi$  for yellow

reduced retardation  $\Gamma$   
because  $n_{\text{eff}} < n_e$   
 $\Gamma = \pi$  for blue

# EXTENDED JONES CALCULUS (OFF-ANGLE)



$$\begin{aligned} \Gamma &= k_{e,z}d - k_{o,z}d \\ &= \frac{2\pi d}{\lambda} (n_e \cos \theta_e - n_o \cos \theta_o) \\ &= \frac{2\pi d}{\lambda} \left( \sqrt{n_e^2 - \sin^2 \theta_a} - \sqrt{n_o^2 - \sin^2 \theta_a} \right) \end{aligned}$$

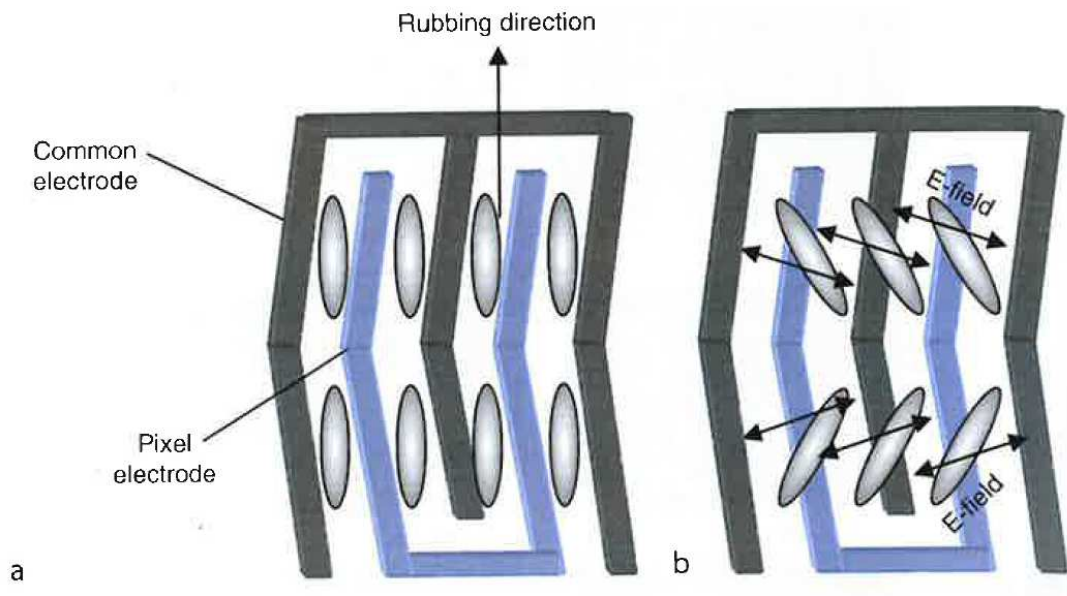
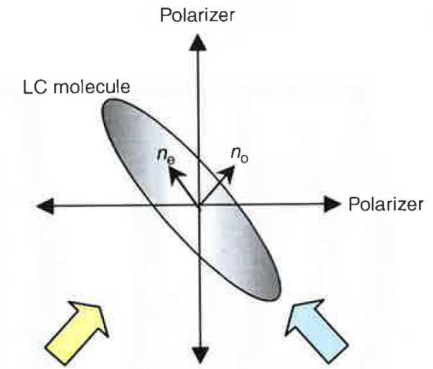
$\Gamma$  increases with  $\theta_a$

$$\begin{aligned} \Gamma &= k_{e,z}d - k_{o,z}d \\ &= \frac{2\pi d}{\lambda} (n_{eff} \cos \theta_e - n_o \cos \theta_o) \\ &= \frac{2\pi d}{\lambda} \left( \sqrt{n_{eff}^2 - \sin^2 \theta_a} - \sqrt{n_o^2 - \sin^2 \theta_a} \right) \end{aligned}$$

$\Gamma$  decreases with  $\theta_a$

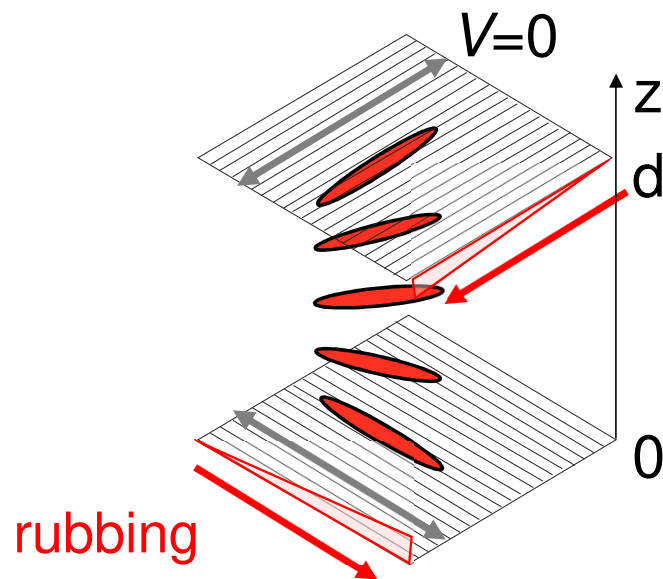
# IN PLANE SWITCHING (IPS) MODE

Multi-domain to solve viewing angle problems

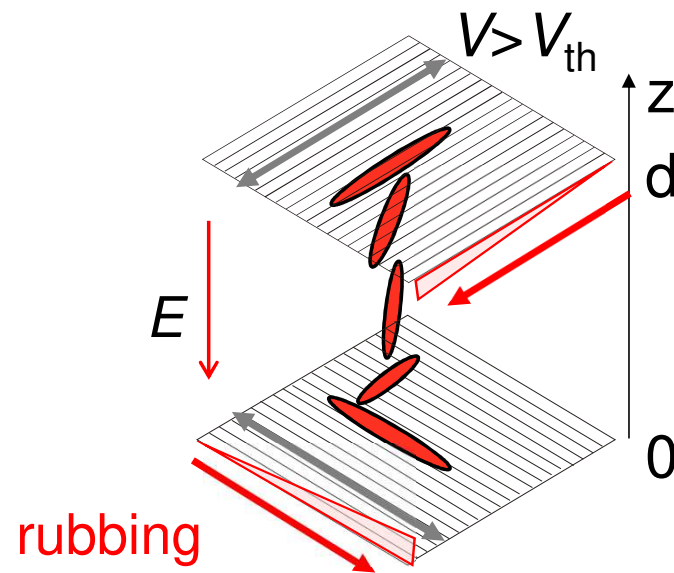


# TWISTED NEMATIC MODE

Cell with initial twist of  $90^\circ$



pure twist  
bright state



LC aligns with the field  
some splay and bend  
dark state

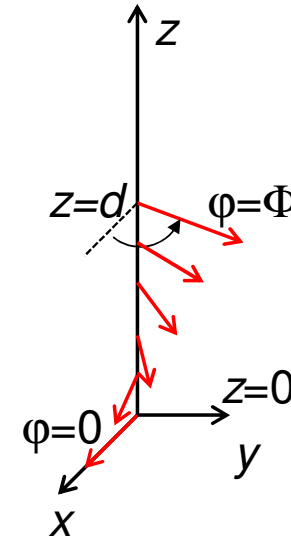


# LAYER WITH TWIST

A layer with total thickness  $d$  and total twist  $\Phi$  can be treated as  $N$  layers each with

- thickness  $d/N$
- retardation  $\Gamma/N$
- azimuth of  $L$ :  $\varphi(z)=\Phi \cdot z/d$

$$\Gamma = \frac{2\pi}{\lambda} (n_{\text{eff}} - n_o) d$$



$$\begin{aligned} \bar{J}_{\text{tot}} &= \bar{J}\left(\Phi, \frac{\Gamma}{N}\right) \cdot \bar{J}\left(\frac{N-1}{N}\Phi, \frac{\Gamma}{N}\right) \cdots \bar{J}\left(\frac{1}{N}\Phi, \frac{\Gamma}{N}\right) \\ &= \prod_{n=1}^N \bar{J}\left(\frac{n}{N}\Phi, \frac{\Gamma}{N}\right) \end{aligned}$$

$$= \prod_{n=1}^N \begin{bmatrix} \cos \frac{n\Phi}{N} & -\sin \frac{n\Phi}{N} \\ \sin \frac{n\Phi}{N} & \cos \frac{n\Phi}{N} \end{bmatrix} \begin{bmatrix} e^{-i\Gamma/2N} & 0 \\ 0 & e^{i\Gamma/2N} \end{bmatrix} \begin{bmatrix} \cos \frac{n\Phi}{N} & \sin \frac{n\Phi}{N} \\ -\sin \frac{n\Phi}{N} & \cos \frac{n\Phi}{N} \end{bmatrix}$$

$R$   $R^{-1}$

# LAYER WITH TWIST

Total Jones matrix for  $N$  layers?

$$\bar{\bar{J}}_{tot} = \prod_{n=1}^N \begin{bmatrix} \cos \frac{n\Phi}{N} & -\sin \frac{n\Phi}{N} \\ \sin \frac{n\Phi}{N} & \cos \frac{n\Phi}{N} \end{bmatrix} \begin{bmatrix} e^{-i\Gamma/2N} & 0 \\ 0 & e^{i\Gamma/2N} \end{bmatrix} \begin{bmatrix} \cos \frac{n\Phi}{N} & \sin \frac{n\Phi}{N} \\ -\sin \frac{n\Phi}{N} & \cos \frac{n\Phi}{N} \end{bmatrix}$$

= ...

$$= \begin{bmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} \left\{ \begin{bmatrix} e^{-i\Gamma/2N} & 0 \\ 0 & e^{i\Gamma/2N} \end{bmatrix} \begin{bmatrix} \cos \frac{\Phi}{N} & \sin \frac{\Phi}{N} \\ -\sin \frac{\Phi}{N} & \cos \frac{\Phi}{N} \end{bmatrix} \right\}^N$$

$$= \begin{bmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} \left\{ \begin{array}{cc} \cos \frac{\Phi}{N} e^{-i\Gamma/2N} & \sin \frac{\Phi}{N} e^{-i\Gamma/2N} \\ -\sin \frac{\Phi}{N} e^{i\Gamma/2N} & \cos \frac{\Phi}{N} e^{i\Gamma/2N} \end{array} \right\}^N$$

Power  $N$  of a matrix?

## LAYER WITH TWIST

Power of a matrix? diagonalize the matrix by finding the **eigenvalues**  $\lambda$  and the eigenvectors

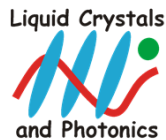
$$\left\{ \begin{array}{cc} \cos \frac{\Phi}{N} e^{-i\Gamma/2N} & \sin \frac{\Phi}{N} e^{-i\Gamma/2N} \\ -\sin \frac{\Phi}{N} e^{i\Gamma/2N} & \cos \frac{\Phi}{N} e^{i\Gamma/2N} \end{array} \right\}^N \quad ? \quad \left| \begin{array}{cc} \cos \frac{\Phi}{N} e^{-i\Gamma/2N} - \lambda & \sin \frac{\Phi}{N} e^{-i\Gamma/2N} \\ -\sin \frac{\Phi}{N} e^{i\Gamma/2N} & \cos \frac{\Phi}{N} e^{i\Gamma/2N} - \lambda \end{array} \right| = 0$$

# LAYER WITH TWIST

eigenvectors of the matrix?

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} \approx \begin{pmatrix} \Phi \\ i\frac{\Gamma}{2} \pm i\Omega \end{pmatrix}$$

$$\Omega = \sqrt{\Phi^2 + \frac{\Gamma^2}{4}}$$



eigenmodes

short pitch CLC

for  $\Phi \gg \Gamma$   $\begin{pmatrix} J_x \\ J_y \end{pmatrix} \sim \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$  circular P

circularly polarized light

RH and LH with different speed

long pitch: Mauguin

for  $\Gamma \gg \Phi$   $\begin{pmatrix} J_x \\ J_y \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  linear P

polarized parallel and perp

to the director

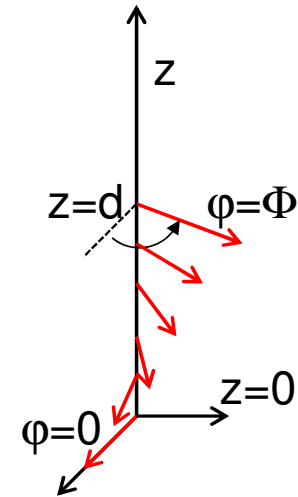
# LAYER WITH TWIST

A layer with total thickness  $d$  and twist  $\Phi$

$$\Gamma = \frac{2\pi}{\lambda} (n_{\text{eff}} - n_o) d \quad \Omega = \sqrt{\Phi^2 + \frac{\Gamma^2}{4}}$$

$$\bar{\bar{J}}_{\text{tot}} = \prod_{n=1}^N \bar{\bar{J}} \left( \frac{n}{N} \Phi, \frac{\Gamma}{N} \right)$$

$$= \bar{\bar{J}}_{\text{tot}} = \begin{bmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} \begin{pmatrix} \cos \Omega - i \frac{\Gamma \sin \Omega}{2 \Omega} & \Phi \frac{\sin \Omega}{\Omega} \\ -\Phi \frac{\sin \Omega}{\Omega} & \cos \Omega + i \frac{\Gamma \sin \Omega}{2 \Omega} \end{pmatrix}$$

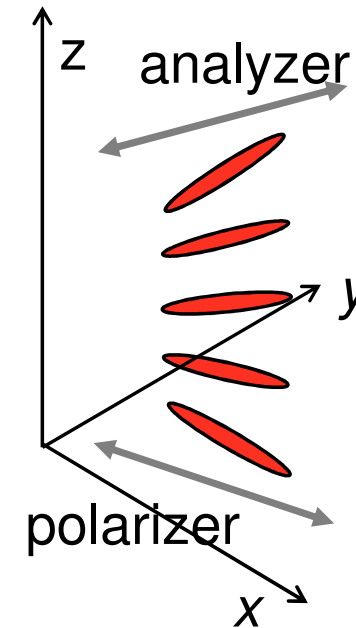
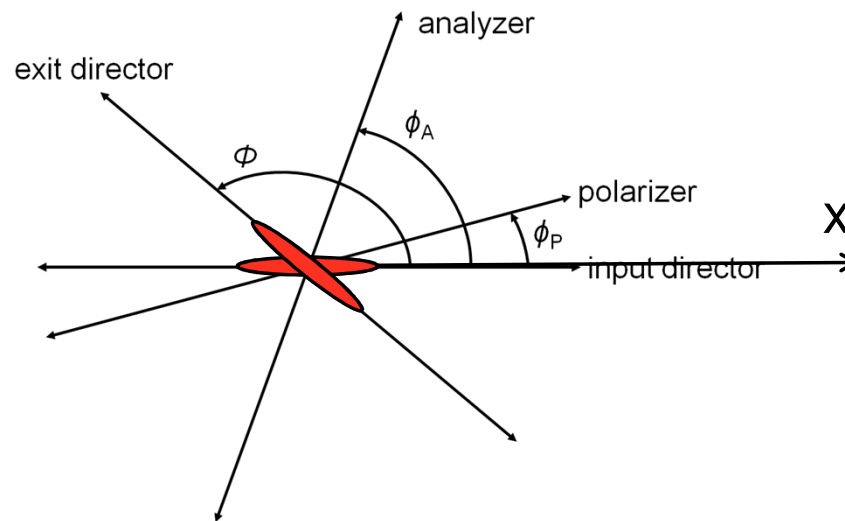


# TWISTED NEMATIC MODE

Transmission in the off state?

$$\bar{\bar{J}}_{TN} = \begin{bmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} \begin{pmatrix} \cos \Omega - i \frac{\Gamma \sin \Omega}{2 \Omega} & \Phi \frac{\sin \Omega}{\Omega} \\ -\Phi \frac{\sin \Omega}{\Omega} & \cos \Omega + i \frac{\Gamma \sin \Omega}{2 \Omega} \end{pmatrix}$$

Jones matrix in the system xy



$$\Gamma = \frac{2\pi \Delta n d}{\lambda}$$

$$\Omega = \frac{\sqrt{\Gamma^2 + 4\Phi^2}}{2}$$

# TWISTED NEMATIC MODE

$$\begin{aligned}
 \begin{pmatrix} E_{out} \\ 0 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \phi_A & \sin \phi_A \\ -\sin \phi_A & \cos \phi_A \end{pmatrix} \bar{J}_{TN} \begin{pmatrix} \cos \phi_P & -\sin \phi_P \\ \sin \phi_P & \cos \phi_P \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} E_{in} \\ 0 \end{pmatrix} \\
 &= \begin{pmatrix} \cos \phi_A & \sin \phi_A \\ 0 & 0 \end{pmatrix} \begin{bmatrix} \cos \Phi & -\sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} \begin{pmatrix} \cos \Omega - i \frac{\Gamma \sin \Omega}{2 \Omega} & \Phi \frac{\sin \Omega}{\Omega} \\ -\Phi \frac{\sin \Omega}{\Omega} & \cos \Omega + i \frac{\Gamma \sin \Omega}{2 \Omega} \end{pmatrix} \begin{pmatrix} \cos \phi_P E_{in} \\ \sin \phi_P E_{in} \end{pmatrix} \\
 &= \begin{pmatrix} \cos(\Phi - \phi_A) & -\sin(\Phi - \phi_A) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \Omega - i \frac{\Gamma \sin \Omega}{2 \Omega} & \Phi \frac{\sin \Omega}{\Omega} \\ -\Phi \frac{\sin \Omega}{\Omega} & \cos \Omega + i \frac{\Gamma \sin \Omega}{2 \Omega} \end{pmatrix} \begin{pmatrix} \cos \phi_P E_{in} \\ \sin \phi_P E_{in} \end{pmatrix} \\
 \frac{E_{out}}{E_{in}} &= \begin{pmatrix} \cos(\Phi - \phi_A) \left( \underline{\cos \Omega - i \frac{\Gamma \sin \Omega}{2 \Omega}} \right) + \sin(\Phi - \phi_A) \underline{\Phi \frac{\sin \Omega}{\Omega}} \right) \cos \phi_P E_{in} \\ + \left( \cos(\Phi - \phi_A) \underline{\Phi \frac{\sin \Omega}{\Omega}} - \sin(\Phi - \phi_A) \left( \underline{\cos \Omega + i \frac{\Gamma \sin \Omega}{2 \Omega}} \right) \right) \sin \phi_P E_{in} \end{pmatrix} \quad \left. \vphantom{\frac{E_{out}}{E_{in}}} \right) \text{ simpson...} \\
 &= \cos(\Phi - \phi_A + \phi_P) \underline{\cos \Omega} + \sin(\Phi - \phi_A + \phi_P) \underline{\Phi \frac{\sin \Omega}{\Omega}} - i \cos(\Phi - \phi_A - \phi_P) \underline{\frac{\Gamma \sin \Omega}{2 \Omega}}
 \end{aligned}$$

# TWISTED NEMATIC MODE

$$\frac{E_{out}}{E_{in}} = \cos(\Phi - \phi_A + \phi_P) \cos \Omega + \sin(\Phi - \phi_A + \phi_P) \Phi \frac{\sin \Omega}{\Omega} - i \cos(\Phi - \phi_A - \phi_P) \frac{\Gamma \sin \Omega}{2 \Omega}$$

$$T = \frac{1}{2} \left[ \cos(\Phi - \phi_A + \phi_P) \cos \Omega + \sin(\Phi - \phi_A + \phi_P) \Phi \frac{\sin \Omega}{\Omega} \right]^2 + \frac{1}{2} \left[ \cos(\Phi - \phi_A - \phi_P) \frac{\Gamma \sin \Omega}{2 \Omega} \right]^2$$

$$\left\{ \begin{array}{l} \Gamma = \frac{2\pi \Delta n d}{\lambda} \\ \Omega = \frac{\sqrt{\Gamma^2 + 4\Phi^2}}{2} \end{array} \right.$$

example: 90° twist device,  $\Phi = 90^\circ$ ,  $\phi_P = 0^\circ$ ,  $\phi_A = 90^\circ$

$$T = \frac{1}{2} [\cos \Omega]^2 + \frac{1}{2} \left[ \frac{\Gamma \sin \Omega}{2 \Omega} \right]^2$$

$$= \frac{1}{2} \left[ \cos^2 \Omega + \frac{\Gamma^2}{\Gamma^2 + 4\Phi^2} \sin^2 \Omega \right] = \frac{1}{2} \left[ 1 - \frac{4\Phi^2 \sin^2 \Omega}{\Gamma^2 + 4\Phi^2} \right]$$

$$\Phi - \phi_A - \phi_P = 0$$

$$\Phi - \phi_A + \phi_P = 0$$



# TWISTED NEMATIC MODE

example: 90° twist device,  $\Phi = 90^\circ$ ,  $\phi_P = 0^\circ$ ,  $\phi_A = 90^\circ$

$$T = \frac{1}{2} \left[ 1 - \frac{4\Phi^2 \sin^2 \Omega}{\Gamma^2 + 4\Phi^2} \right]$$

Maximum transmission  $T=1/2$  when:  $\sin \Omega = 0$

$$\Omega = \frac{\sqrt{\Gamma^2 + 4\Phi^2}}{2} = m\pi$$

$$\Rightarrow \Gamma^2 = 4\pi^2 \left( m^2 - \frac{1}{4} \right)$$

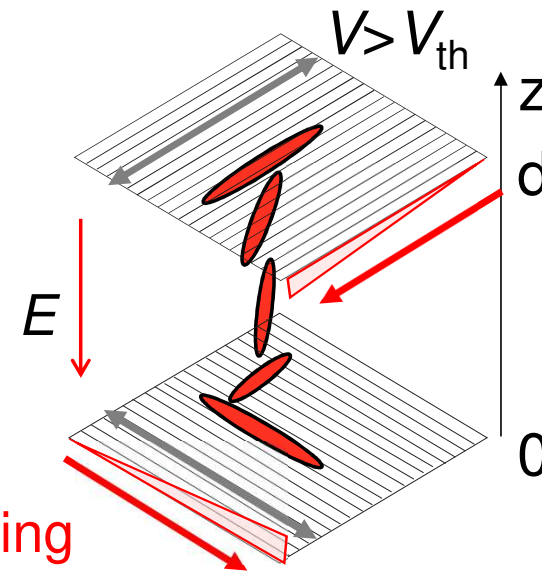
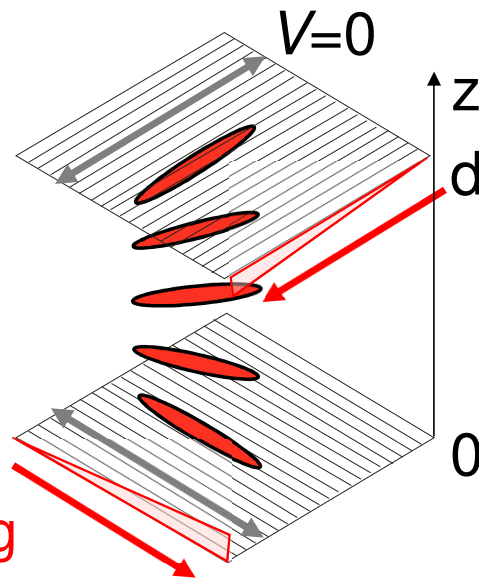
$$\Rightarrow \frac{\Delta n d}{\lambda} = \sqrt{m^2 - \frac{1}{4}}$$

Condition of Gooch and Tarry  
chosen for  $\lambda=550$  nm,  $m=1$

example: 
$$d = \sqrt{m^2 - \frac{1}{4}} \frac{\lambda}{\Delta n} = \sqrt{\frac{3}{4}} \frac{550 \text{ nm}}{0.2} = 2.38 \mu\text{m}$$

# TWISTED NEMATIC MODE

switching: threshold voltage?

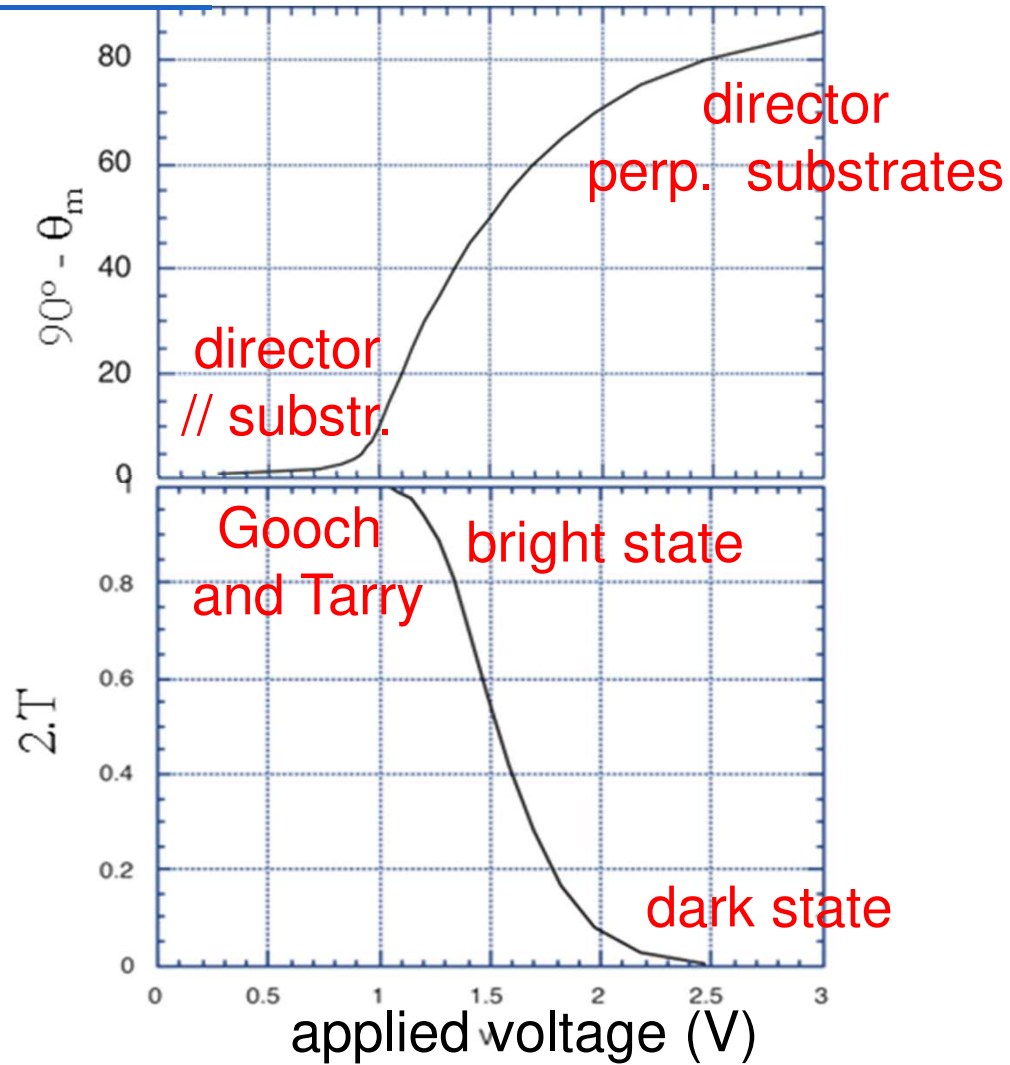


$$V < V_{th} = \pi \sqrt{\frac{K_{11} - \frac{1}{2} K_{23} + \frac{1}{4} K_{33}}{\Delta \epsilon}}$$

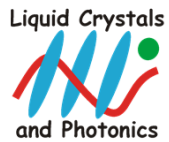
$$V > V_{th}$$

# TWISTED NEMATIC MODE

mid-plane tilt

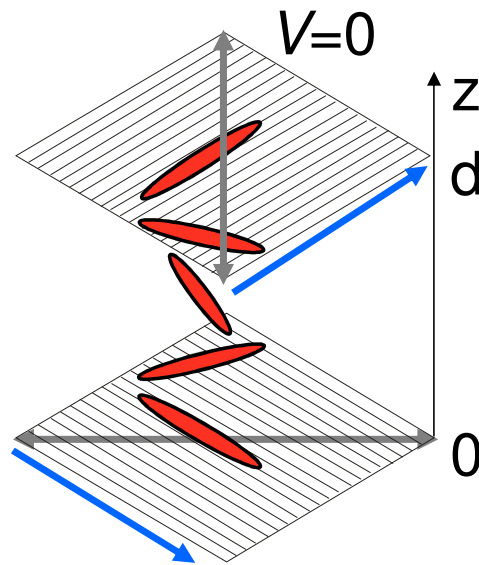


transmission

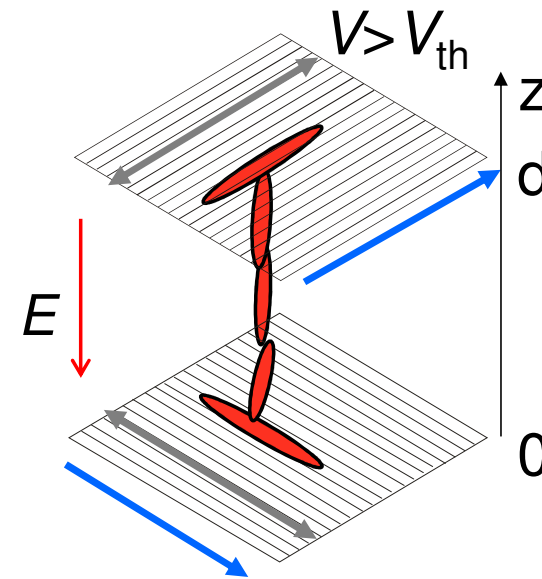


# SUPER TWISTED NEMATIC MODE

Cell with larger initial twist (ex: 270°)



pure twist  
bright state  
(chiral dopant needed)



LC aligns with the field  
some splay and bend  
dark state

# SUPER TWISTED NEMATIC MODE

example: 270° twist device,  $\Phi = 270^\circ$ ,  $\phi_P = 45^\circ$ ,  $\phi_A = 315^\circ$

$$T = \frac{1}{2} \left[ \cos(\Phi - \phi_A + \phi_P) \cos \Omega + \sin(\Phi - \phi_A + \phi_P) \Phi \frac{\sin \Omega}{\Omega} \right]^2$$

$$+ \frac{1}{2} \left[ \cos(\Phi - \phi_A - \phi_P) \frac{\Gamma \sin \Omega}{2 \Omega} \right]^2$$

$$\begin{cases} \phi_P = 45^\circ \\ \Phi - \phi_A = -45^\circ \end{cases} \Rightarrow \begin{cases} \Phi - \phi_A + \phi_P = 0 \\ \Phi - \phi_A - \phi_P = -90^\circ \end{cases}$$

$$\Rightarrow T = \frac{1}{2} \cos^2 \Omega = \frac{1}{2} \cos^2 \sqrt{\left(\frac{\pi \Delta n d}{\lambda}\right)^2 + \Phi^2}$$

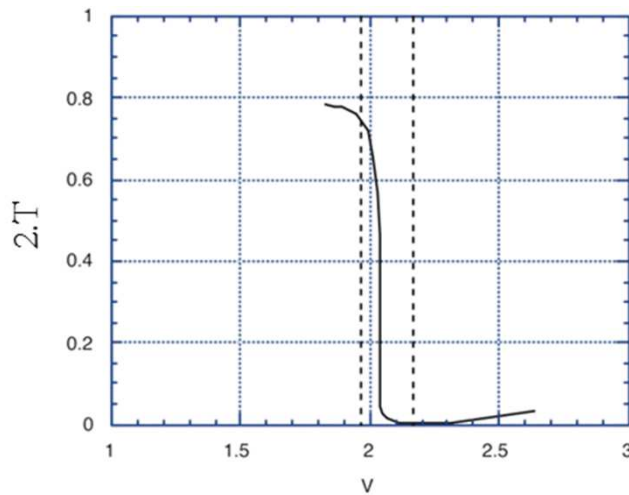
Maximum transmission (bright)  $T=1/2$  when

$$\Omega = \frac{\sqrt{\Gamma^2 + 4\Phi^2}}{2} = \sqrt{\left(\frac{\pi \Delta n d}{\lambda}\right)^2 + \Phi^2} = m\pi \quad \Rightarrow \quad \frac{\Delta n d}{\lambda} = \sqrt{m^2 - \left(\frac{\Phi}{\pi}\right)^2}$$

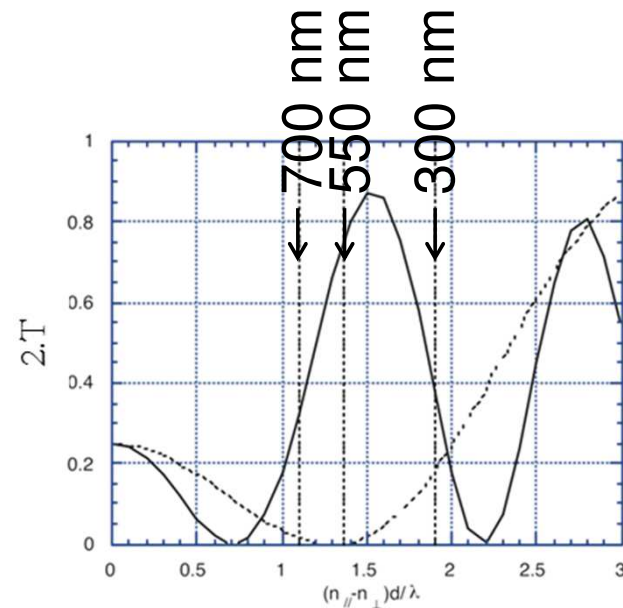
# SUPER TWISTED NEMATIC MODE

example: 270° twist device,  $\Phi = 270^\circ$ ,  $\phi_P = -30^\circ$ ,  $\phi_A = 210^\circ$

different values to obtain good dark state (and  $2T_{\text{bright}} < 1$ )



steep  $T$ - $V$  characteristic



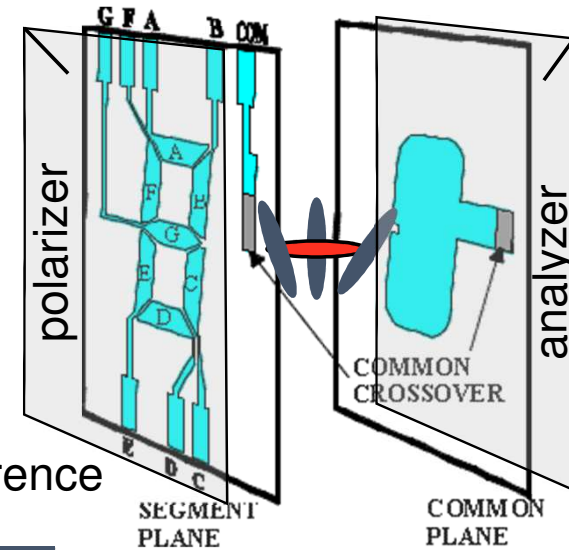
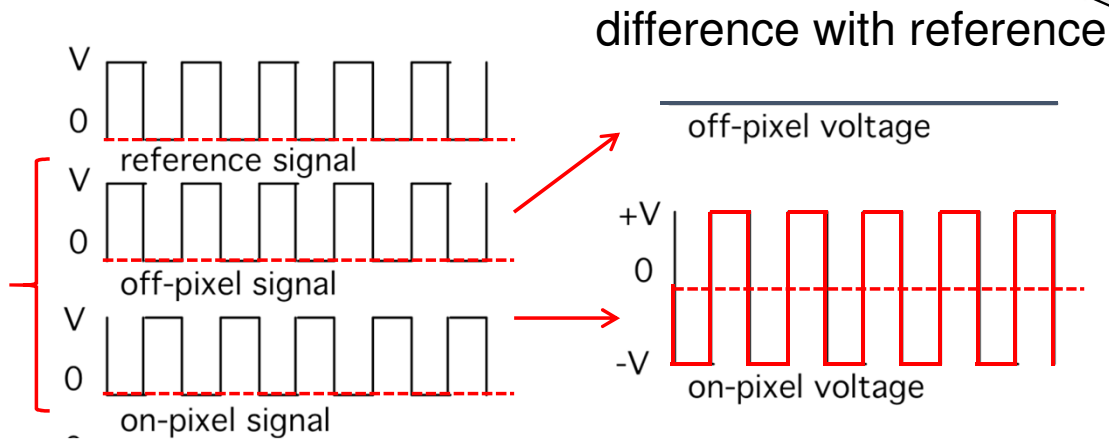
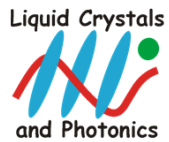
green bright state  
purple dark state

# DIRECT DRIVING

LCD reacts to  $|V^2|$

AC driving to avoid chemical reactions in the LC

one electrode per segment  
reflective display



TN – LCD  
off: bright state  
on: dark state