Nonlinear Dynamics *for* Engineering Design:

Good Vibrations?

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Key Facts

- Founded 1495
- 55,000 alumni in135 countries
- 16,500 Students
- 3,705 Staff (2691 fte)
- Turnover: £224m
- Research awards: £58m
- 5 Nobel Prize winners

UNIVERSITY OF ABERDEEN

CADR

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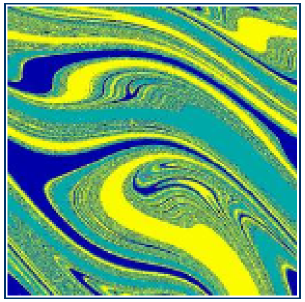

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About CADR

Founded in 2003, the **Centre for Applied Dynamics Research (CADR)** at the University of Aberdeen is a multi-disciplinary research group with a strong focus on application theory of dynamical systems to science and engineering.

The aim of the Centre is:

- to facilitate and enhance interactions among applied mathematicians, scientists, engineers and medical researchers at the University
- increase interaction between the University on one hand and other universities, industry, and national laboratories on the other
- establish international partnerships, collaborations and alliances
- increase the visibility of the University as a focal point for academic excellence.

systems, condition monitoring, renewable energy, MEMS, neural networks and neural dynamics, and experimental methods.

Latest News

21/01/2013: Professor Wiercigroch appointed a Joint-Editor-in-Chief of the International Journal of Mechanical Sciences.

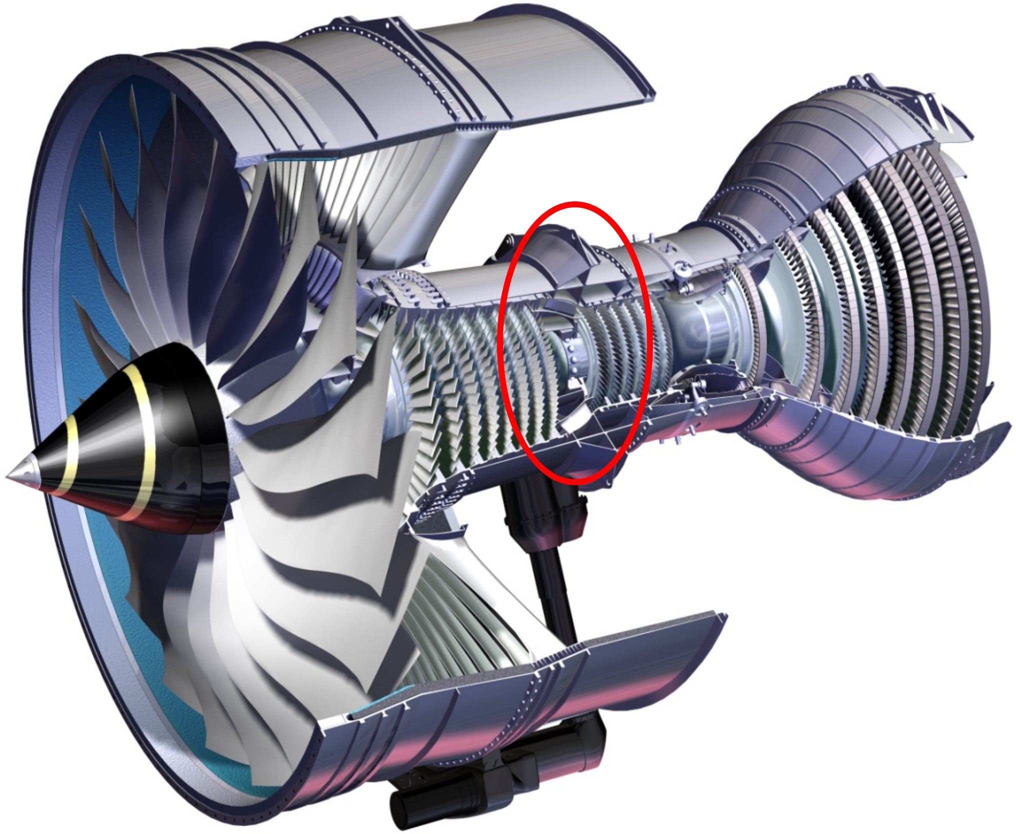
16/01/2013: CADR Seminar by P Perlikowski and P Brzeski.

11/12/2012: CADR Seminar by Yao Yan on Chatters in Plunge and Transverse Grinding.

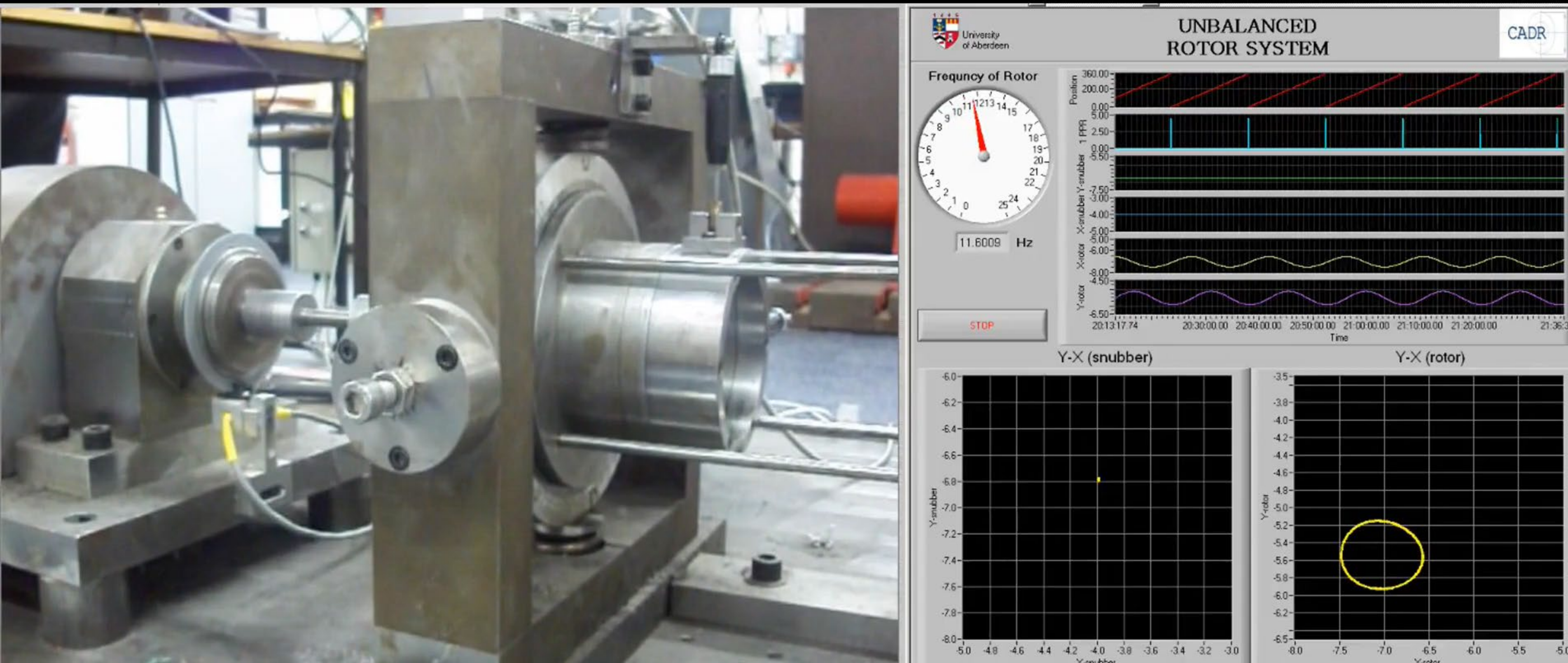
20/10/2012: CADR Seminar by Dr Zi-Qiang Lang on Exploitation of Nonlinear Damping for Structural Systems Vibration Control.

» more news

Rolls-Royce Jet Engine



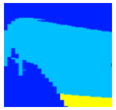
Rolls-Royce Jet Engine



Presentation Outline

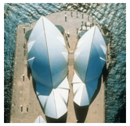
Nonlinear dynamics for engineering design: Introduction

Rotor dynamics



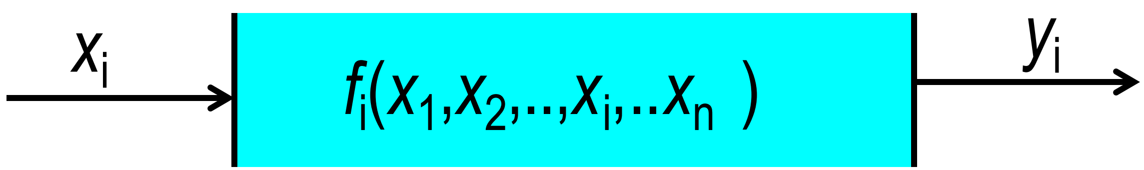
Drill-string vibration

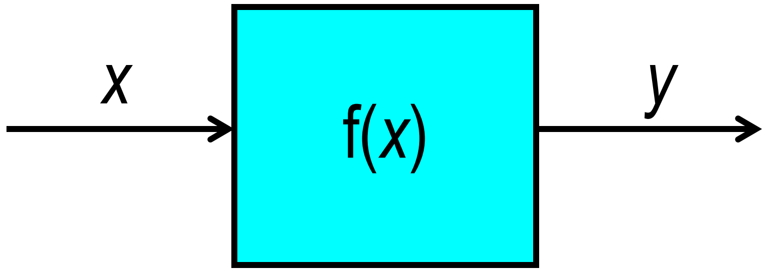
Resonance Enhanced Drilling (RED)



Closing remarks

Input System Output



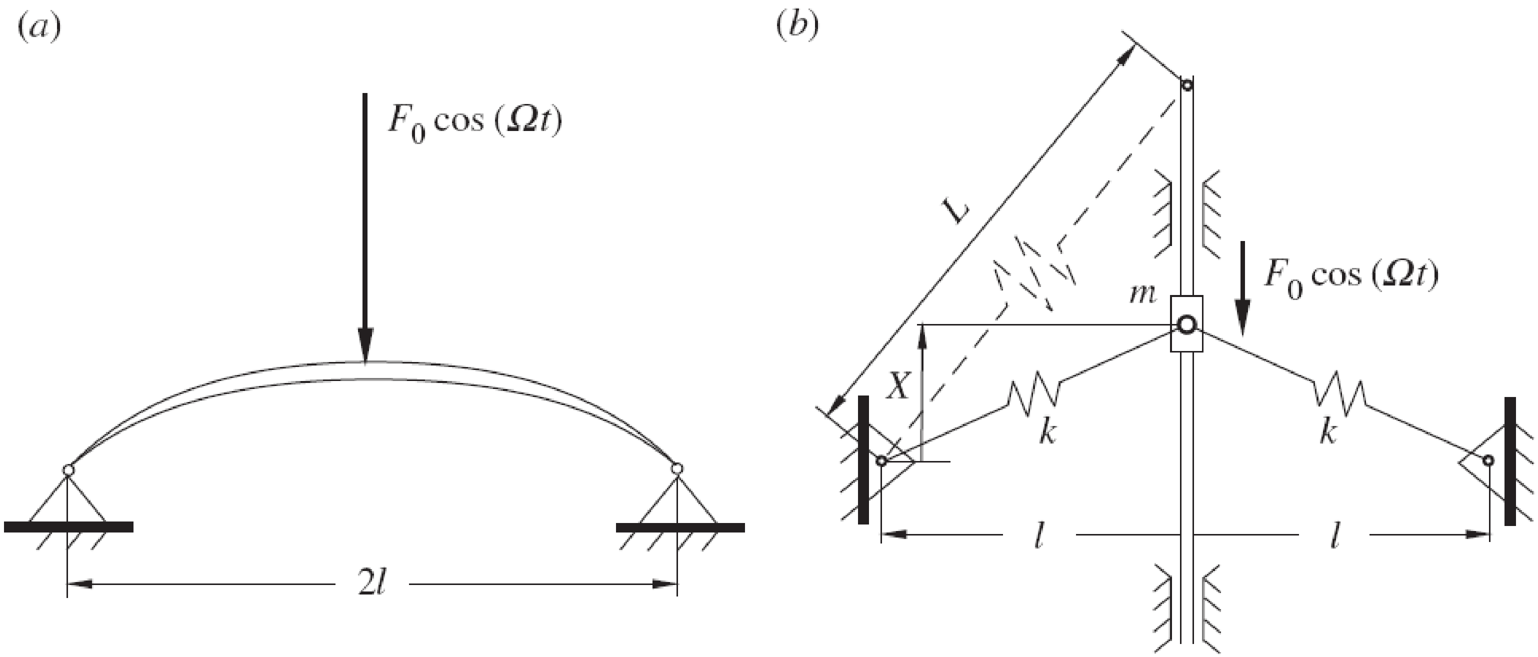


$$y = f(x)$$

$$f(x) = c\,x$$ Linear system

$$f(x) \neq c\,x$$ Nonlinear system

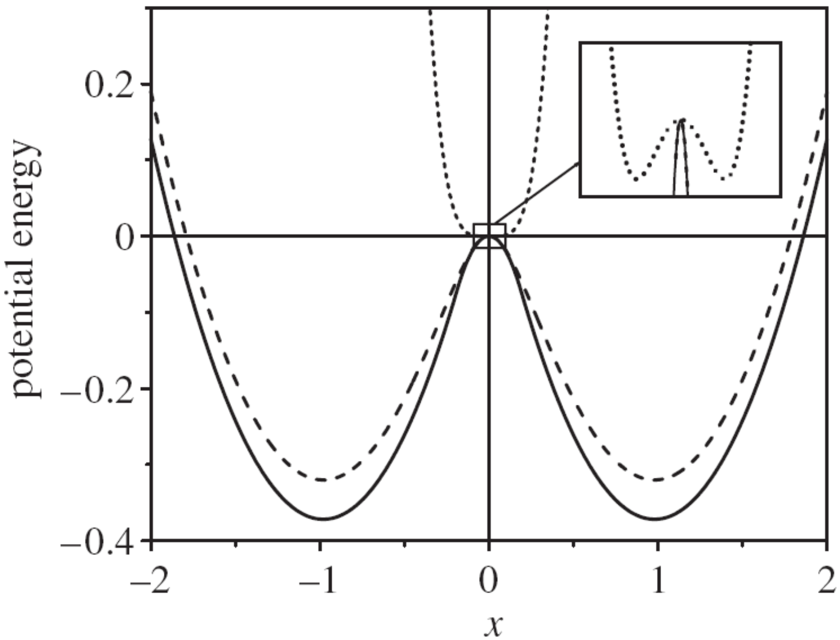
Shallow Arch – Model of Double-well Potential

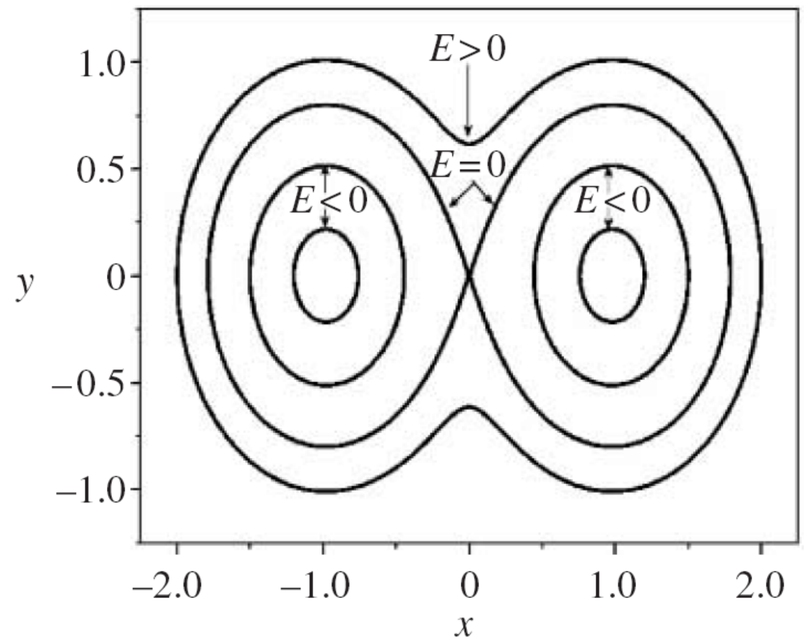


$$x'' + 2\xi x' + x\left(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\right) = f_0 \cos \omega\tau$$

[Cao Q, Wiercigroch M, Pavlovskaia E, Thompson JMT, Grebogi Physical Review E (2006) **74**, 046218]

Double-well Potential - Trajectories





[Cao Q, Wiercigroch M, Pavlovskaia E, Thompson JMT, Grebogi C *Phil T Roy Soc A* (2008) **366**, 365-392]

ND*f*ED: Introduction

Engineering design is **not** a science as science aims to find general laws.

Engineering design makes use of these laws to solve practical problems. In this context, it more closely related to art, as in art, problems are under-defined, and they have many solutions, good, bad and indifferent. The art is by a synthesis of ends and means, to arrive with a good solution. This is activity involving **imagination**, **creativity** and **a deliberate choice**.

(Ove Arup)

Engineering design is the process of devising a system, component, or process to meet desired needs. It is a decision-making process (often iterative), in which the basic science and mathematics and engineering sciences are applied to convert resources optimally to meet a stated objective. Among the fundamental elements of the design process are the establishment of objectives and criteria, synthesis, analysis, construction, testing and evaluation.

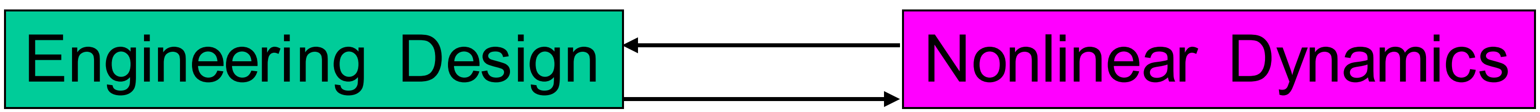
(ABET)

ND*f*ED: Introduction

‘Nonlinear dynamics is inter and multi disciplinary **science** involving mathematics, physics, chemistry, biology, medicine and engineering.

It aims to **advance fundamental understanding** of complex phenomena occurring in dynamical systems and to **apply** its findings to a real world.’

(MW)



The basic idea behind ***Nonlinear Dynamics for Engineering Design*** is to **harvest nonlinearities** [or **nonlinear interactions]** to enhance performance of newly designed systems and to provide support for exiting designs by exploring untypical ranges of the system parameters. Phenomena such as chaos, bifurcation behaviour, nonlinear resonances and co-existence of attractors are to be utilized.

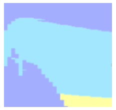
Objectives

- To establish a basis for a unified framework for harnessing nonlinear interactions
- To systematically classify various nonlinear phenomena
- To investigate how various nonlinearities can be introduced to physical systems and processes
- To investigate a number of benchmark systems and processes
- To synthesise the obtained results and methodologies into unified strategies, criteria and procedures for design of new devices, sensors and machines for control, manufacturing and remote sensing

Presentation Outline

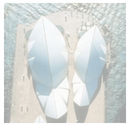
Nonlinear dynamics for engineering design: Introduction

Rotor dynamics



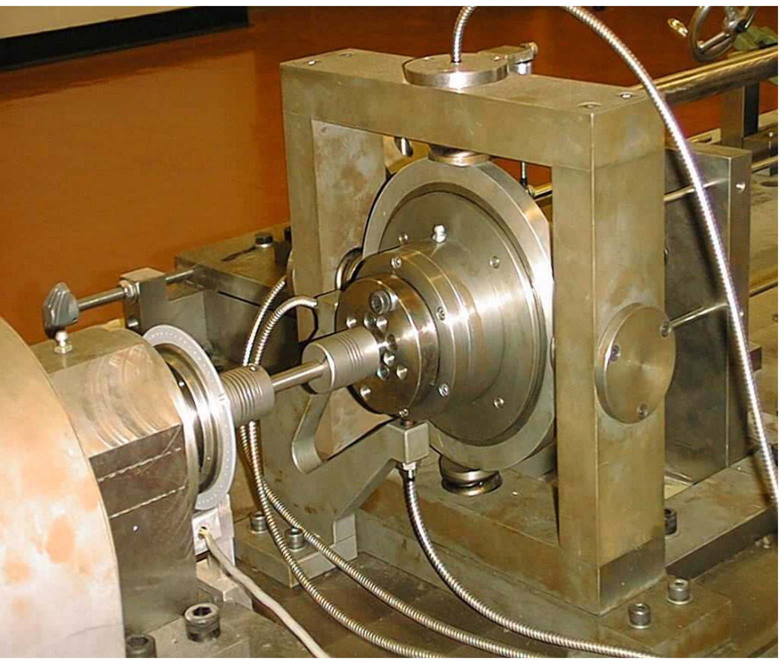
Drill-string vibration

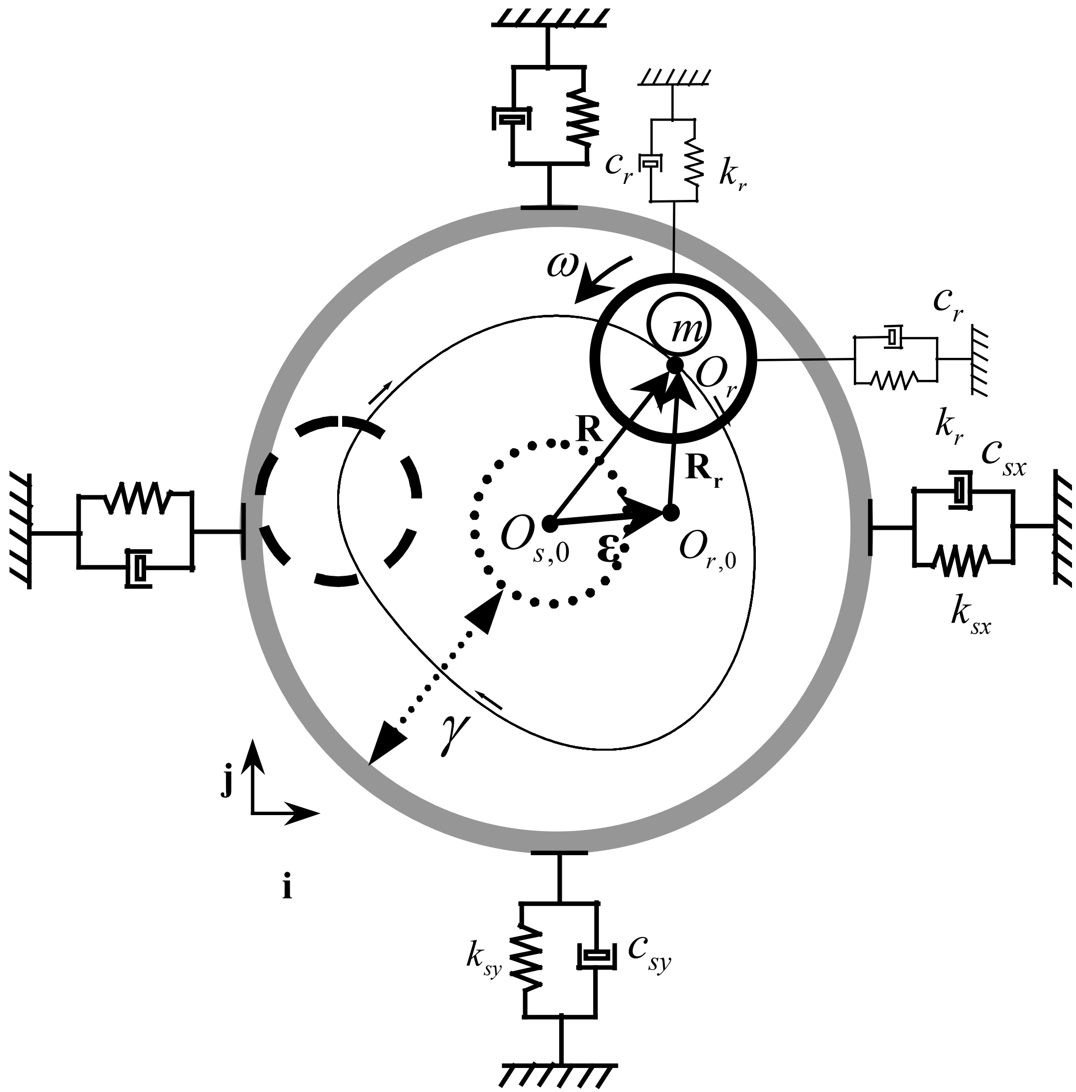
Resonance Enhanced Drilling (RED)



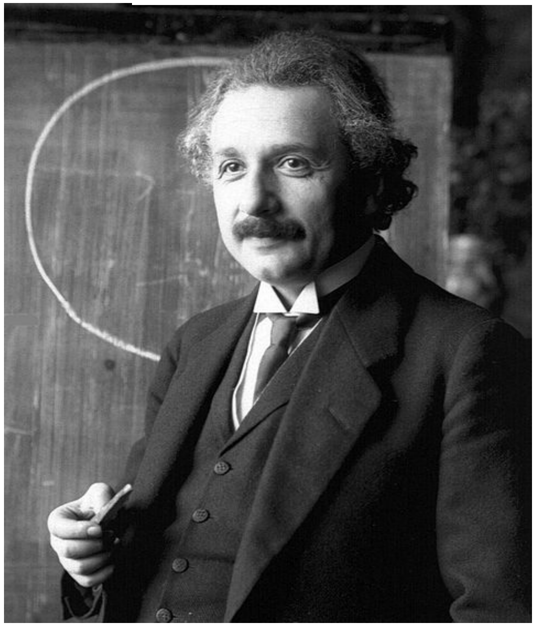
Closing remarks

Physical Models





Einstein's Tips on Modelling



'Everything should be **as simple as it is, but not simpler**.'

'Everything that **can be counted** does **not** necessarily **count**; everything that **counts, cannot** necessarily **be counted**.'

'Any intelligent fool can make things bigger and more complex. It takes a touch of genius and a lot of courage to move in the opposite direction.'

Main assumptions:

Massless snubber ring.

No dry friction between rotor and snubber ring.

Snubber ring with viscoelastic support.

No gyroscopic forces.

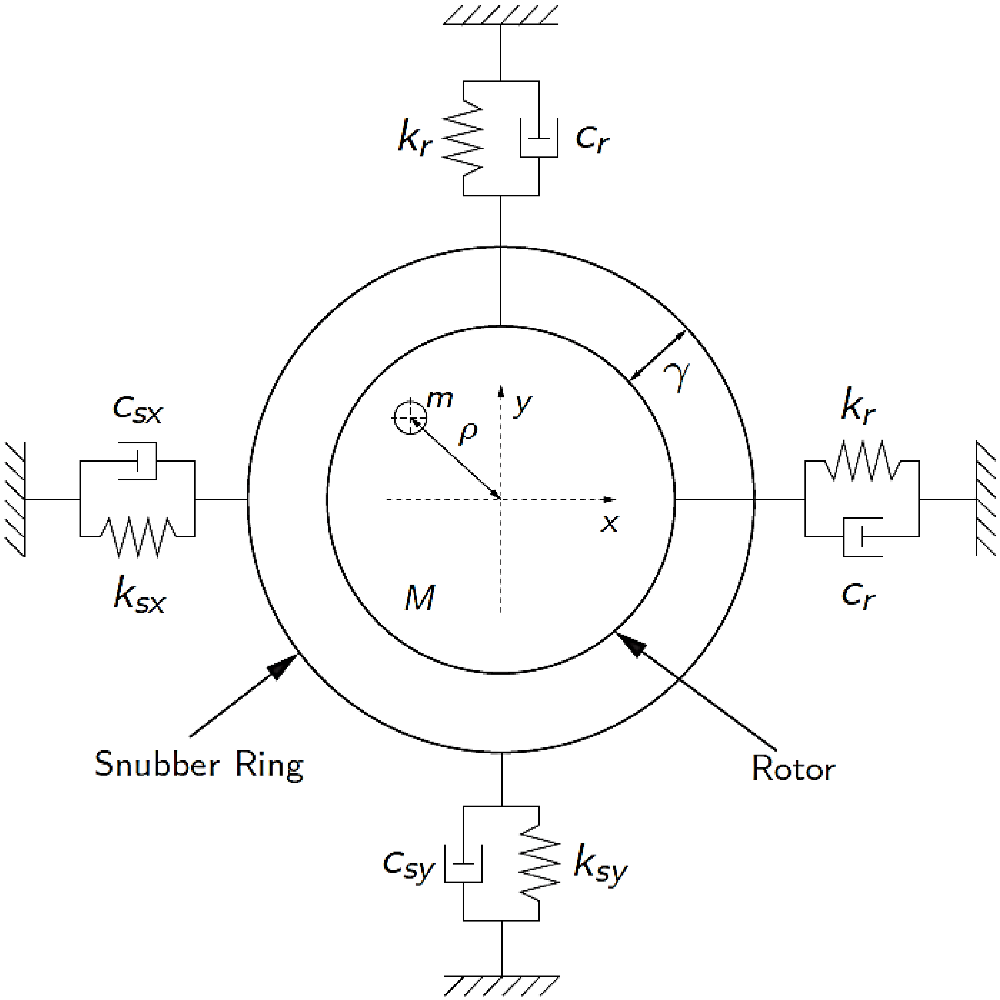
Gravity loads are neglected.

Imbalance caused by:

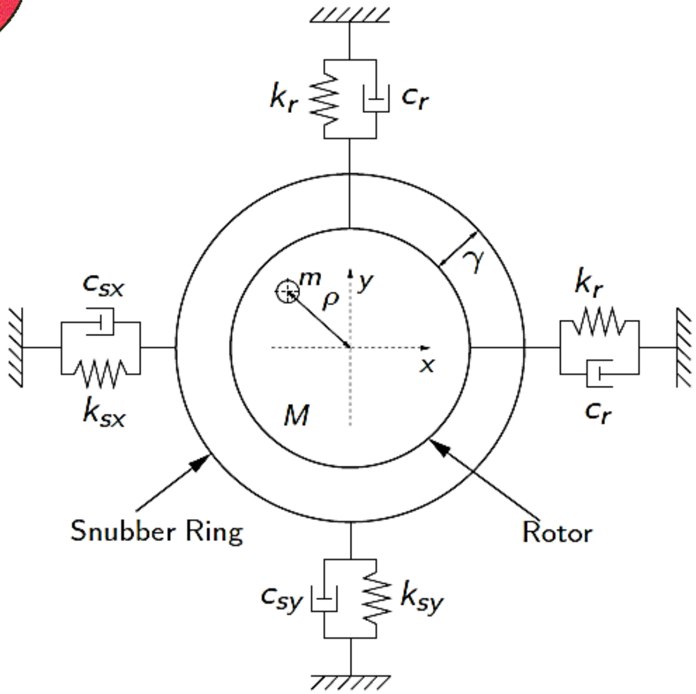
Thermal deformation.

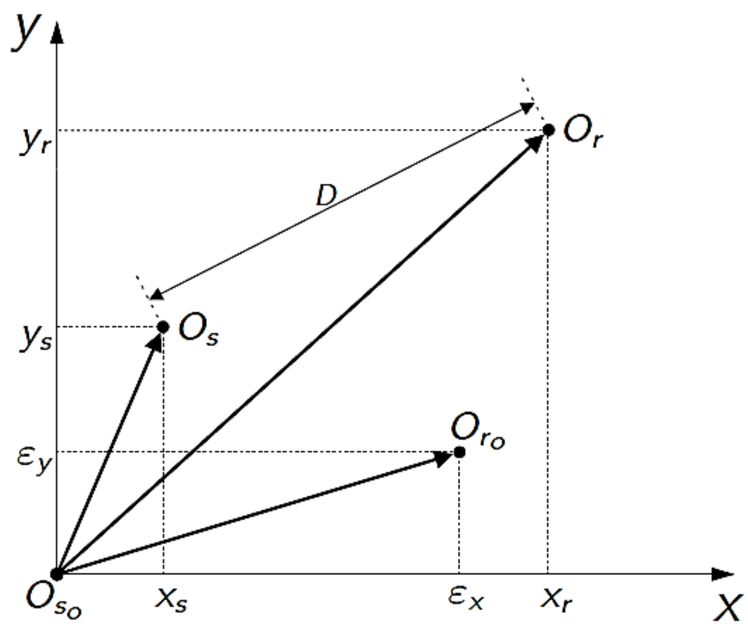
Blade/tooth breakage.

$\Rightarrow$ Serious malfunctions.



Mathematical Models





No contact ($D < \gamma$):

$$M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) = m\rho\omega^2 \cos(\omega t + \theta_0),$$

$$M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) = m\rho\omega^2 \sin(\omega t + \theta_0),$$

$$k_{sx}x_s + c_{sx}\dot{x}_s = 0, \quad k_{sy}y_s + c_{sy}\dot{y}_s = 0.$$

Contact ($D \geq \gamma$):

$$M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) + N_{sx} = m\rho\omega^2 \cos(\omega t + \theta_0),$$

$$M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) + N_{sy} = m\rho\omega^2 \sin(\omega t + \theta_0),$$

$$\dot{x}_s = \chi_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K), \quad \dot{y}_s = \Upsilon_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K), \quad K = \frac{k_{sx}}{k_{sy}}.$$

Mathematical Models

For $D < \gamma$ (no contact), we integrate the system

$$
\begin{aligned}
&M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) = m\rho\omega^2\cos(\omega t + \theta_0),\\
&M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) = m\rho\omega^2\sin(\omega t + \theta_0),\\
&k_{sx}x_s + c_{sx}\dot{x}_s = 0, \quad k_{sy}y_s + c_{sy}\dot{y}_s = 0.
\end{aligned}
$$

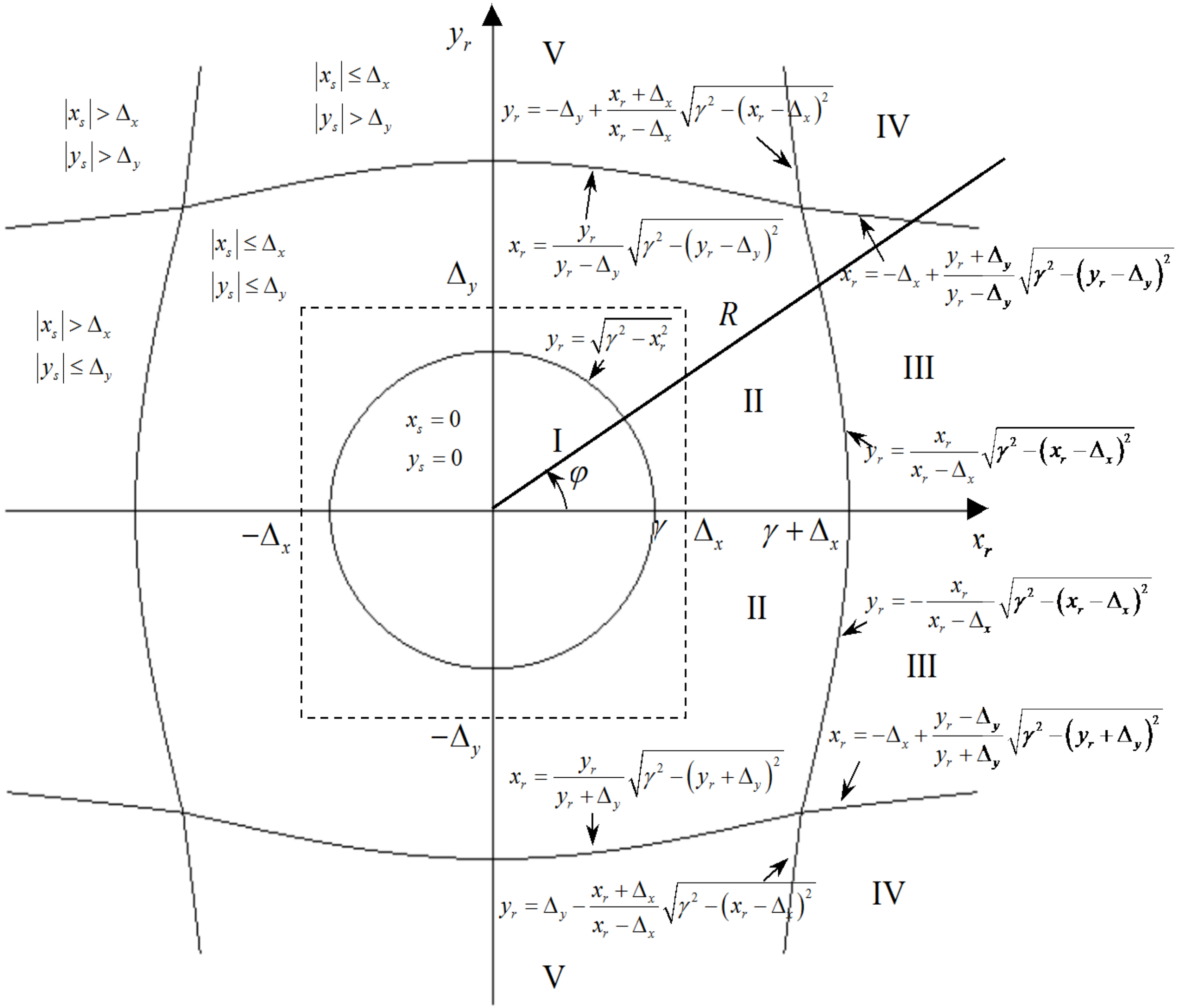
At the moment when $D = \gamma$ ($t = t_{\text{imp}}$), an impact with the snubber has occurred and the motion of the system is obtained by integrating the equations

$$
\begin{aligned}
&M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) + N_{sx} = m\rho\omega^2\cos(\omega t + \theta_0),\\
&M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) + N_{sy} = m\rho\omega^2\sin(\omega t + \theta_0),\\
&\dot{x}_s = \chi_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K), \quad \dot{y}_s = \Upsilon_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K),
\end{aligned}
$$

with the initial position of the snubber ring $x_s(t_{\text{imp}}^+) = \chi(x_r(t_{\text{imp}}), y_r(t_{\text{imp}}), K, \gamma)$, $y_s(t_{\text{imp}}^+) = \Upsilon(x_r(t_{\text{imp}}), y_r(t_{\text{imp}}), K, \gamma)$.

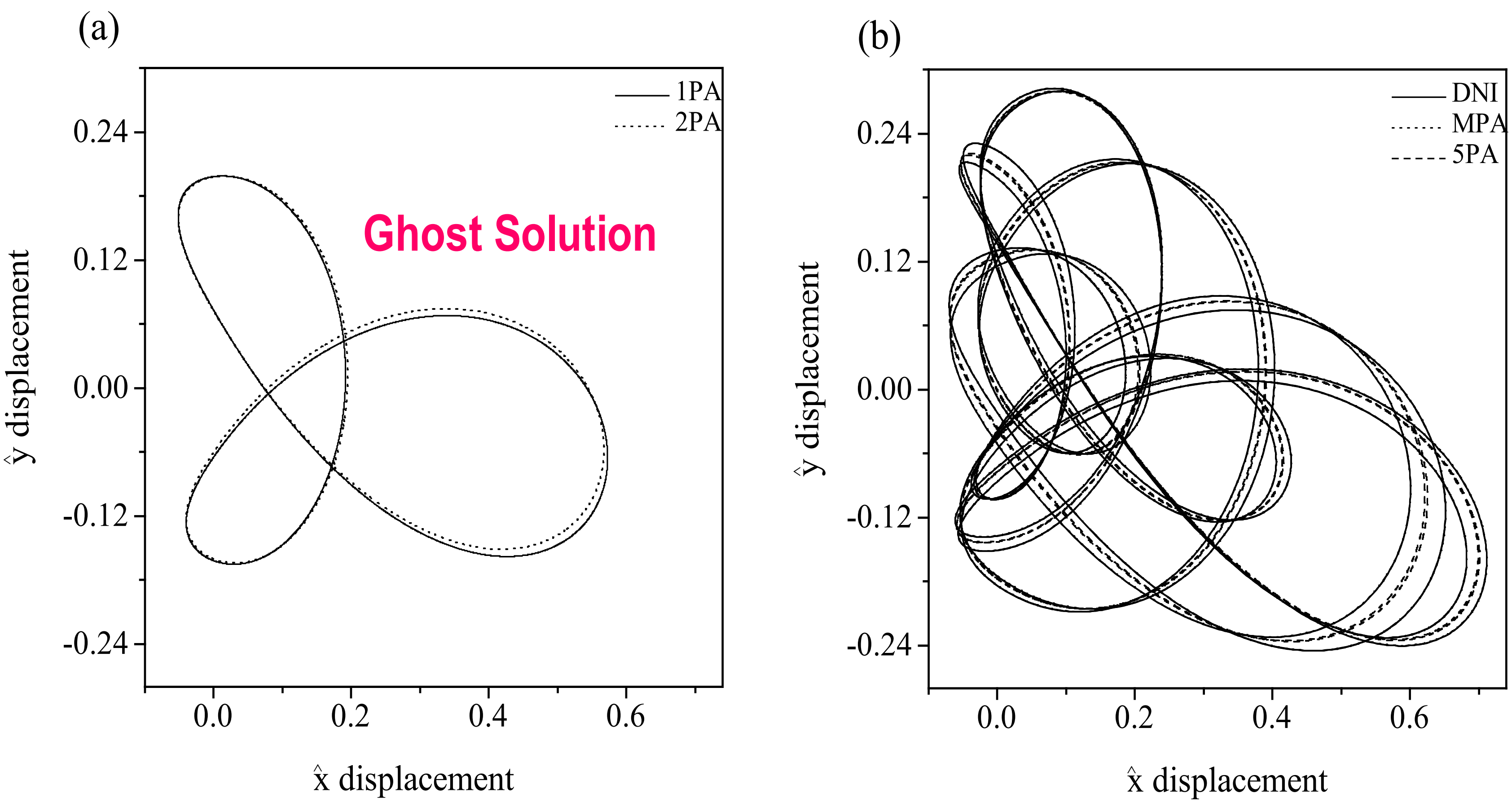
The system returns to the no contact regime when $\langle F_s, \widehat{N}\rangle \leq 0$.

Mathematical Models



[CADR, Journal of Sound and Vibration **276** (2004) 361-379]

Nonlinear Dynamics Analysis

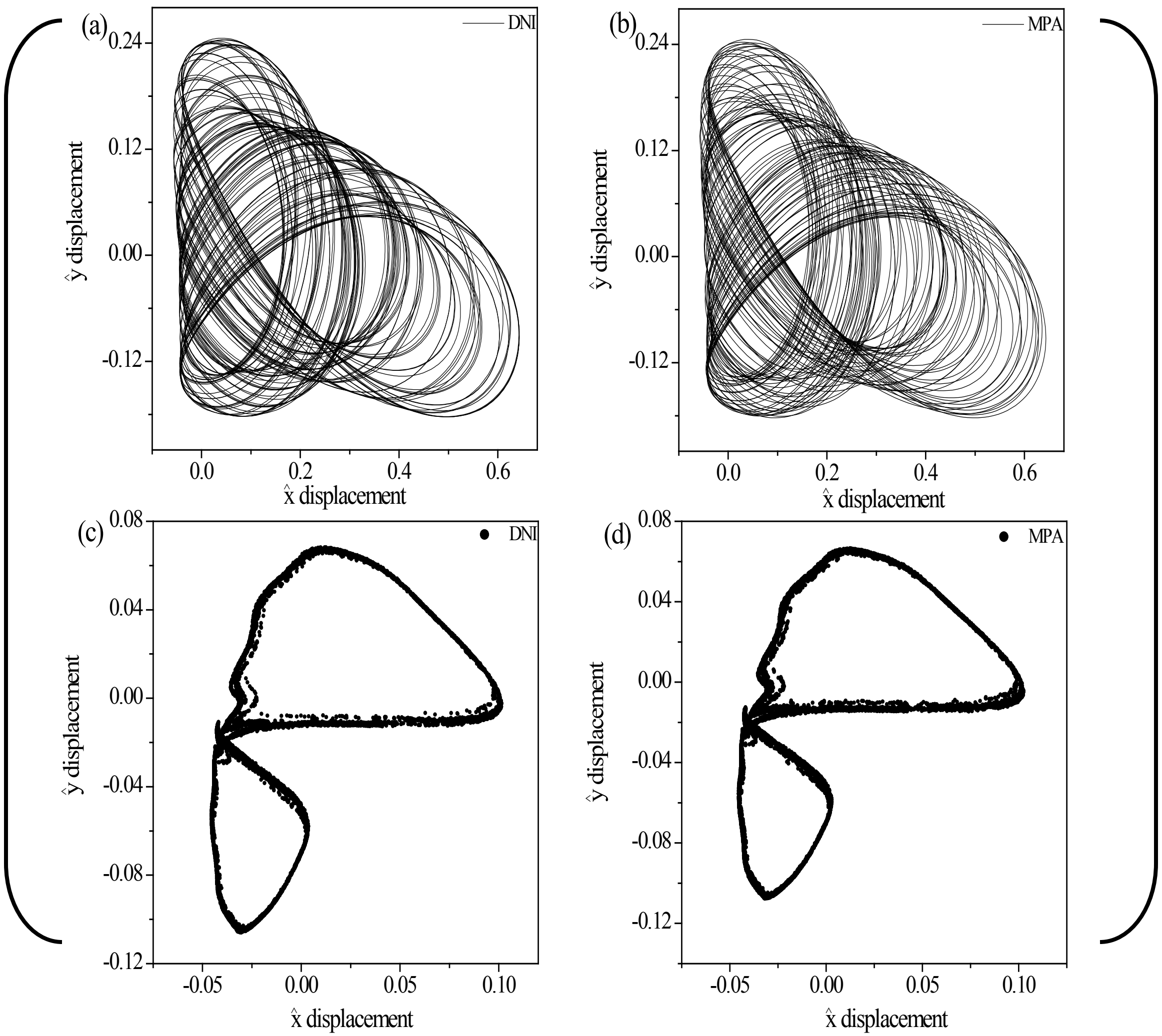


[CADR, International Journal of Mechanical Sciences **44** (2002) 475-488]

Nonlinear Dynamics Analysis

Numerical

(1056s to compute)

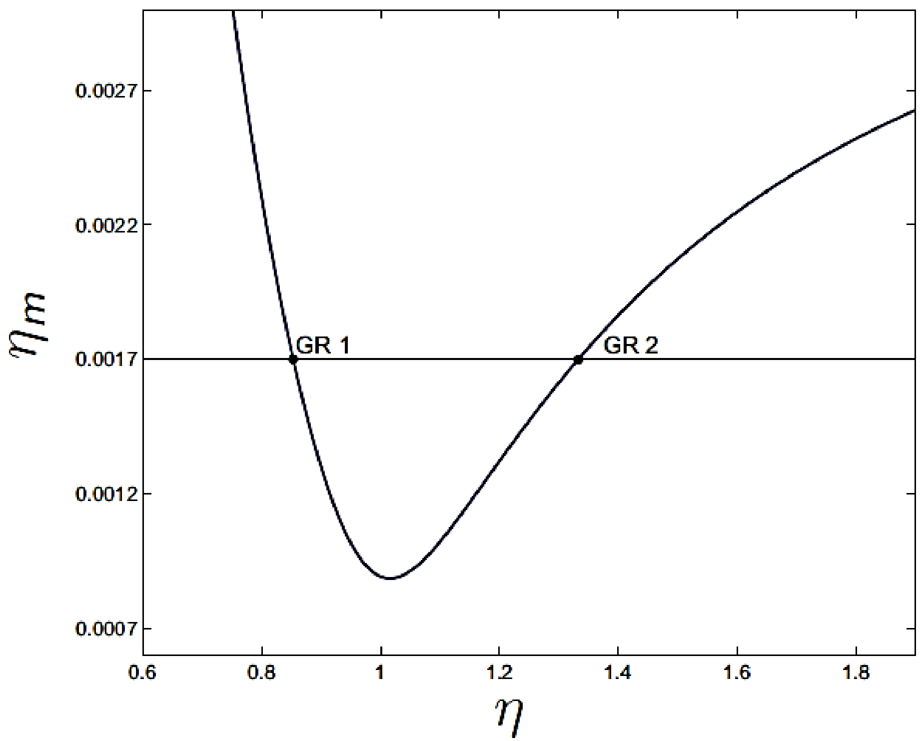


Analytical

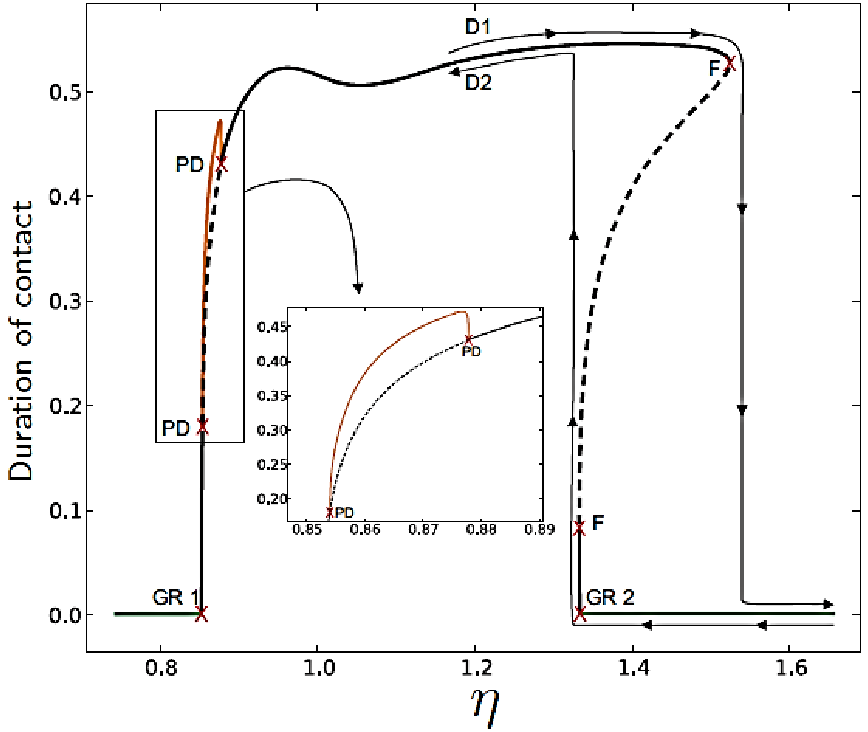
(8s to compute)

Nonlinear Dynamics Analysis

Grazing curve

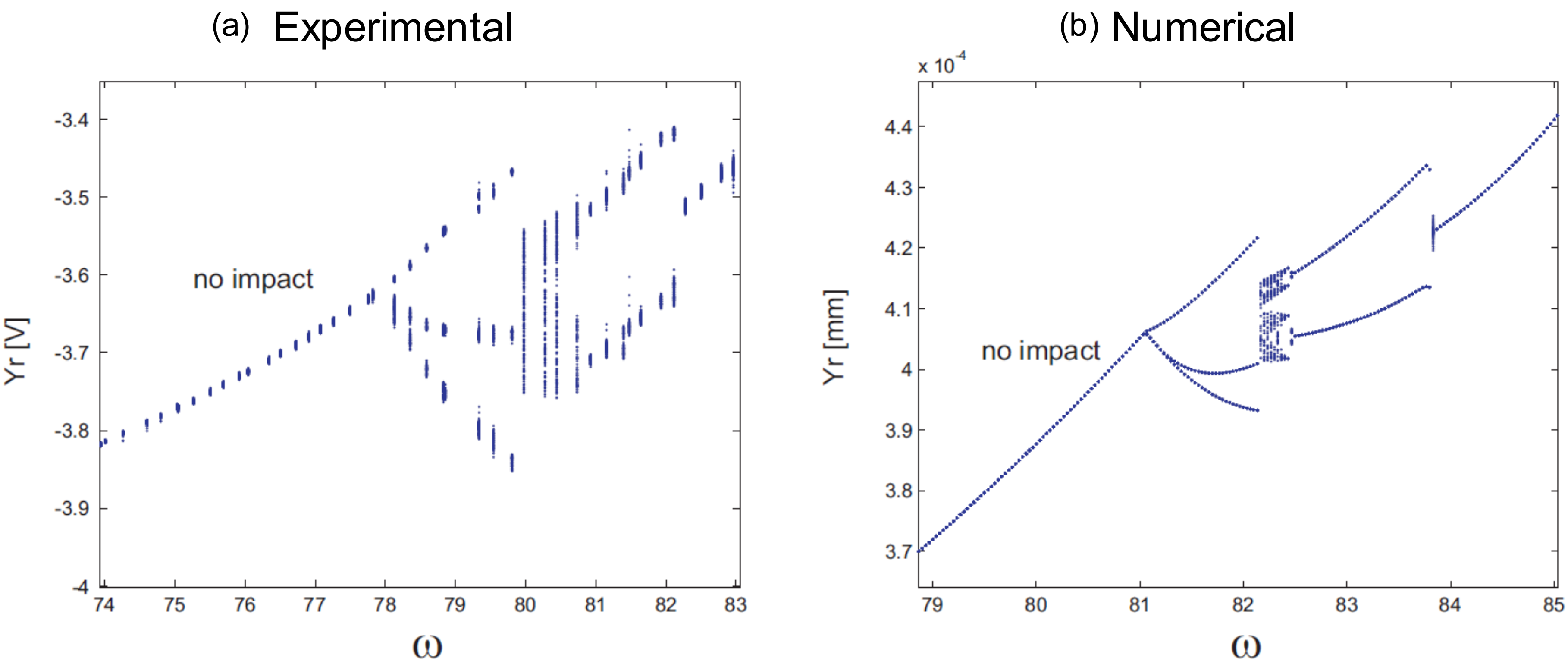


One-parameter continuation w.r.t. η



[CADR, Communication s in Nonlinear Science and Numerical Simulation **18** (2013) 2571–2580]

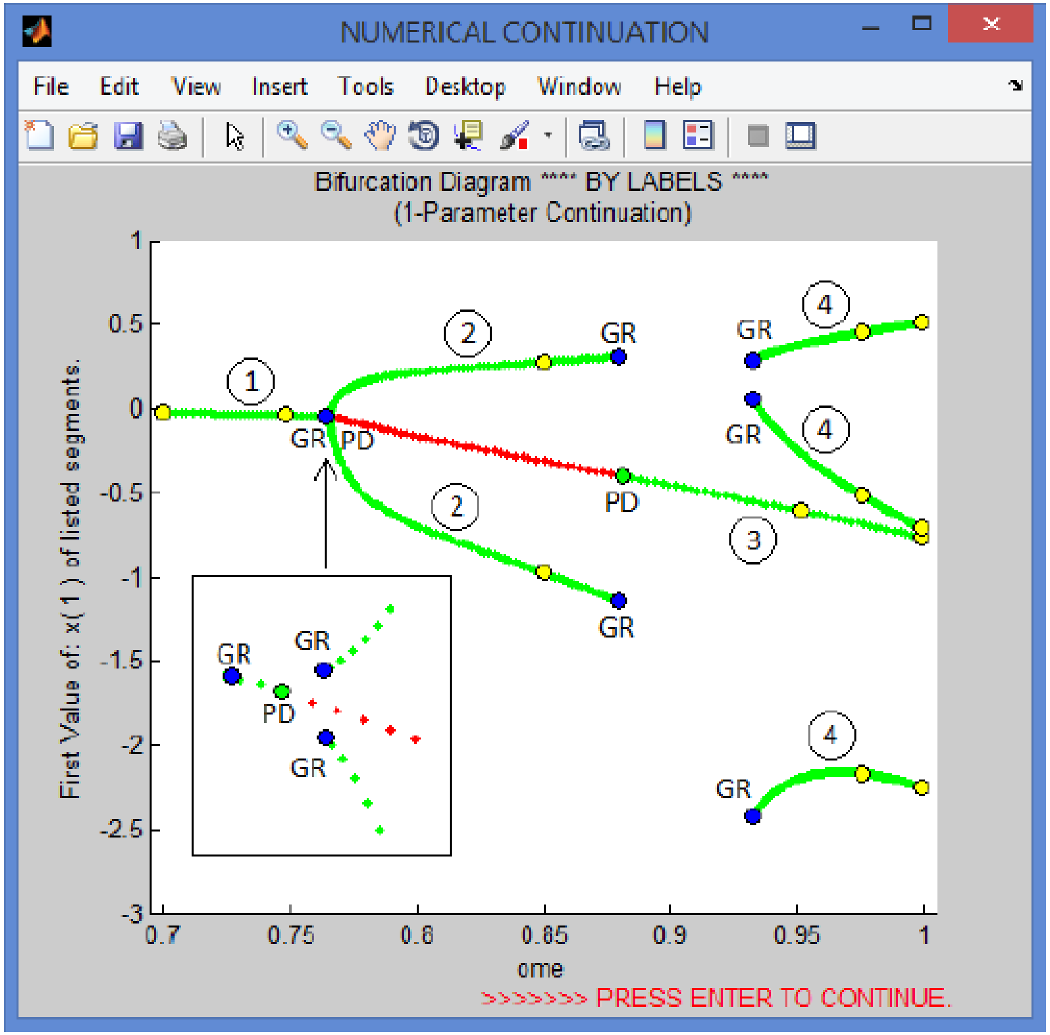
Experimental Studies

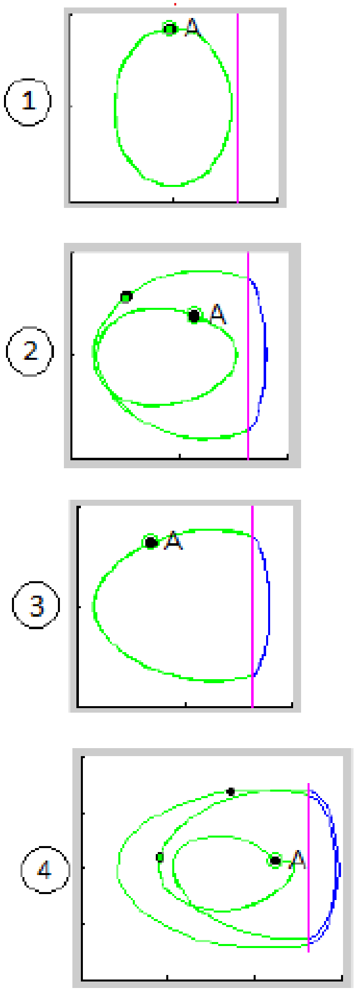


(a) Experimental and (b) theoretical bifurcation diagrams. The following sequence can be identified: no impact (period-1), impact (period-3), impact (period-2), impact (period-1).

[CADR, Journal of Sound and Vibration **334** (2015) 86-97]

Grazing Induced Bifurcations





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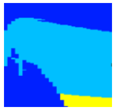
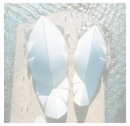


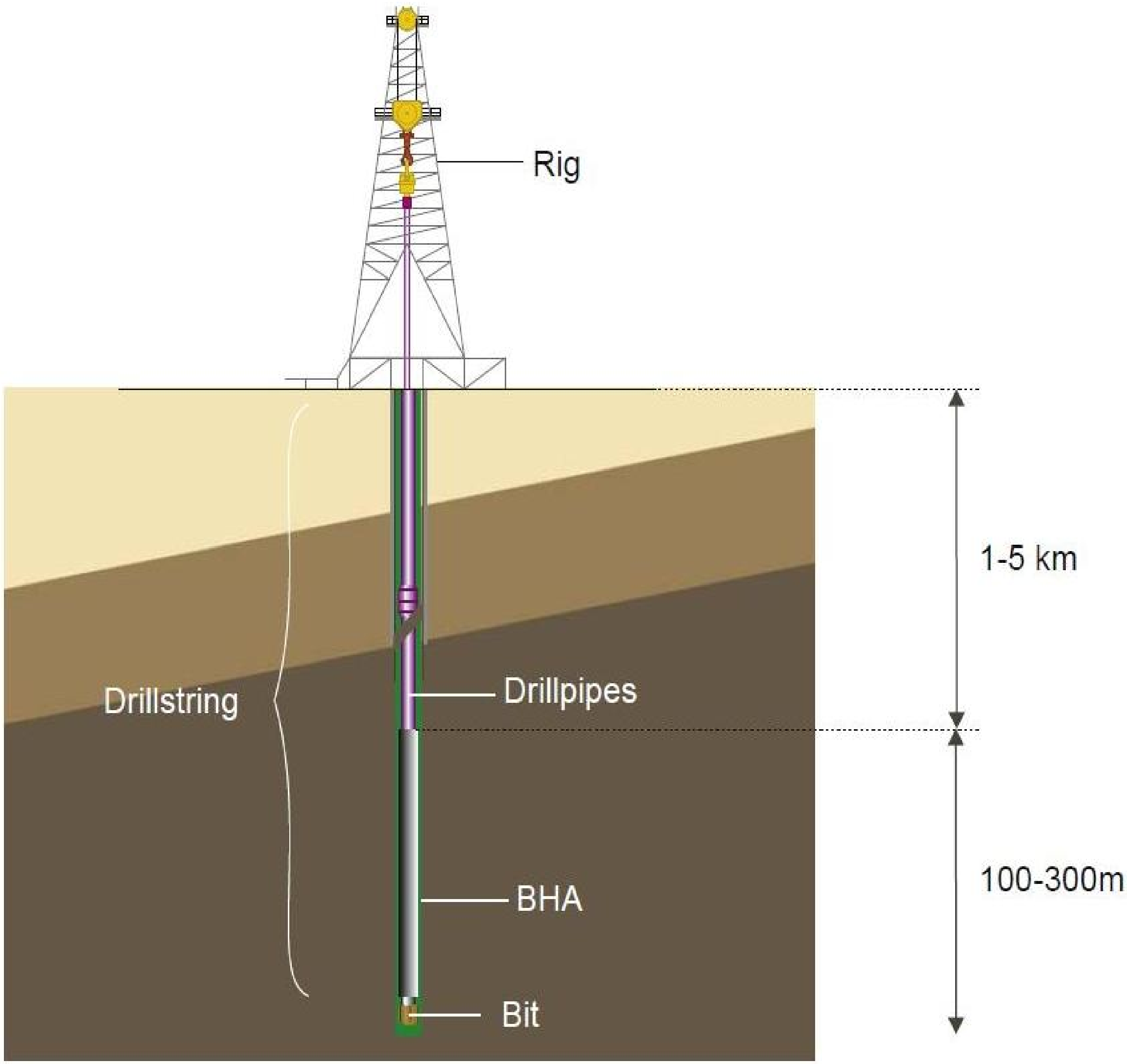
Drill-string vibration

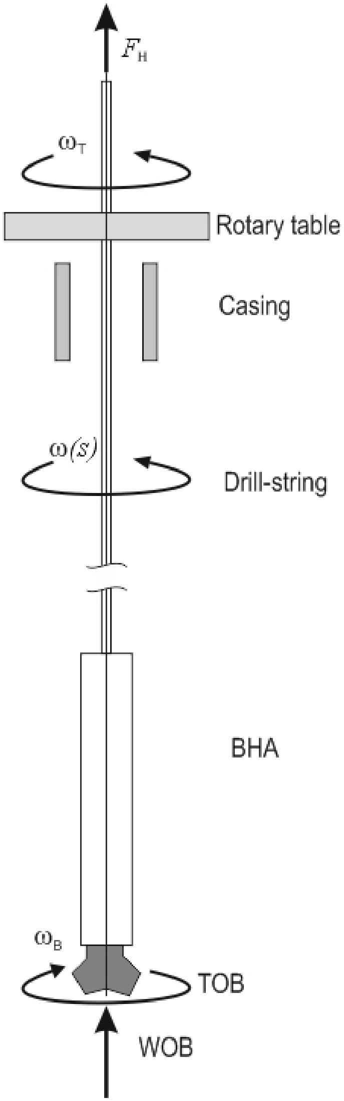
Resonance Enhanced Drilling (RED)



Closing remarks

Drill-string Vibration: Background



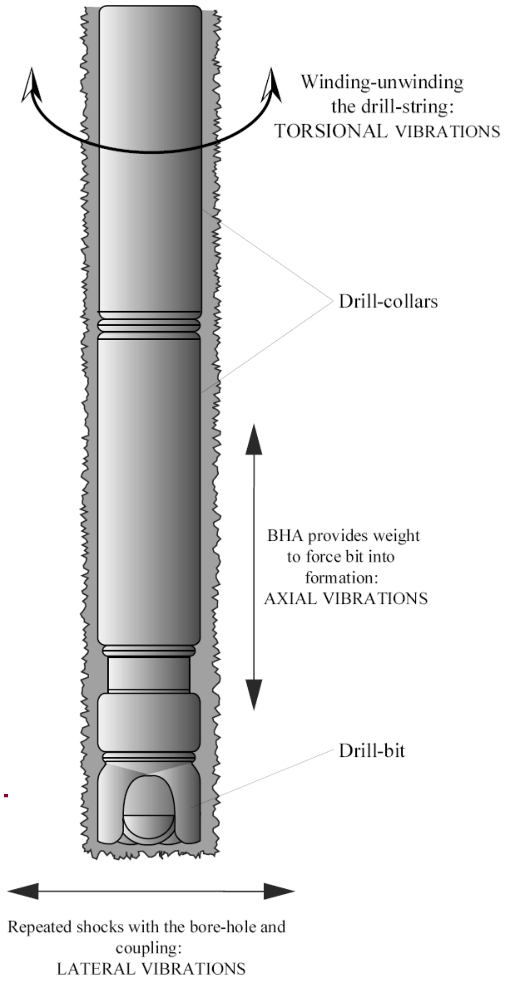


Drill-string Vibration: Background

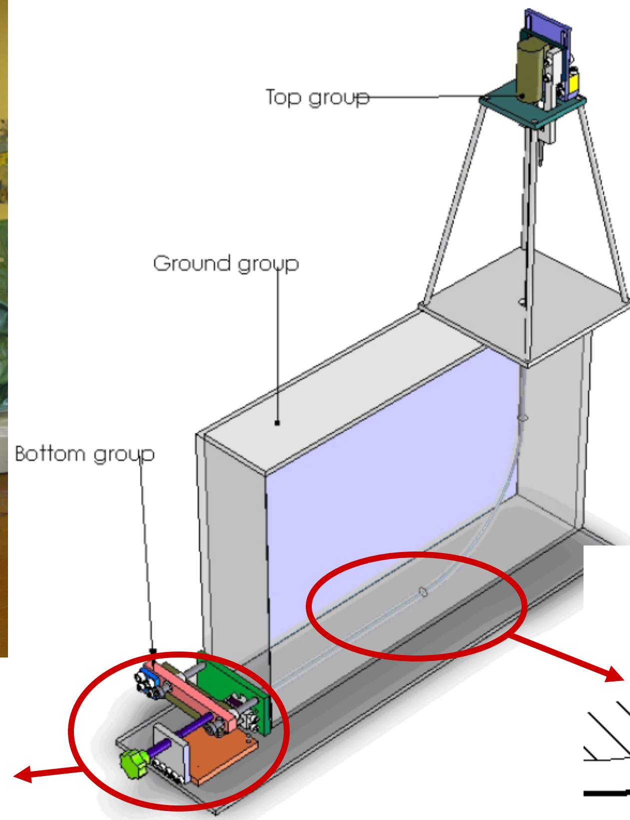
Axial vibration	→	Bit-bounce
Lateral vibration	→	Bit-whirl
Torsional vibration	→	Stick-slip

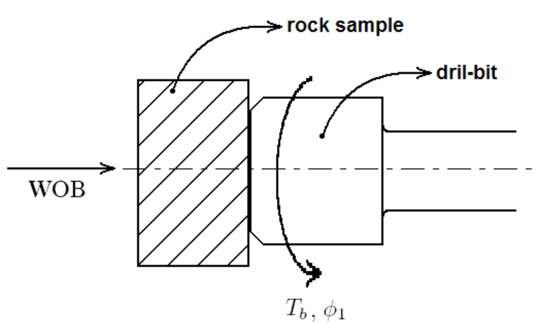
Stick-slip phenomenon: Occurs as much as 50% of the drilling time.

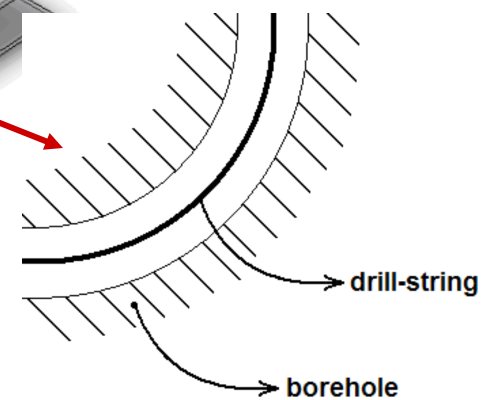
- A drill-string stores torsional, axial and bending energy.
- A drill-bit may come to standstill because of a sudden WOB increase.



Drill-string Vibration: Small Scale Experiments

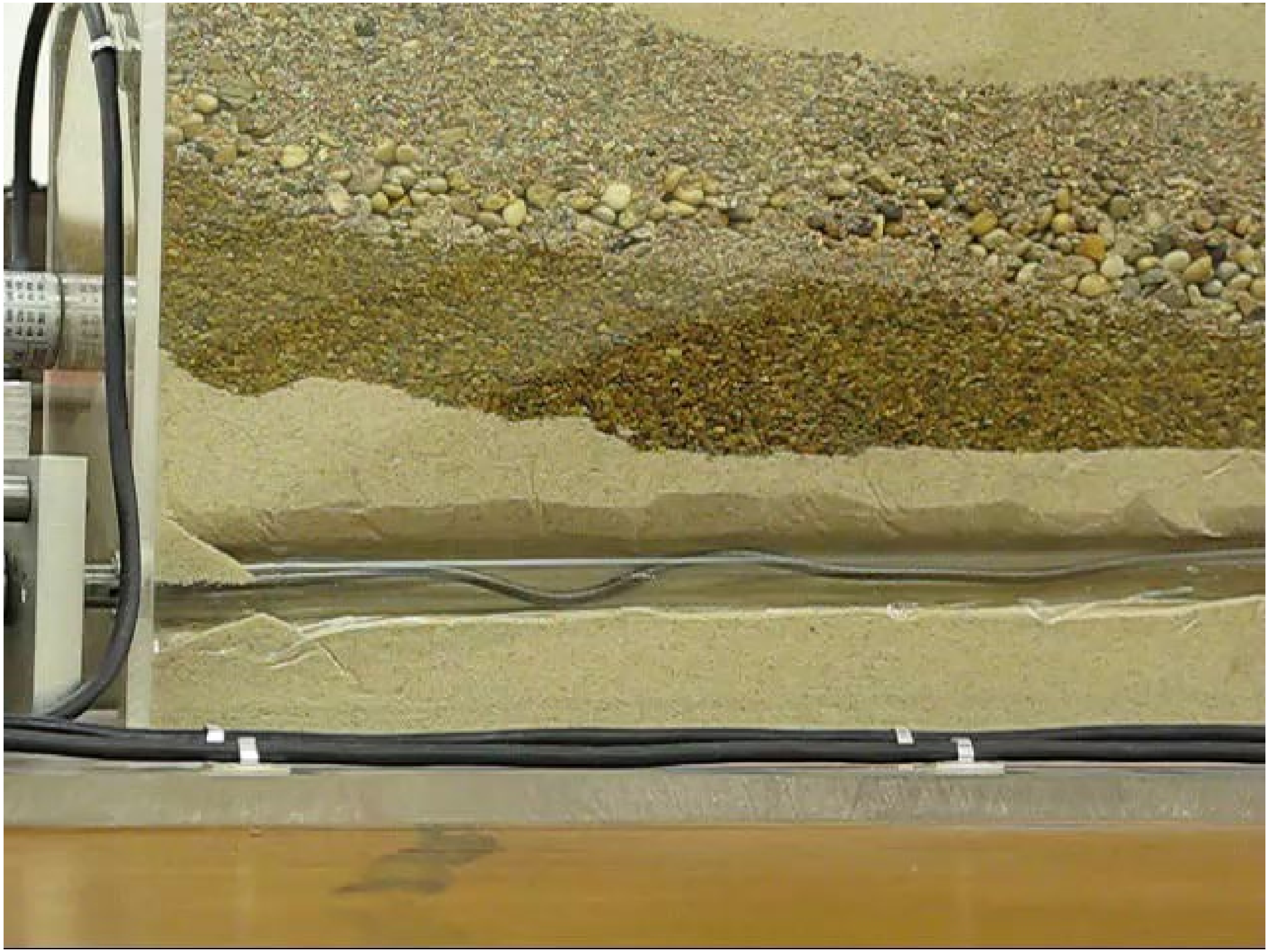




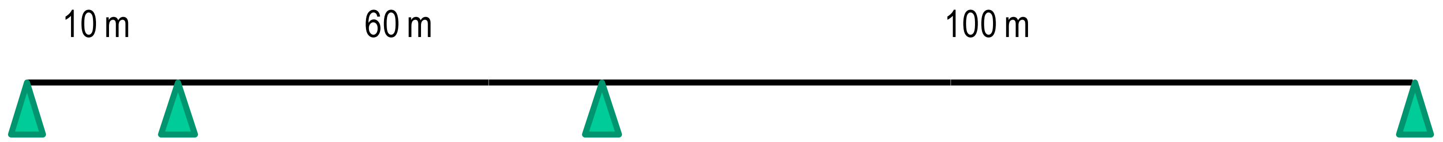


Drill-string Vibration: Small Scale Experiments

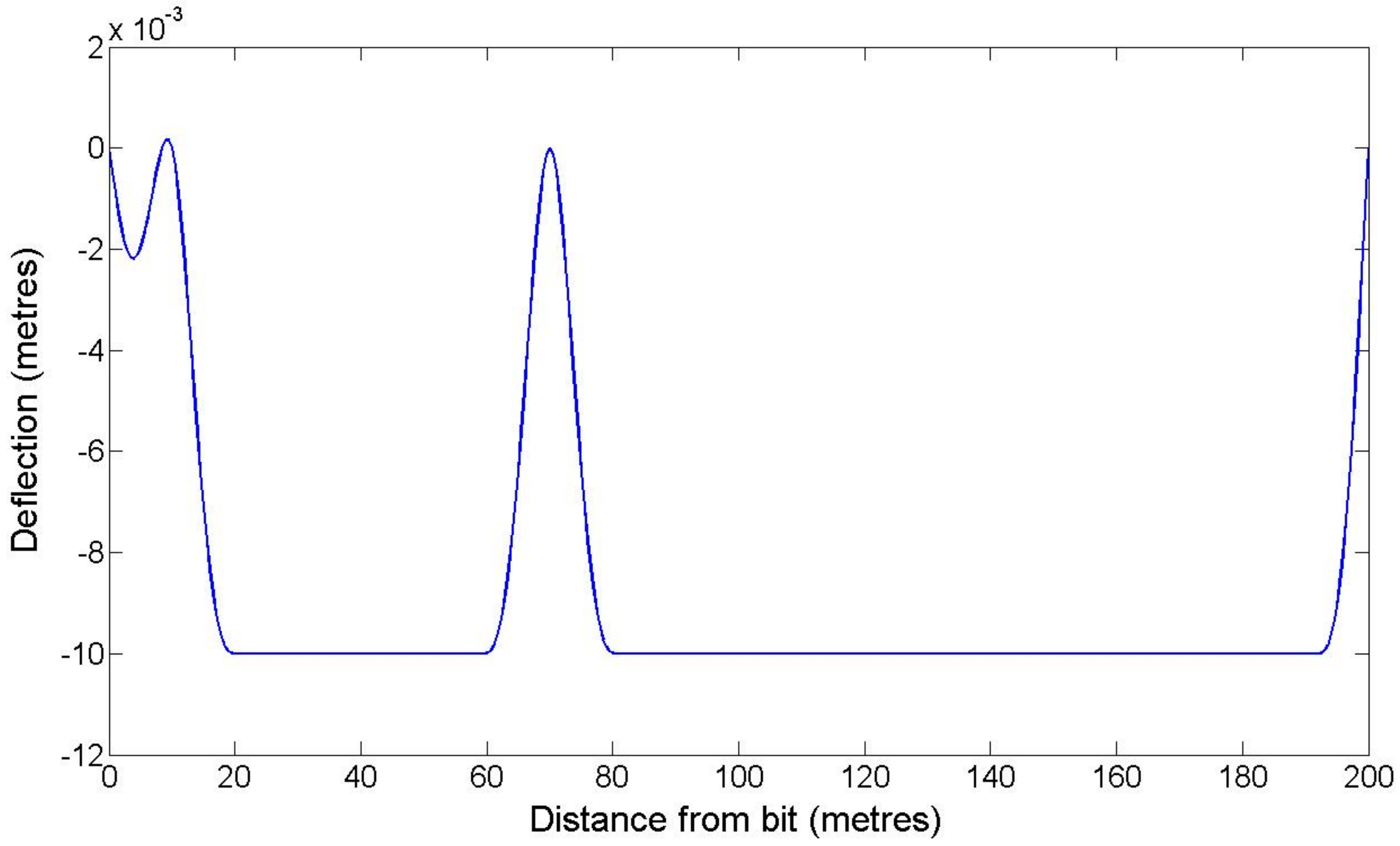
Moderate velocity of the motor



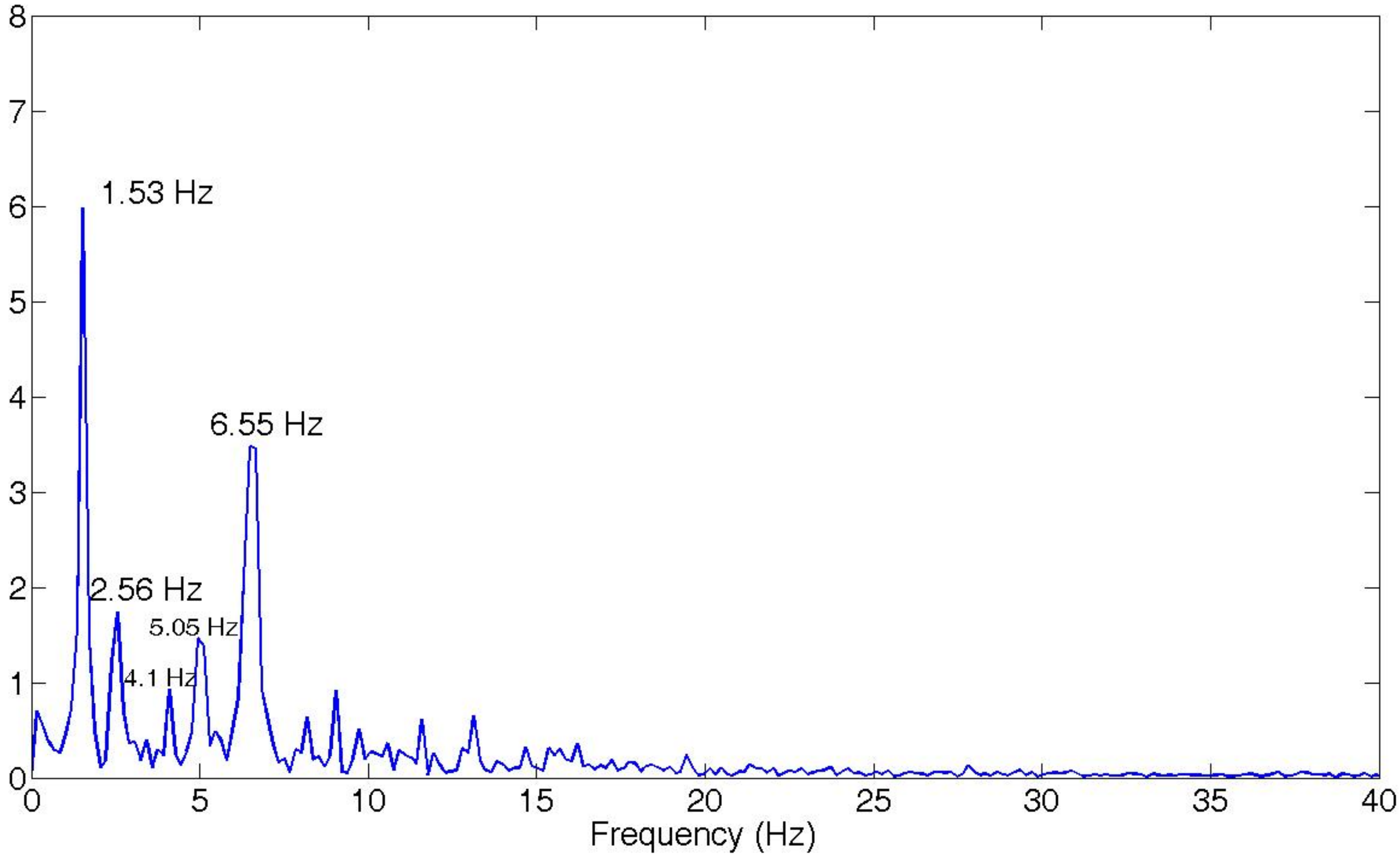
Drill-string Vibration: Modelling



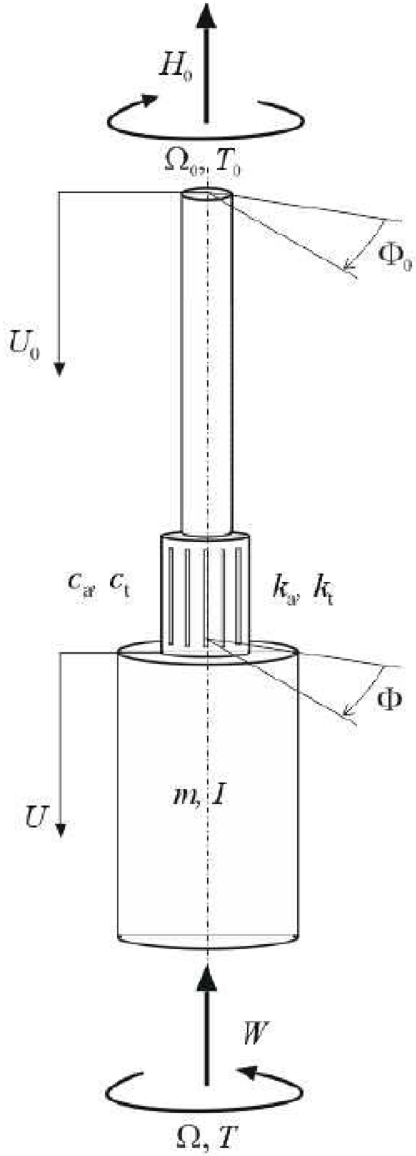
45 degrees inclined bore hole; Clearance – 0.01 metres

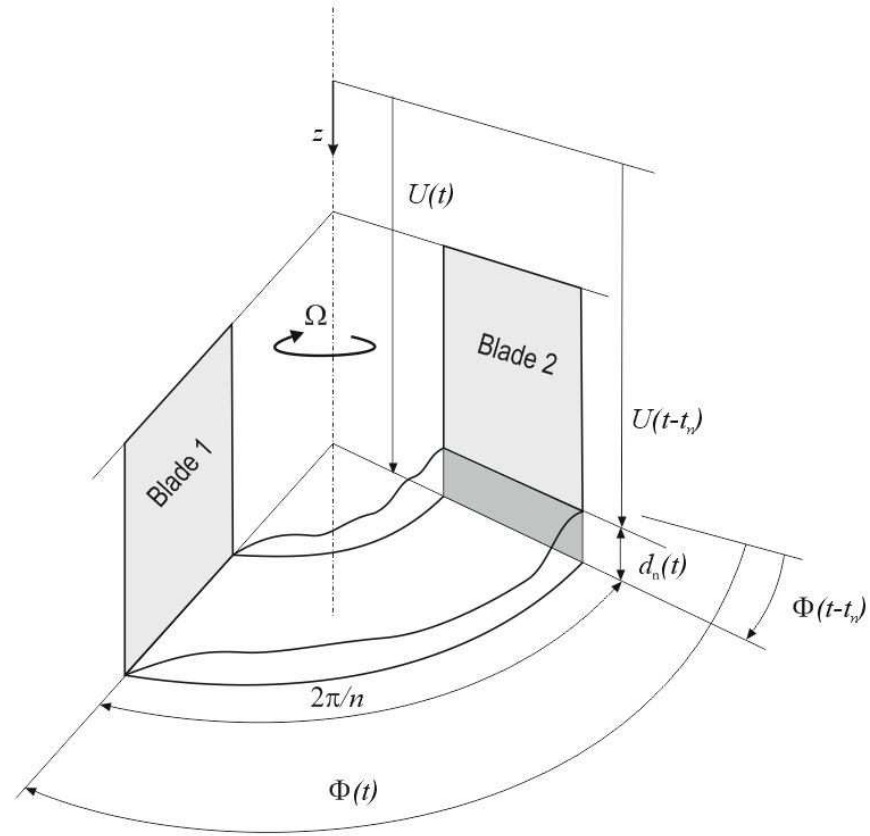


FFT of time domain response of node at 34.5 m from left end.



Drill-string Vibration: Modelling





[CADR, Journal of Sound and Vibration **332** (2013) 2575-2592]

Drill-string Vibration: Modelling

$$M\ddot{U}+C_a\dot{U}+K_a(U-V_0t-l_0)=W_0-\left[\xi\, a\,\varepsilon\, d\, H(\dot{\Phi})\, H(d)+W_f\, H(d)\, H(\dot{U})\right]$$

$$I\ddot{\Phi}+C_t\,\dot{\Phi}+K_t\,(\Phi-\Omega_0\, t)=-\left[\frac{a^2\varepsilon\, d}{2}H(\dot{\Phi})+\frac{\mu\,\gamma\, a\, W_f}{2}sign(\dot{\Phi})H(\dot{U})\right]H(d)$$

$$d=n\,(U(t)-U(t-t_n))$$

$$\Phi(t)-\Phi(t-t_n)=\frac{2\pi}{n}$$

Linearized EOMs for steady drilling

$$\ddot{x}+2\,\zeta\,\beta\,\dot{x}+\beta^2 x=-n\,\psi\left[-\frac{v_0}{\omega_0}\{\phi-\phi\,(\tau-\hat{\tau})\}+\{x-x\,(\tau-\hat{\tau})\}\right]$$

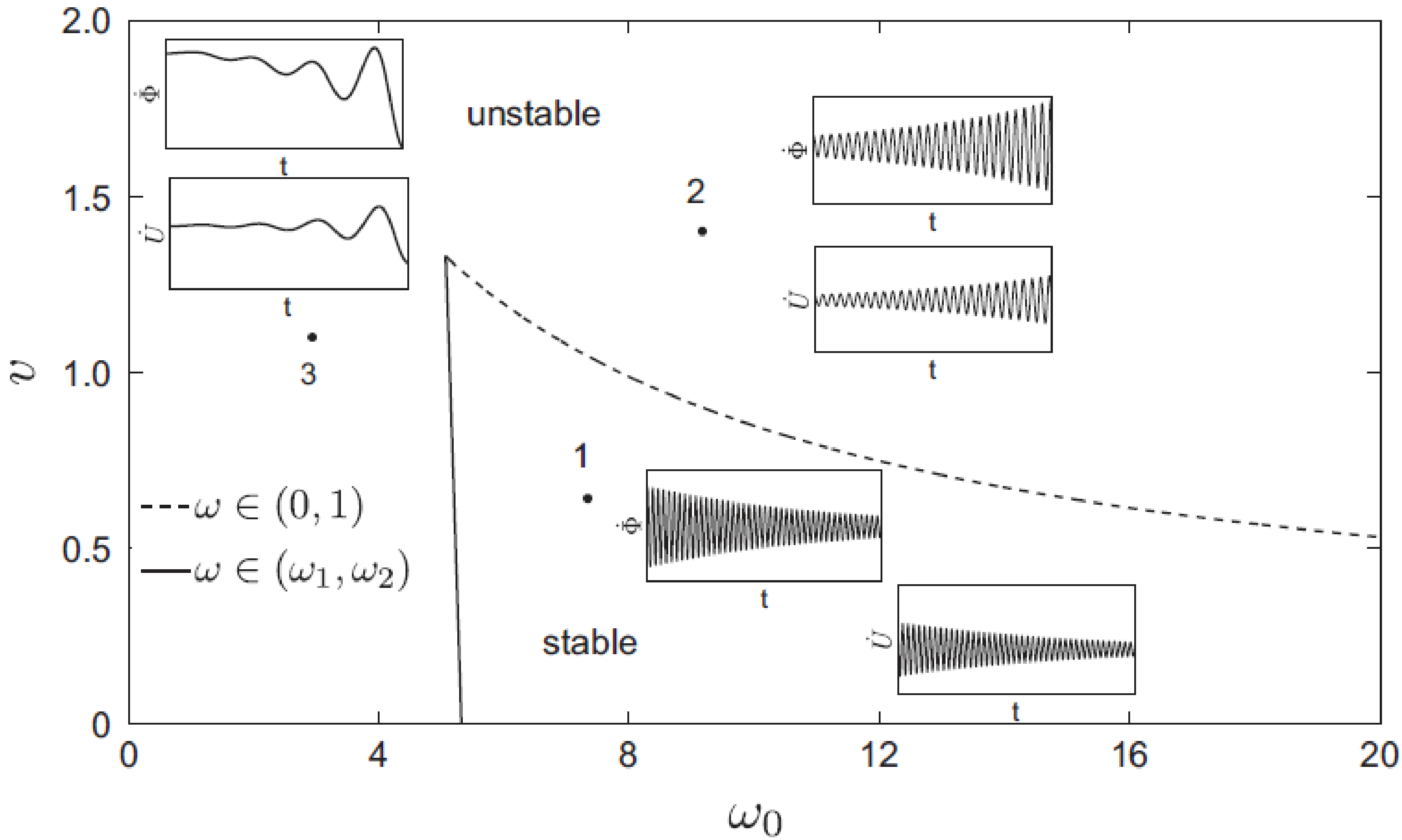
$$\ddot{\phi}+2\,\kappa\,\dot{\phi}+\phi=-n\left[-\frac{v_0}{\omega_0}\{\phi-\phi\,(\tau-\hat{\tau})\}+\{x-x\,(\tau-\hat{\tau})\}\right]$$

[CADR, Applied Mathematical Modelling **334** (2013) 1705-1722]

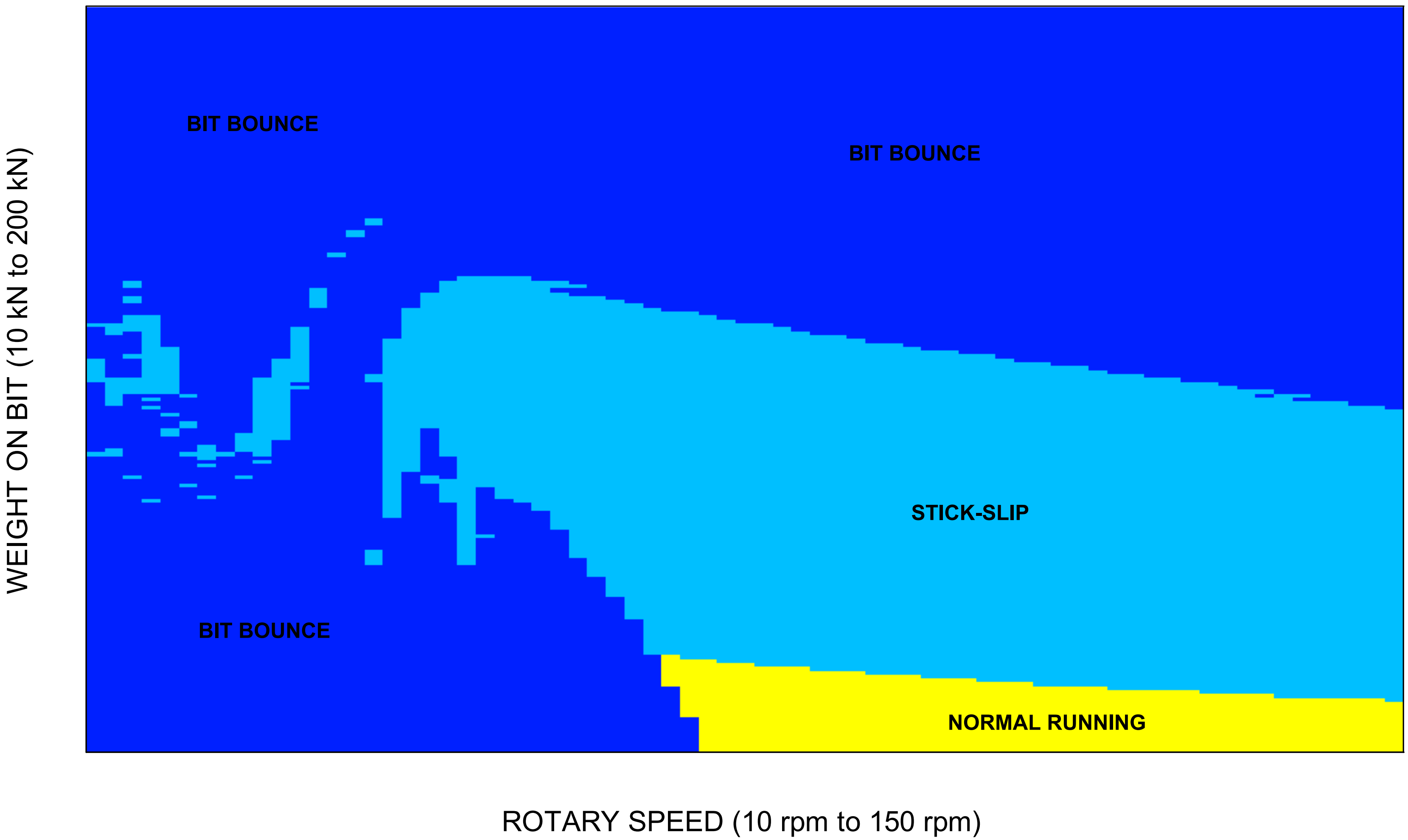
Table 1 List of parameter values for stability chart

Parameter	Symbol	Value	Units
Drillpipe axial stiffness	K_a	7e5	N/m
Drillpipe torsional stiffness	K_t	940	N m/rad.
Vibrational mass	M	34333	Kg
Vibrational mass moment of inertia	I	115.3	Kg m^2
Radius of bit	a	0.108	m
Wear flat length	l	0.0012	m
Rock specific strength	ε	60	MPa
Rock contact stress	σ	60	MPa
Coefficient of friction	μ	0.6	-
-	ξ	0.6	-
-	γ	1	-
Axial damping coefficient	ζ	0.01	-
Torsional damping coefficient	κ	0.01	-
Number of blades	n	4	-

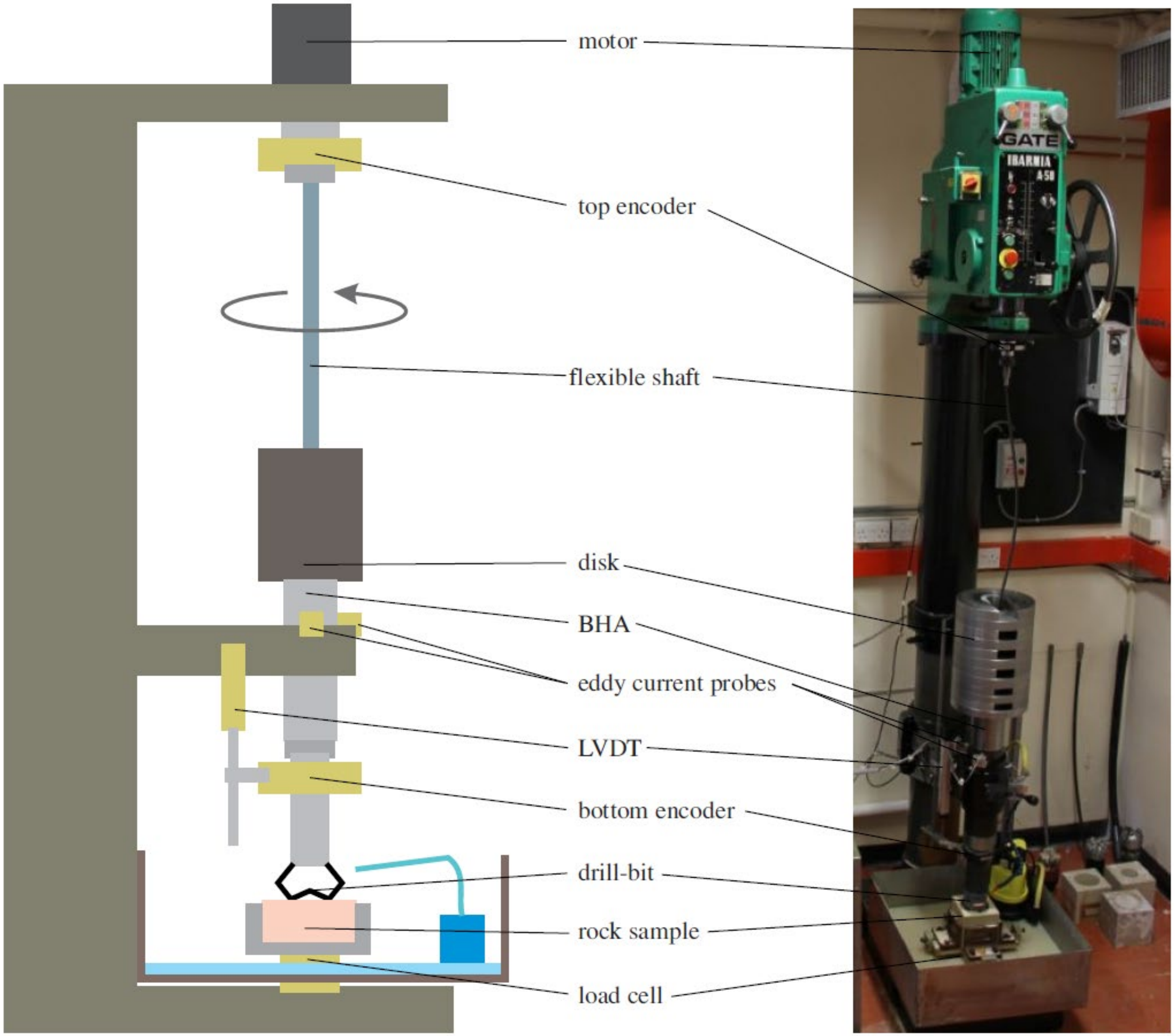
Complete Stability Boundary



Parameter map with initial disturbance 50% in excess of rotary speed



Drill-string Vibration: Large Scale Experiments



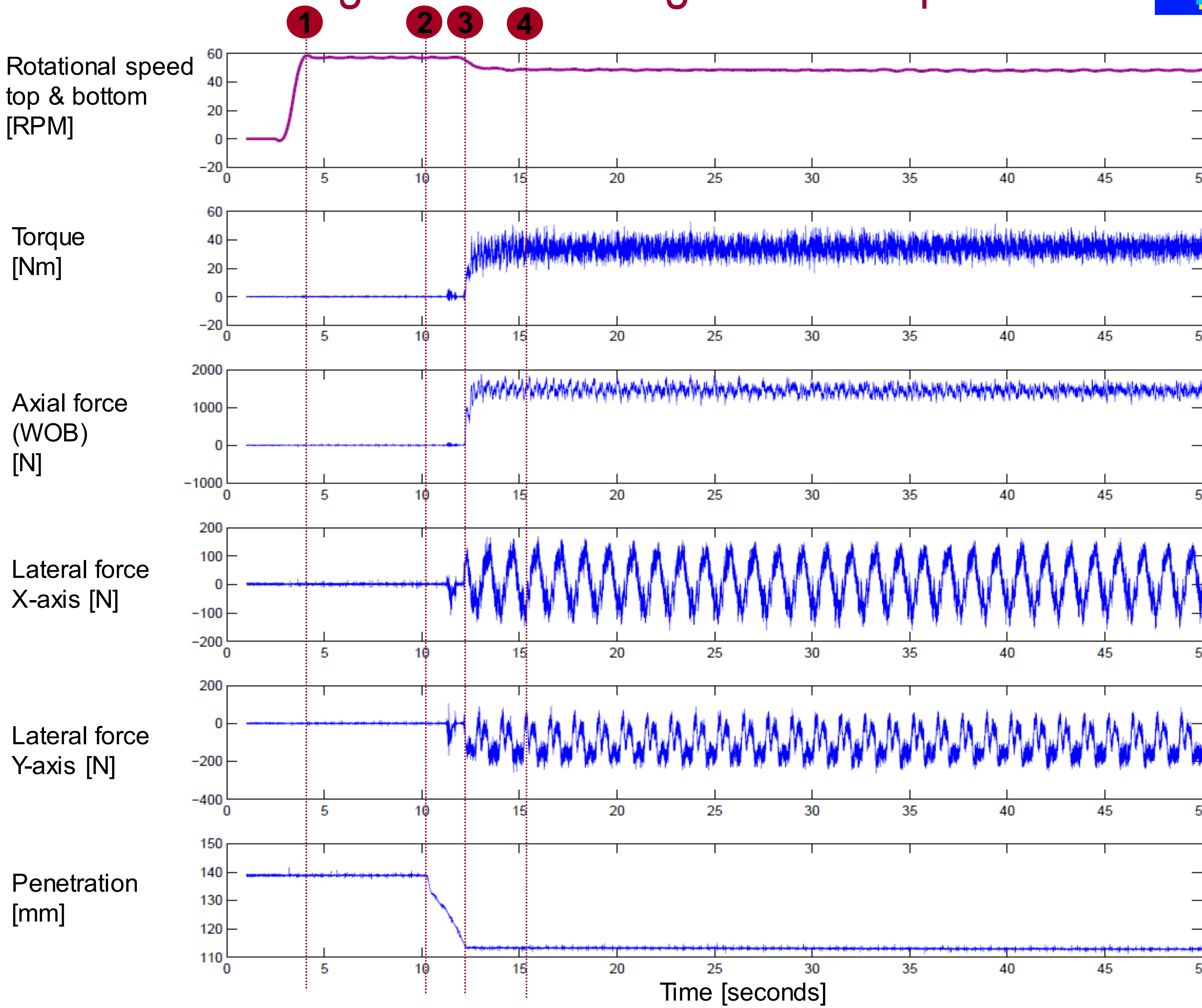
Drill-string Vibration: Large Scale Experiments

23/4" PDC bit

+

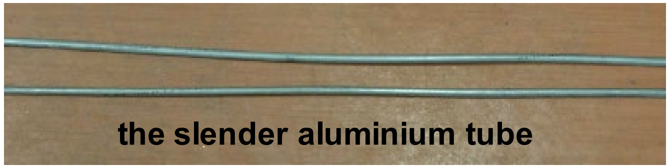
sandstone

+



Drill-string Vibration: Large Scale Experiments

+



+

Drill-string Vibration: Large Scale Experiments

$3_{7/8}$"PDC bit

+

+

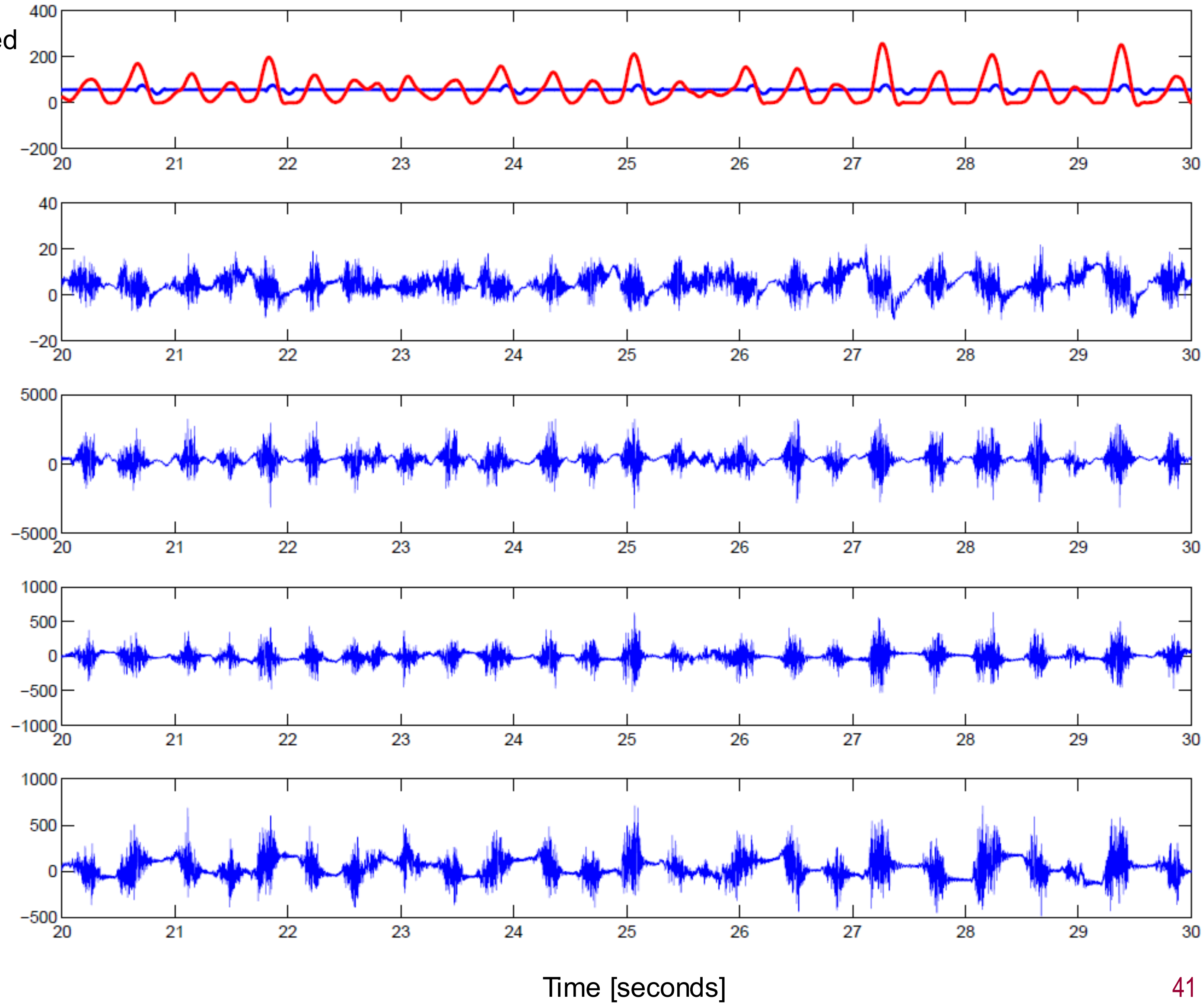
Rotational speed top & bottom [RPM]

Torque [Nm]

Axial force [N]

Lateral force X-axis [N]

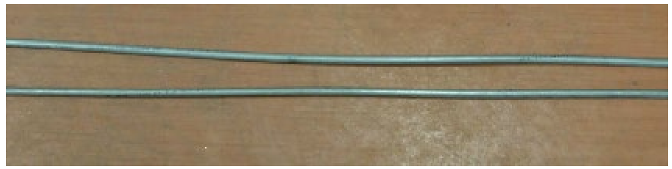
Lateral force Y-axis [N]



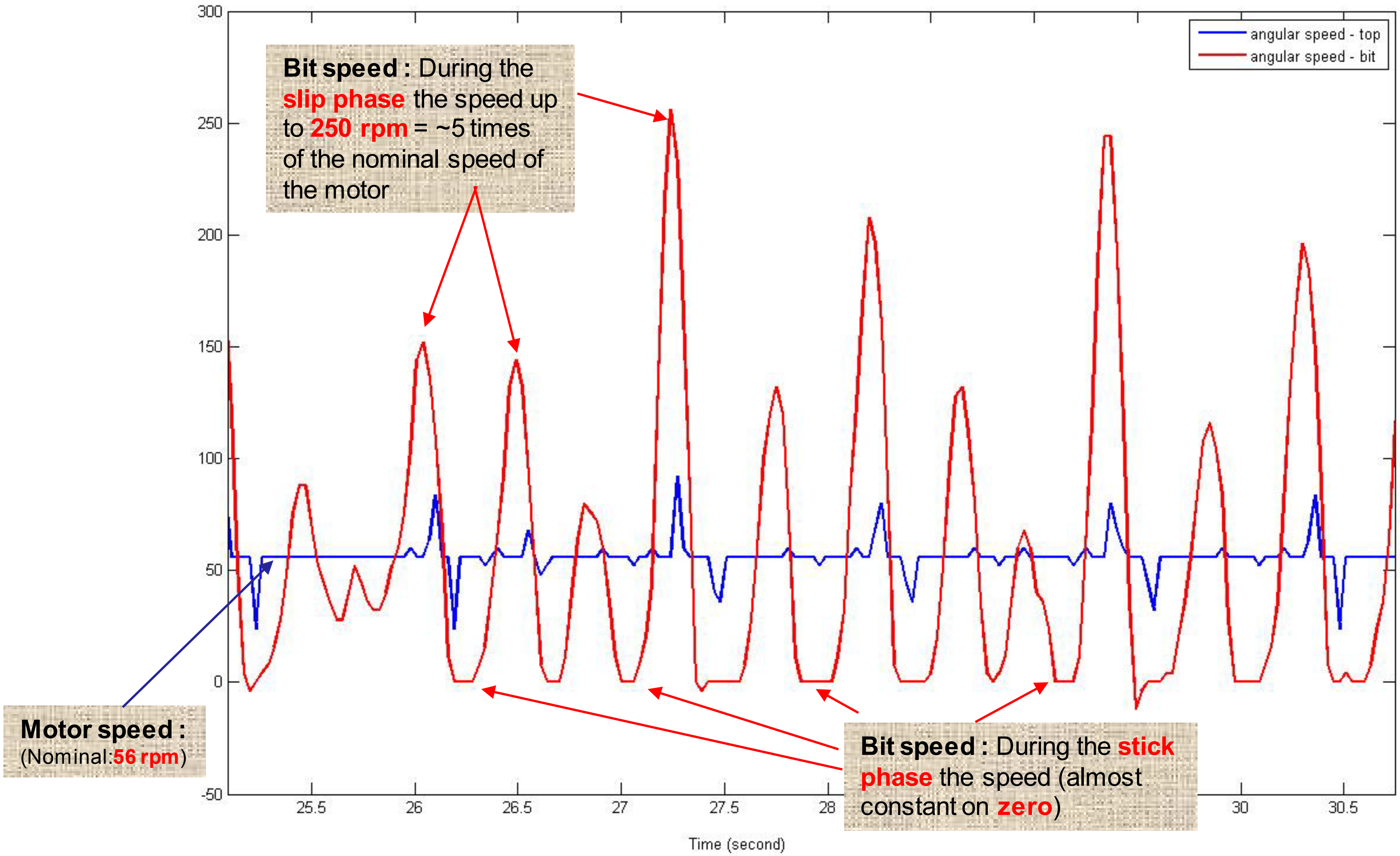
Drill-string Vibration: Large Scale Experiments

granite

the slender aluminium tube



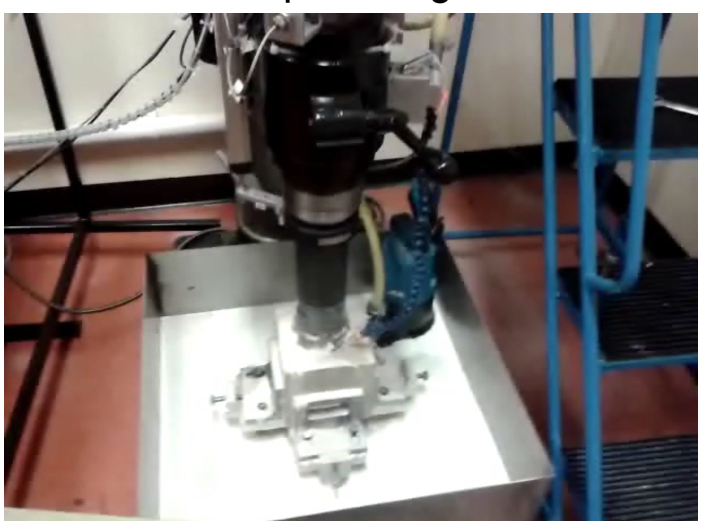
$3_{7/8}$"PDC bit



Drill-string Vibration: Large Scale Experiments

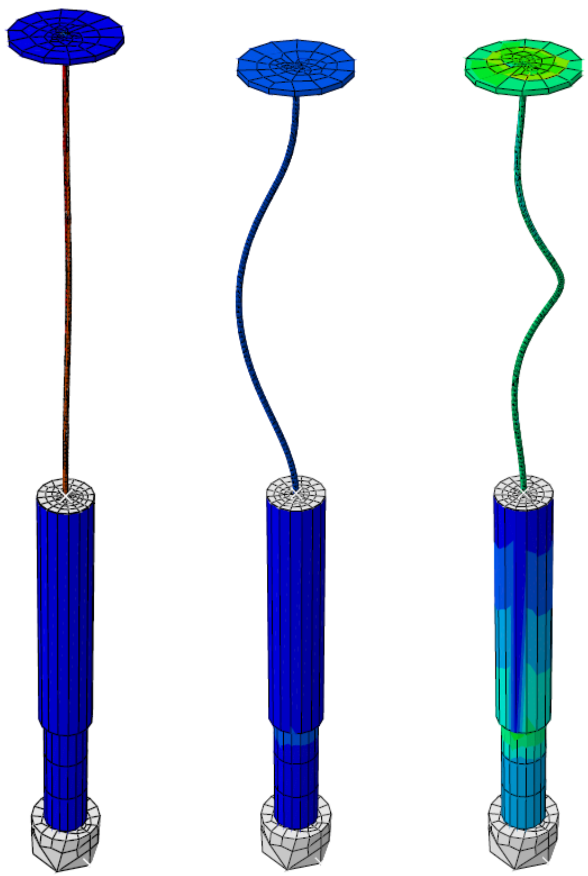
High speed, rigid shaft

Low speed, rigid shaft



Helical buckling
Flexi shaft, 3 7/8” PDC bit

Drill-string Vibration: Modelling

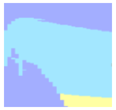


[CADR, IJMS **101-102** (2015) 324-337]

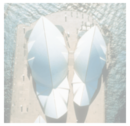
Presentation Outline


Nonlinear dynamics for engineering design: Introduction


Rotor dynamics

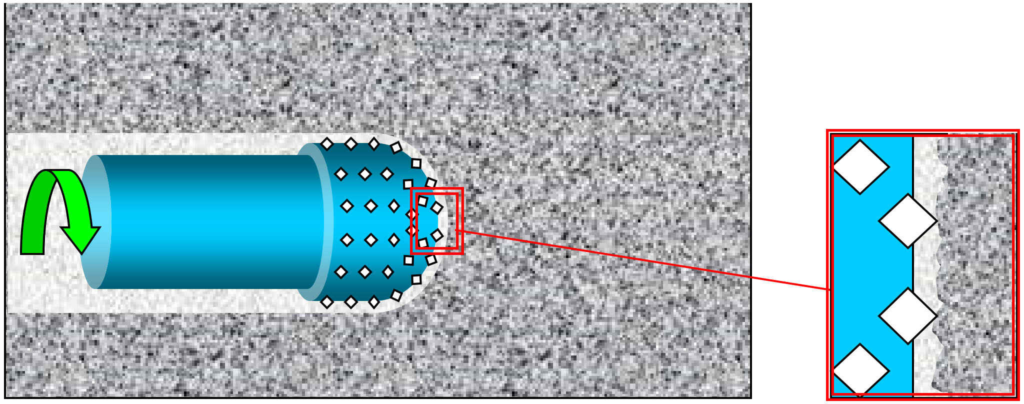

Drill-string vibration

Resonance Enhanced Drilling (RED)

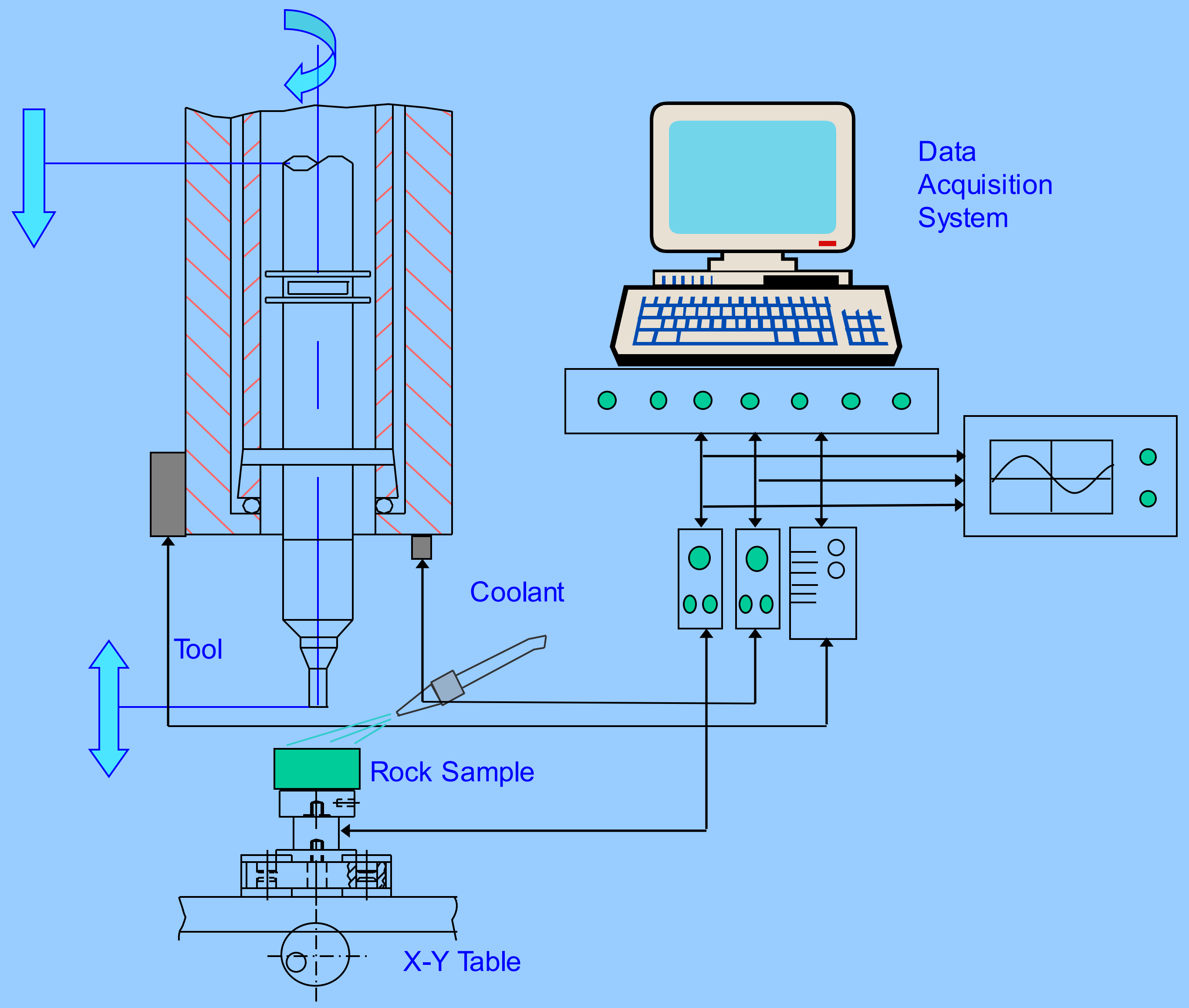

Closing remarks

Resonance Enhanced Drilling (RED)

How Does It Work?

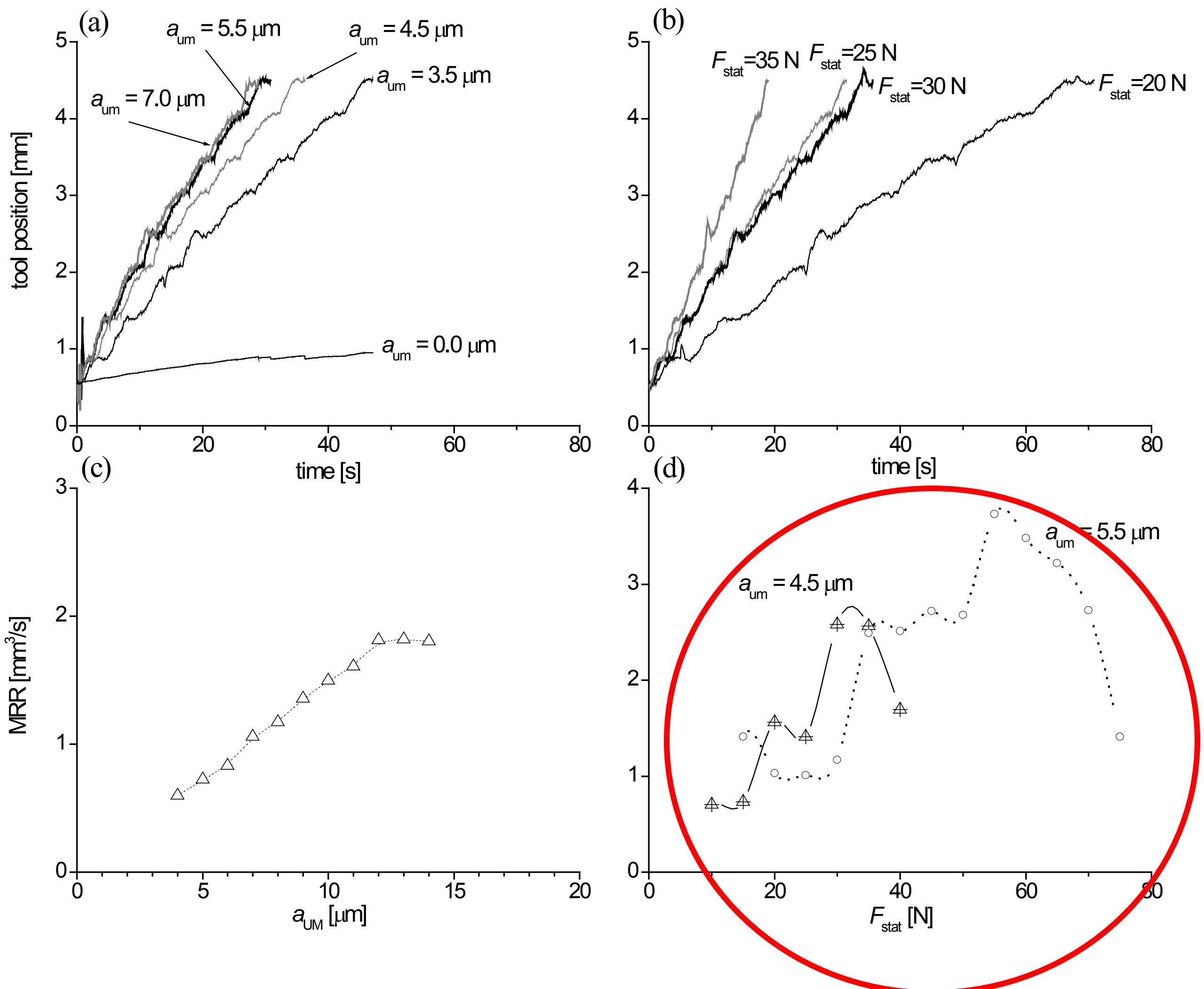


Small Scale Experimental Studies

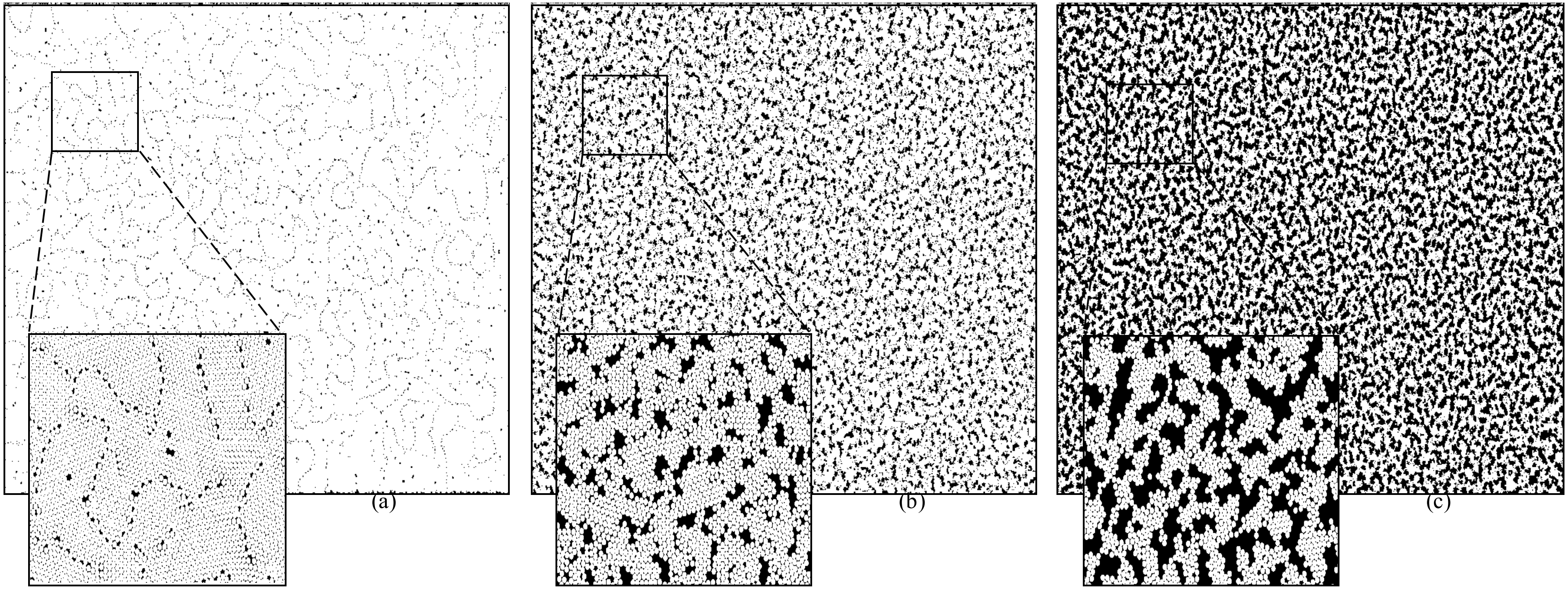


Small Scale Experimental Studies

Small Scale Experimental Studies

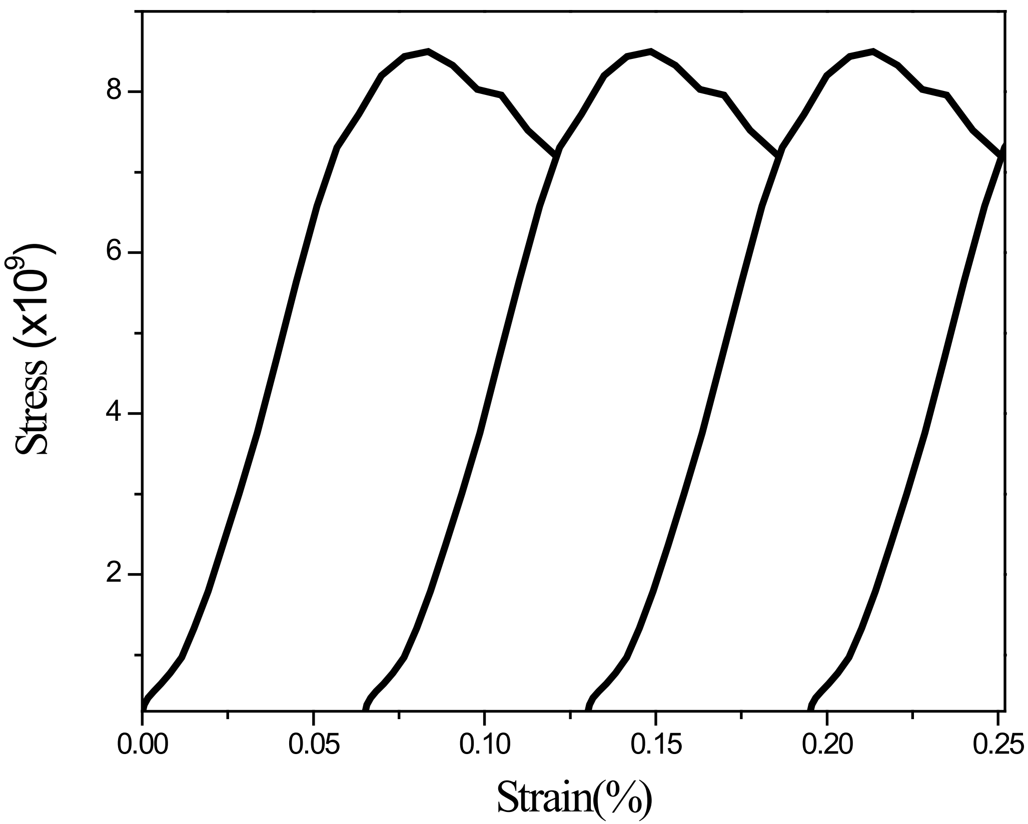


[CADR, Journal of Sound and Vibration **280** (2005) 739-757]

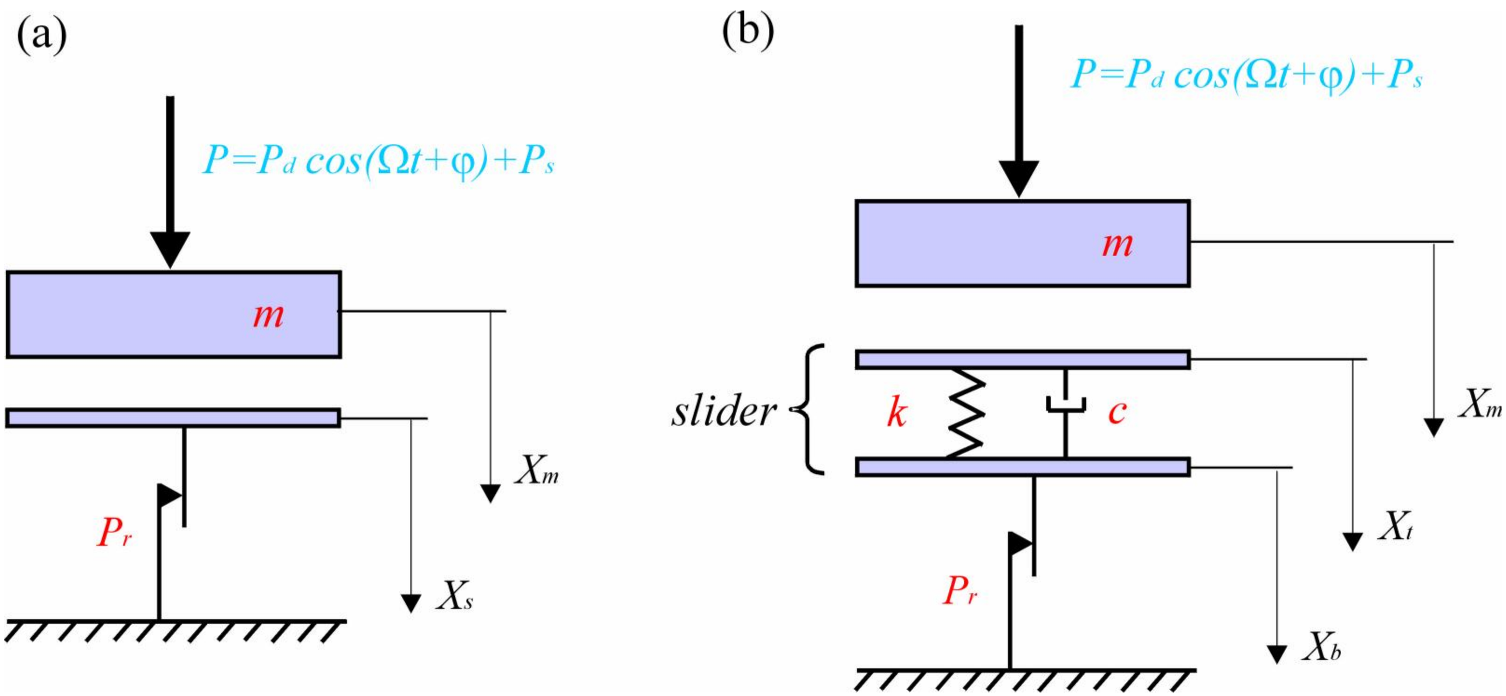


Material patterns with different porosity; simulation with 100 thousands particles; (a) 1% porosity, (b) 10% porosity, (c) 20% porosity.

Stress-Strain Curves from Particle Dynamics Simulation



Modelling of Impact Dynamics



(a) Krivtsov A., Wiercigroch M. (1999) *Meccanica* **34**, 425-434.

(b) Pavlovskaia E., Wiercigroch M., Grebogi C. (2001) *Physical Review E* **64**, 056224.

- No contact

$$x < z + g$$
$$x' = y$$
$$y' = a\cos(\omega\tau + \varphi) + b$$
$$z' = -\frac{1}{2\xi}(z - v)$$
$$v' = 0$$

→

$$p < q + g$$
$$p' = y$$
$$y' = a\cos(\omega\tau + \varphi) + b$$
$$q' = -\frac{1}{2\xi}q$$
$$v' = 0$$

- Contact without progression

$$x \ge z + g \quad \text{and} \quad 0 < 2\xi y + (z - v) < 1$$
$$x' = y$$
$$y' = -2\xi y - (z - v) + a\cos(\omega\tau + \varphi) + b$$
$$z' = y$$
$$v' = 0$$

→

$$p \ge q + g \quad \text{and} \quad 0 < 2\xi y + q < 1$$
$$p' = y$$
$$y' = -2\xi y - q + a\cos(\omega\tau + \varphi) + b$$
$$q' = y$$
$$v' = 0$$

- Contact with progression

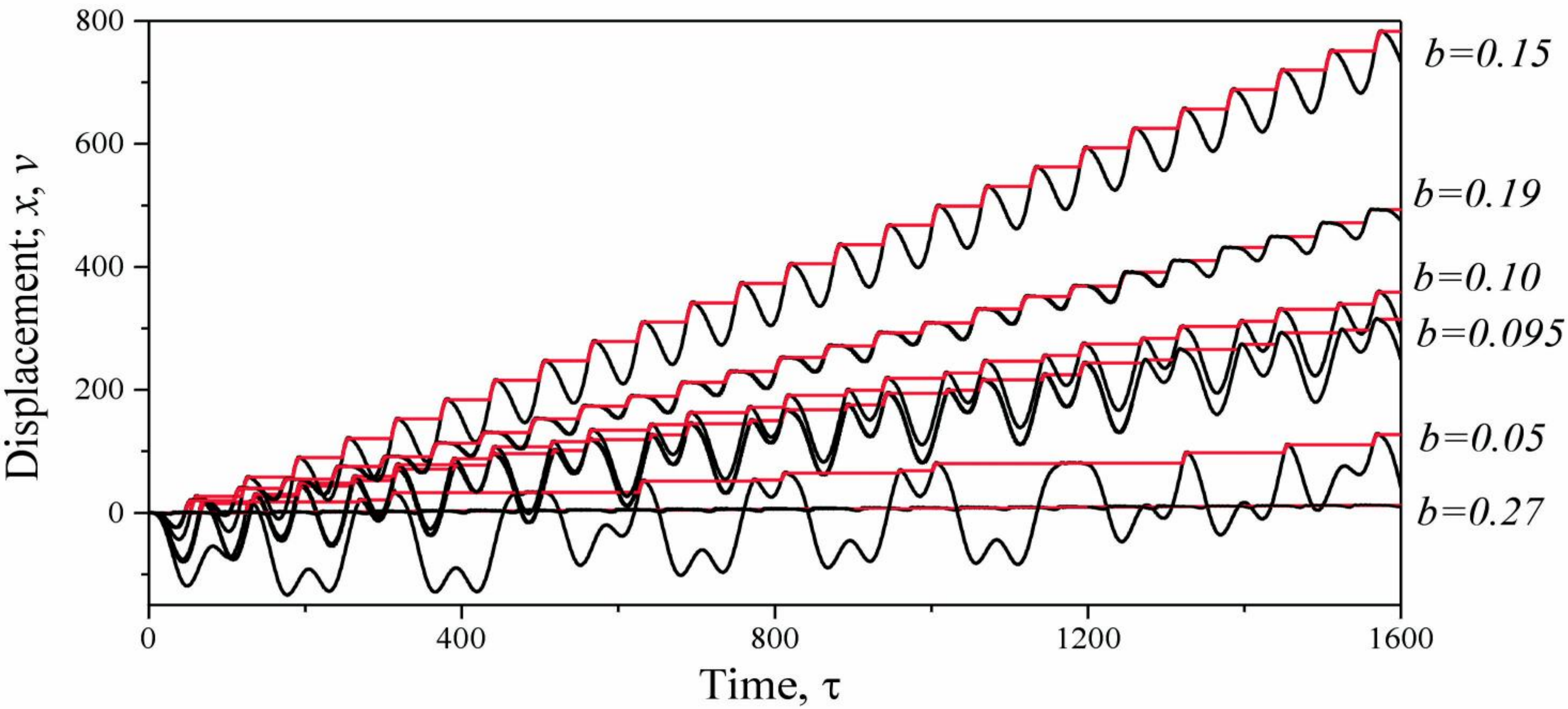
$$x \geq z + g \quad \text{and} \quad 2\xi z' + (z - v) \geq 1$$

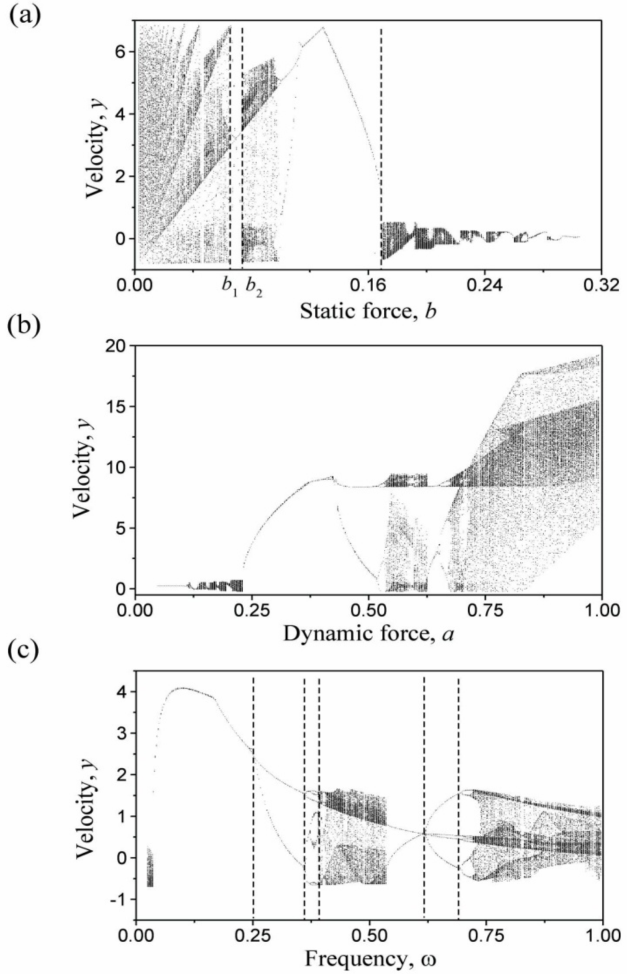
$$x' = y$$
$$y' = a\cos(\omega\tau + \varphi) + b - 1$$
$$z' = v$$
$$v' = y + \frac{1}{2\xi}(z - v - 1)$$

$\Rightarrow$

$$p \geq q + g \quad \text{and} \quad 2\xi y + q \geq 1$$

$$p' = -\frac{1}{2\xi}(q - 1)$$
$$y' = a\cos(\omega\tau + \varphi) + b - 1$$
$$q' = -\frac{1}{2\xi}(q - 1)$$
$$v' = y + \frac{1}{2\xi}(q - 1)$$

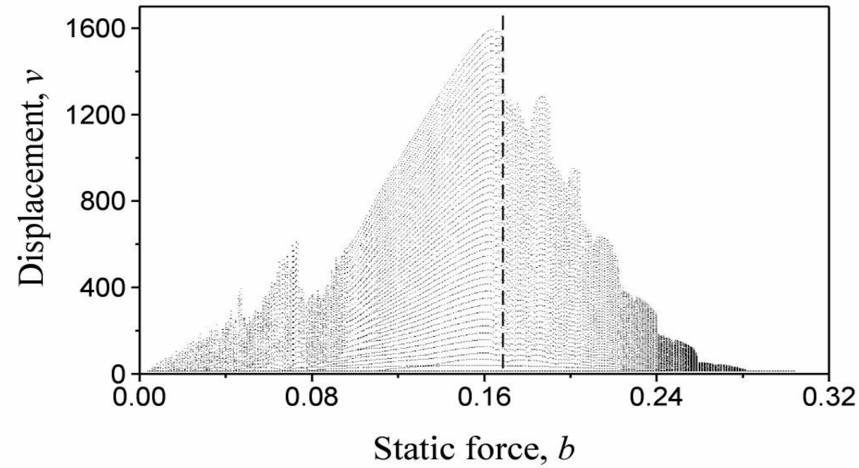

800
600
400
200
0
Displacement; x, v
0
400
800
1200
1600
Time, τ
b=0.15
b=0.19
b=0.10
b=0.095
b=0.05
b=0.27

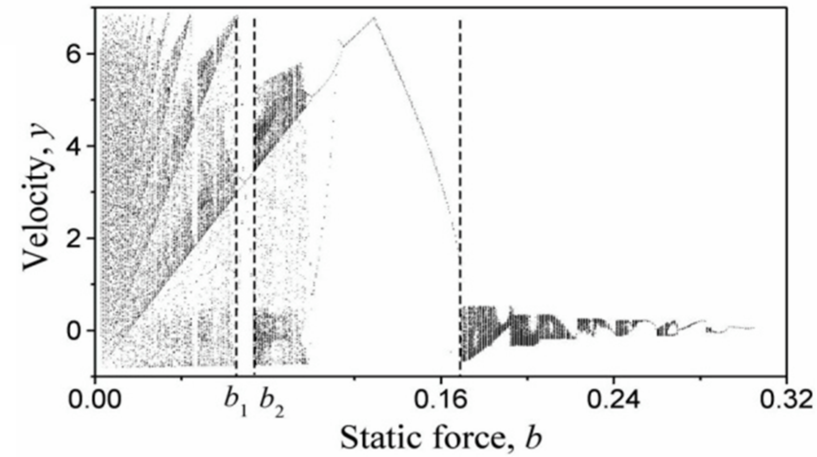


$a = 0.3$
$\xi = 0.05$
$\omega = 0.1$

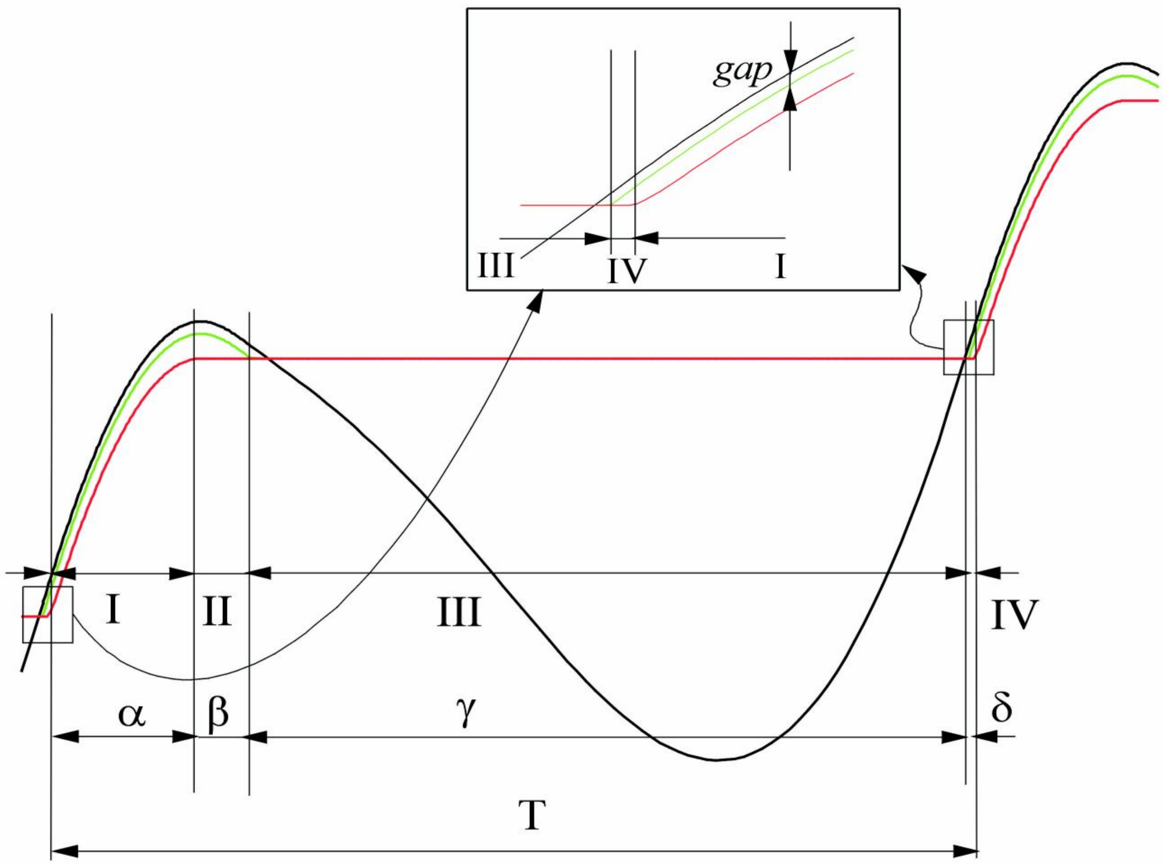
$b = 0.15$
$\xi = 0.05$
$\omega = 0.1$

$a = 0.3$
$b = 0.15$
$\xi = 0.05$





Reconstruction of Periodic Orbits

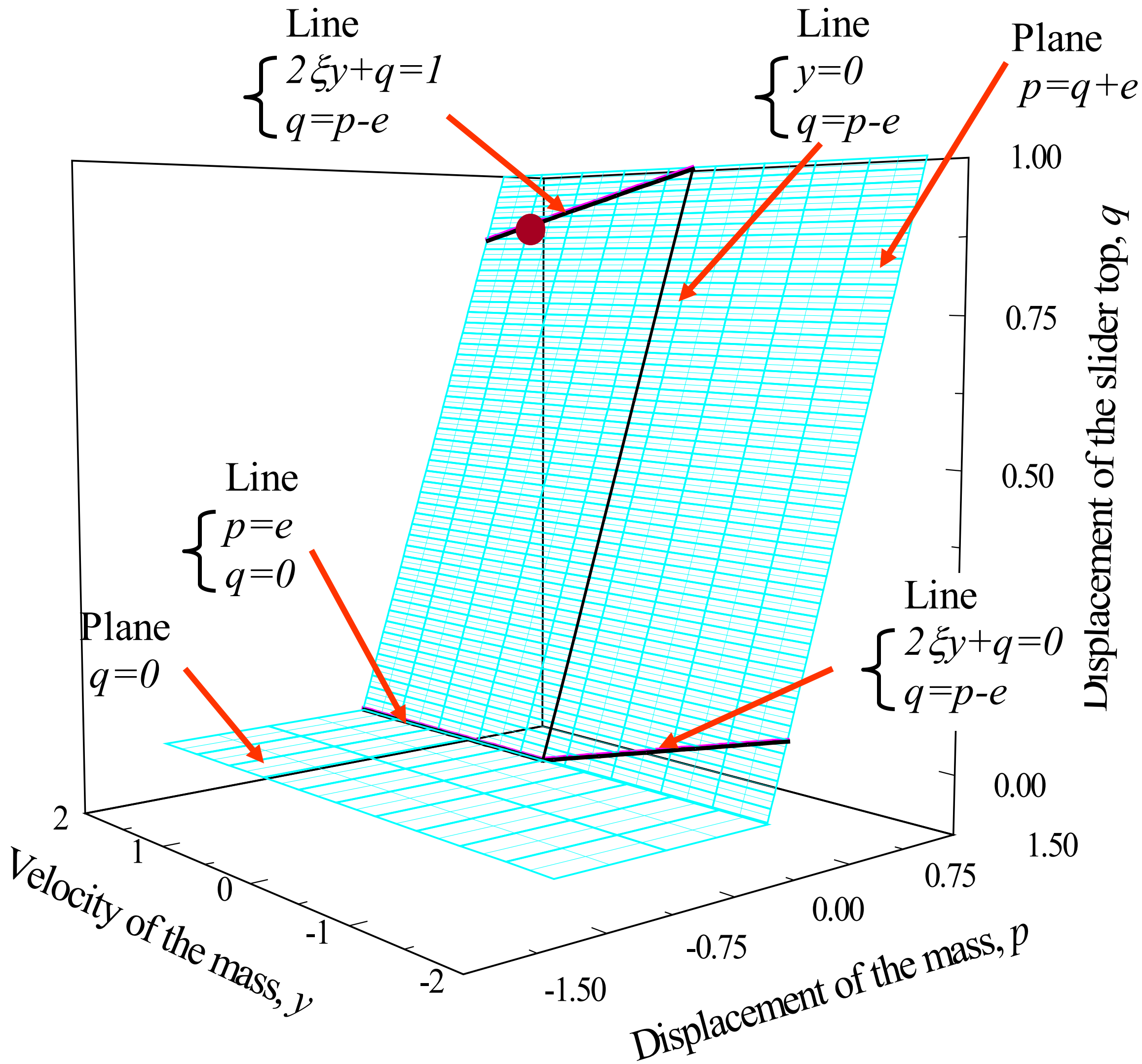


This four dimensional flow:

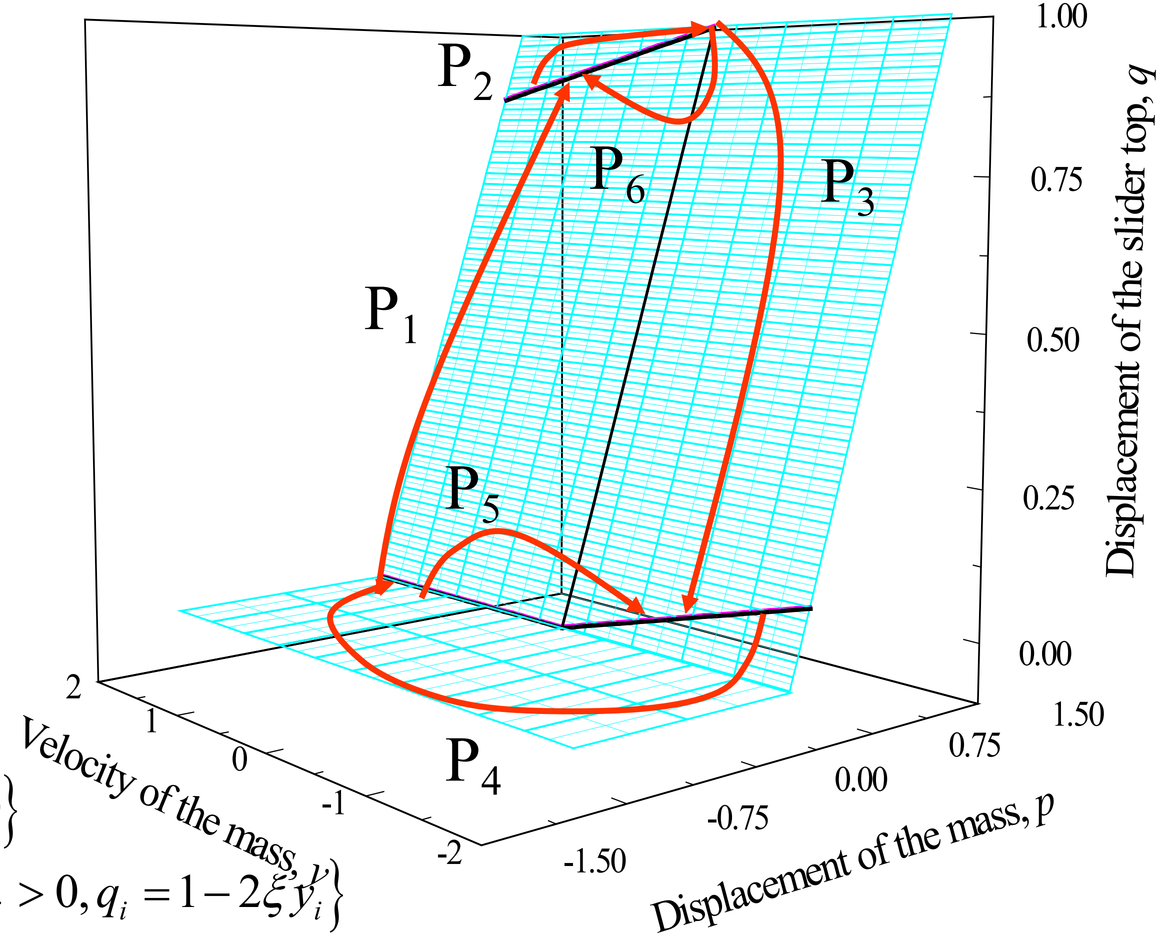
$$(\tau; p, y, q)$$

can be locally three dimensional (trajectories belong to the planes). Cross-section of the flow with phase plane borders (the lines) gives further possibilities of reduction

to 2D discrete maps



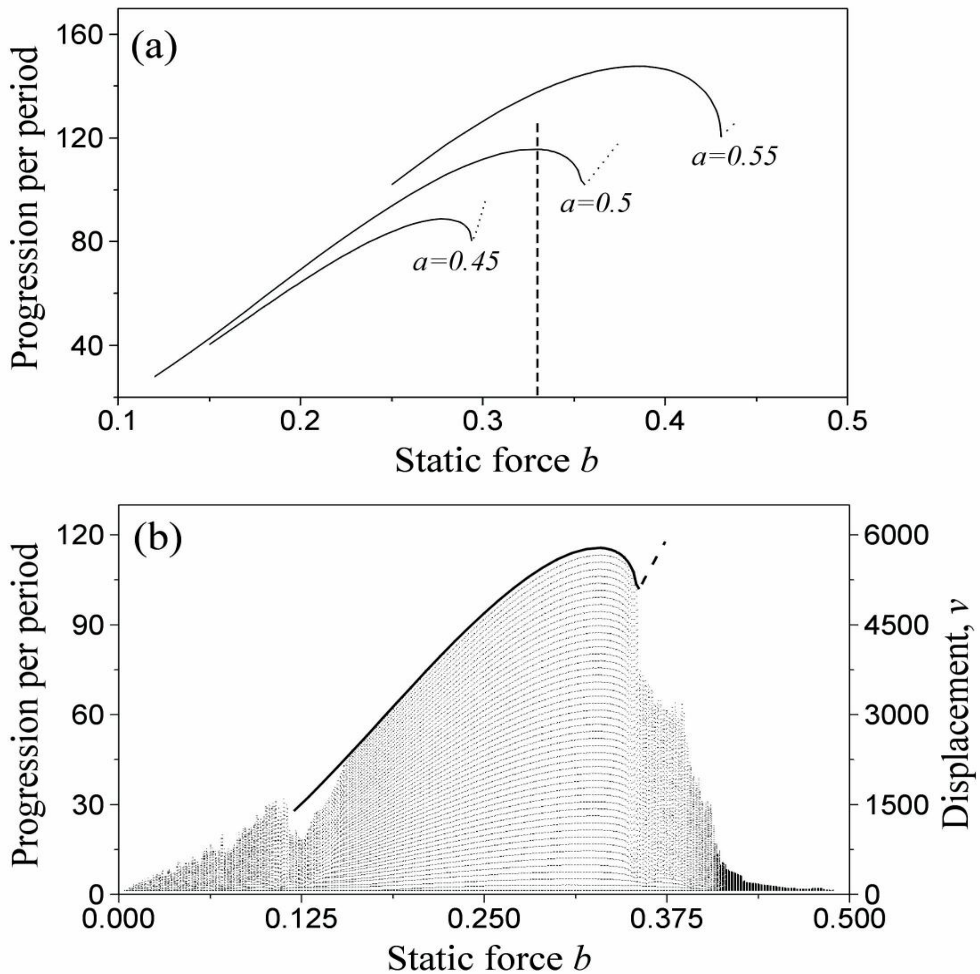
$$P_1: \quad \Sigma_1 \to \Sigma_2$$
$$P_2: \quad \Sigma_2 \to \Sigma_3$$
$$P_3: \quad \Sigma_3 \to \Sigma_4$$
$$P_4: \quad \Sigma_4 \to \Sigma_1$$
$$P_5: \quad \Sigma_1 \to \Sigma_4$$
$$P_6: \quad \Sigma_3 \to \Sigma_2$$



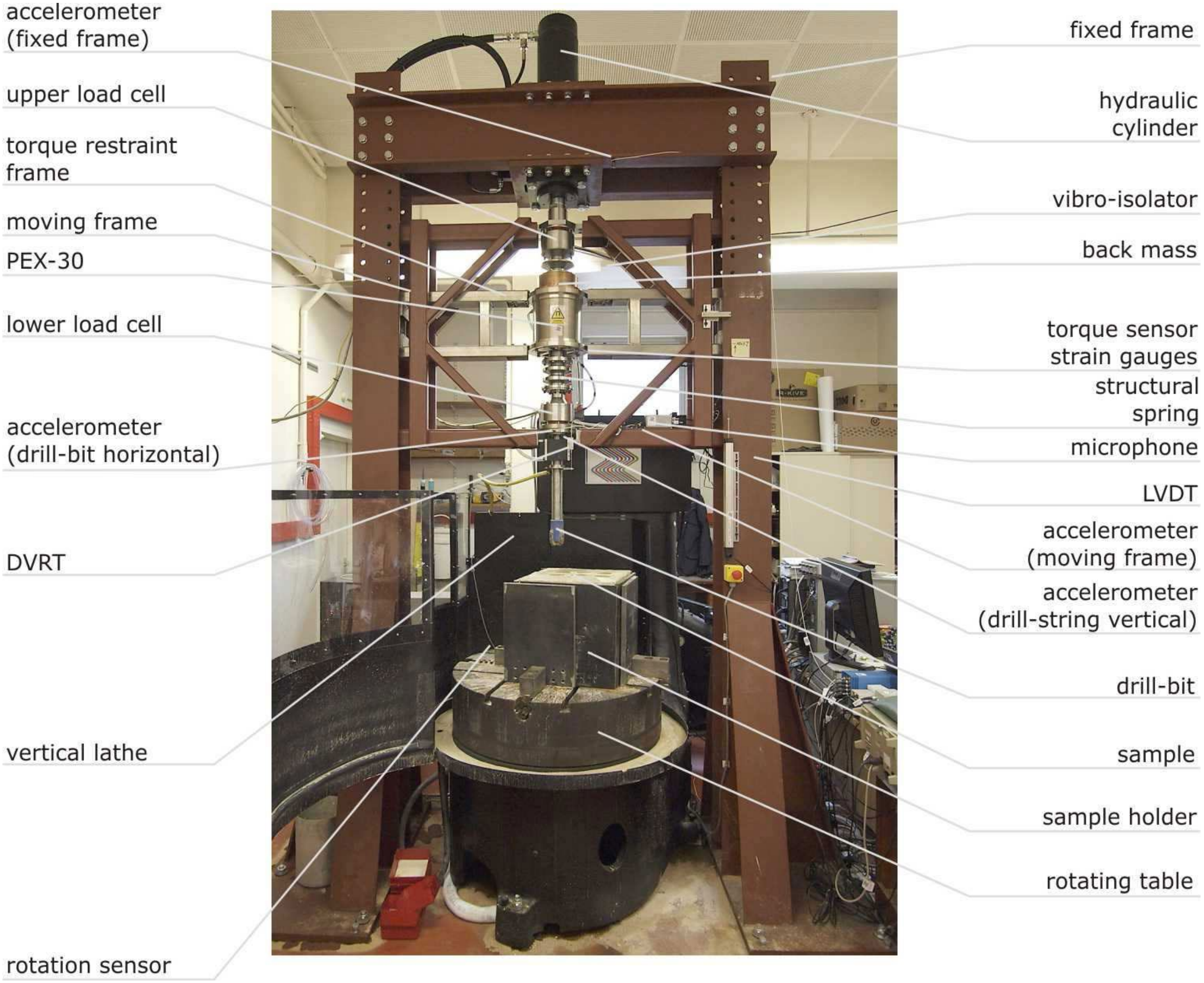
$$\Sigma_1 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = e, y_i > 0, q_i = 0 \right\}$$
$$\Sigma_2 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 + e - 2\xi y_i, y_i > 0, q_i = 1 - 2\xi y_i \right\}$$
$$\Sigma_3 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 + e - 2\xi y_i, y_i < 0, q_i = 1 - 2\xi y_i \right\}$$
$$\Sigma_4 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 - 2\xi y_i, y_i < 0, q_i = -2\xi y_i \right\}$$

[CADR, Physical Review E **70** (2004) 036201]

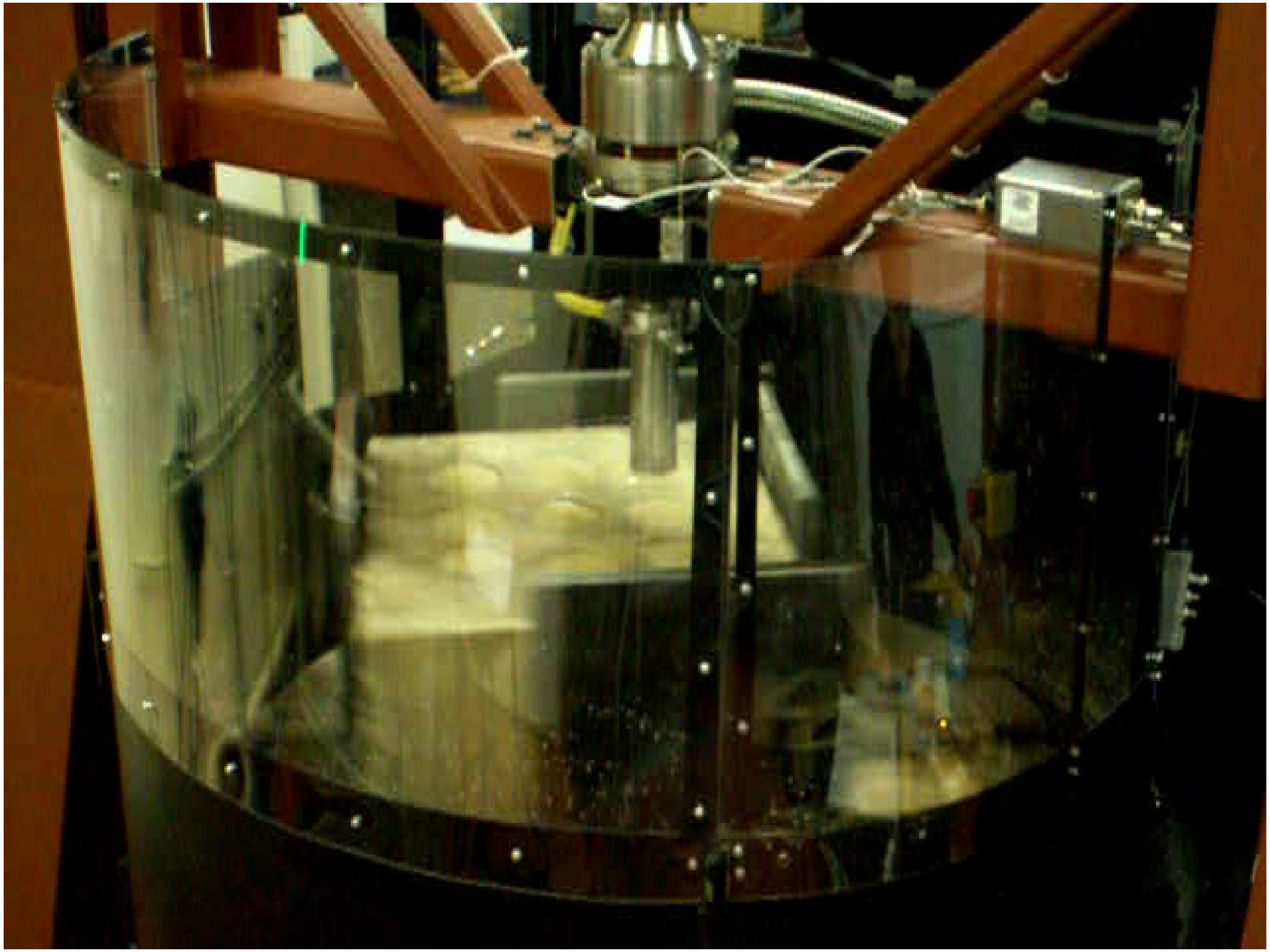
Predictions via 1D Map



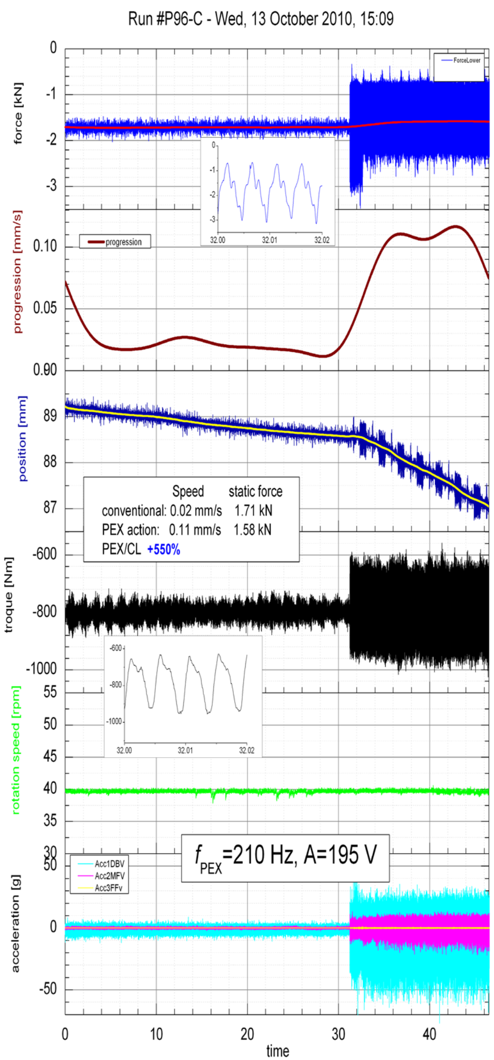
Large Scale Experimental Rig

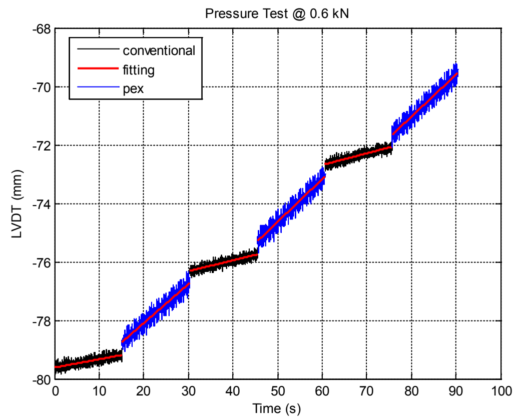


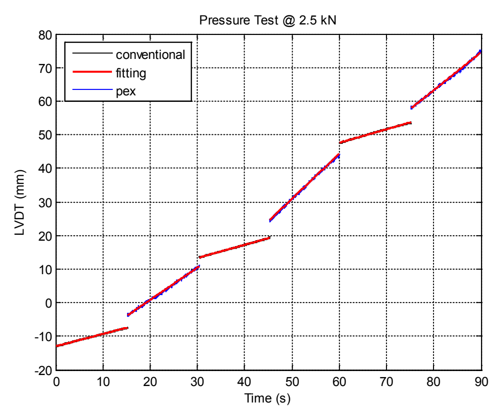
Large Scale Experimental Studies



Large Scale Experimental Studies



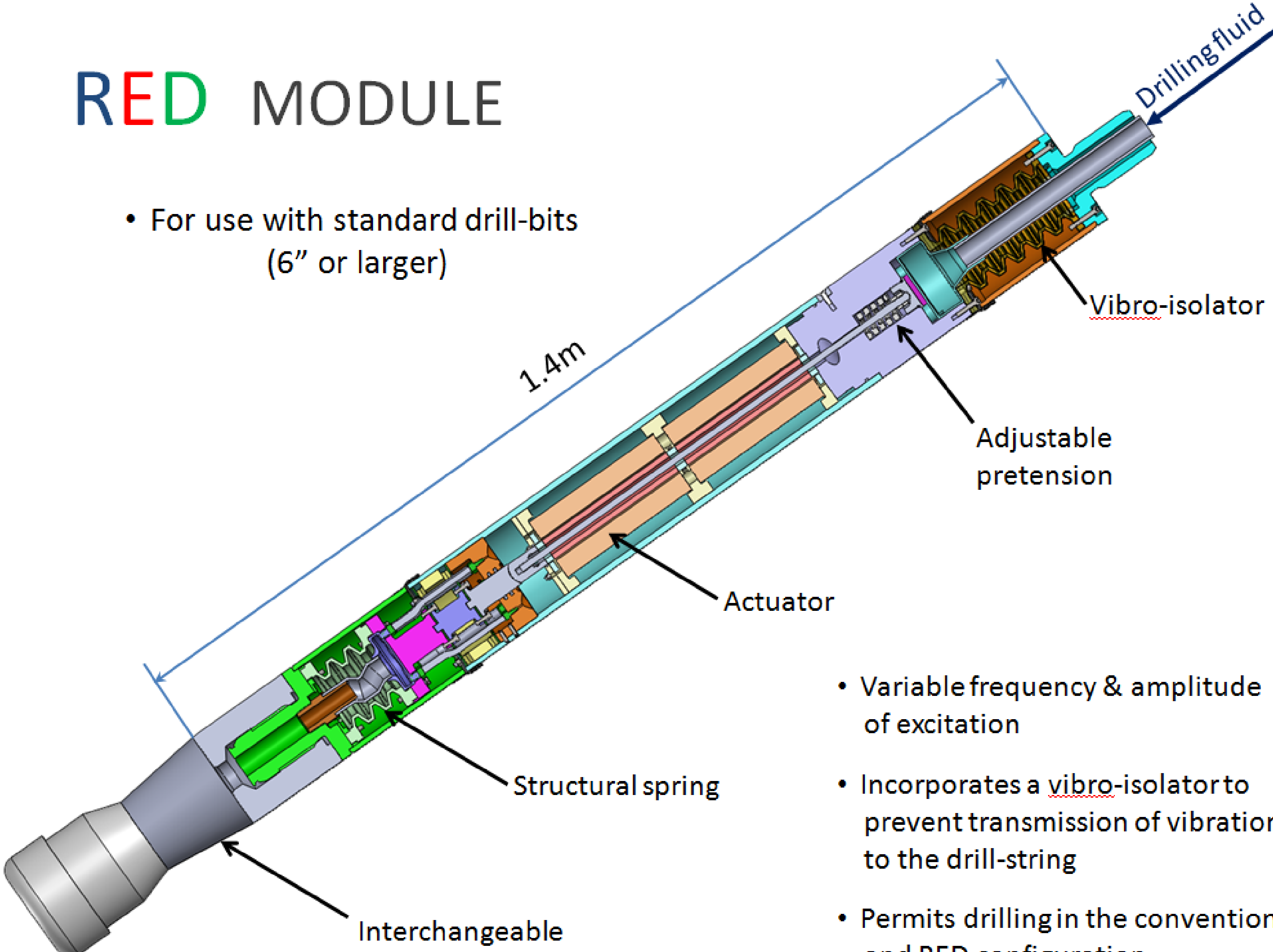




Experimental Studies – Drag Bit

Run No	Step Duration (s)	WOB (approx. in kN)	Conventional ROP (mm/s)	RED ROP (mm/s)	Improvement Factor (%)
P	20	0.6	0.0283	0.1331	470.7
			0.0371	0.146	393.8
			0.0399	0.1437	360.5
P97b	20	0.5	0.0225	0.1135	504.8
			0.0275	0.1095	398.1
			0.0299	0.1032	345.7
P	20	2	0.5128	1.2218	238.3
			0.3423	0.8047	235.1
			0.3062	0.9084	296.6
P97d	20	2.5	0.0903	0.2682	297.2
			0.0608	0.1306	214.8
			0.0222	0.0647	291.1
P	20	0.5	0.0199	0.1003	504.9
			0.0257	0.1059	411.2
			0.0252	0.1117	442.4
P98b	20	0.6	0.0286	0.1221	426.8
			0.0225	0.1092	486.1
			0.0226	0.1158	512.7
P	20	2	0.4797	1.2105	252.3
			0.354	1.3675	386.3
			0.5622	1.159	206.2
P98d	20	2.5	0.3685	0.9619	261
			0.3917	1.3388	341.8
			0.404	1.1245	278.4

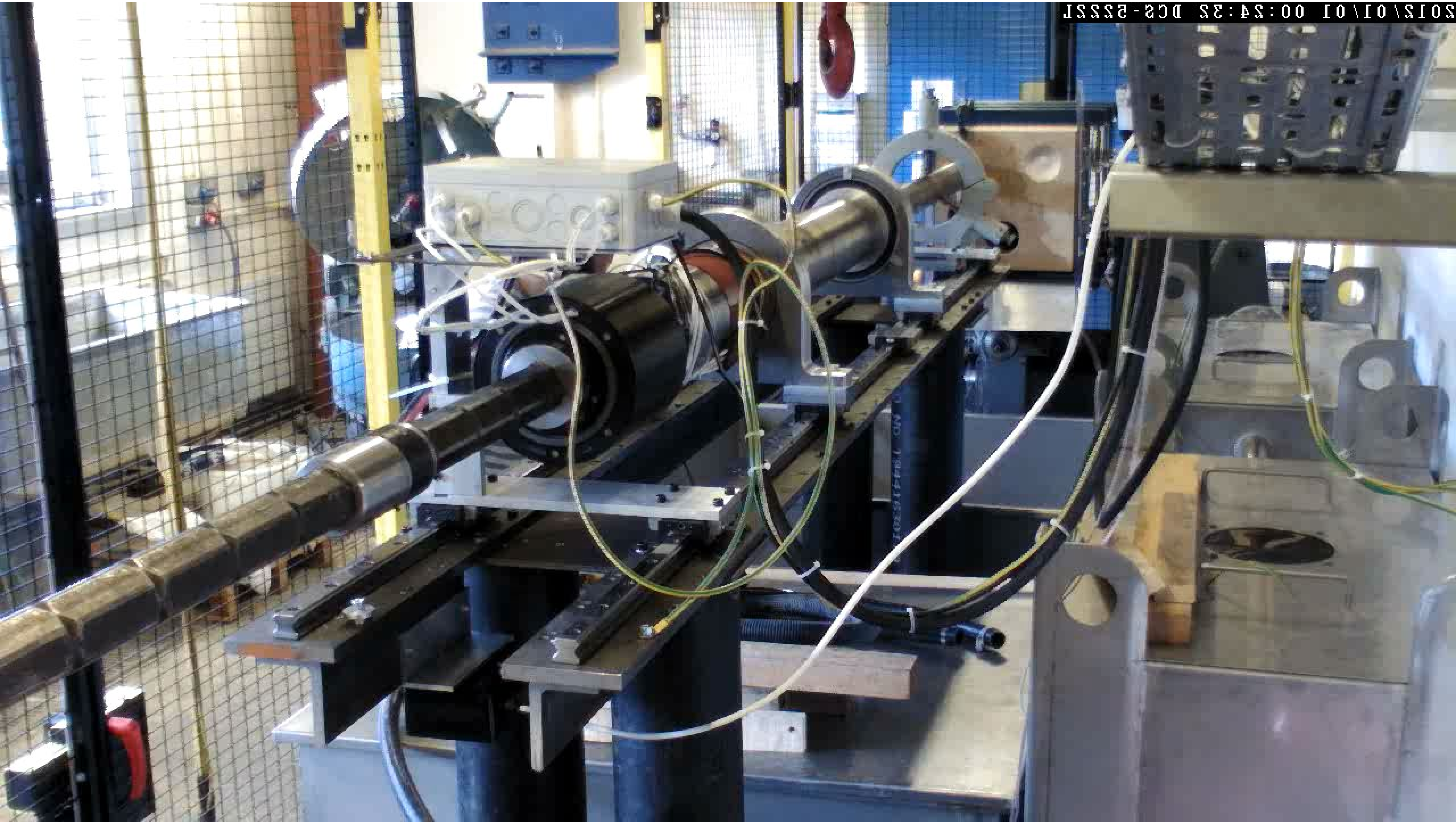
RED Module Design



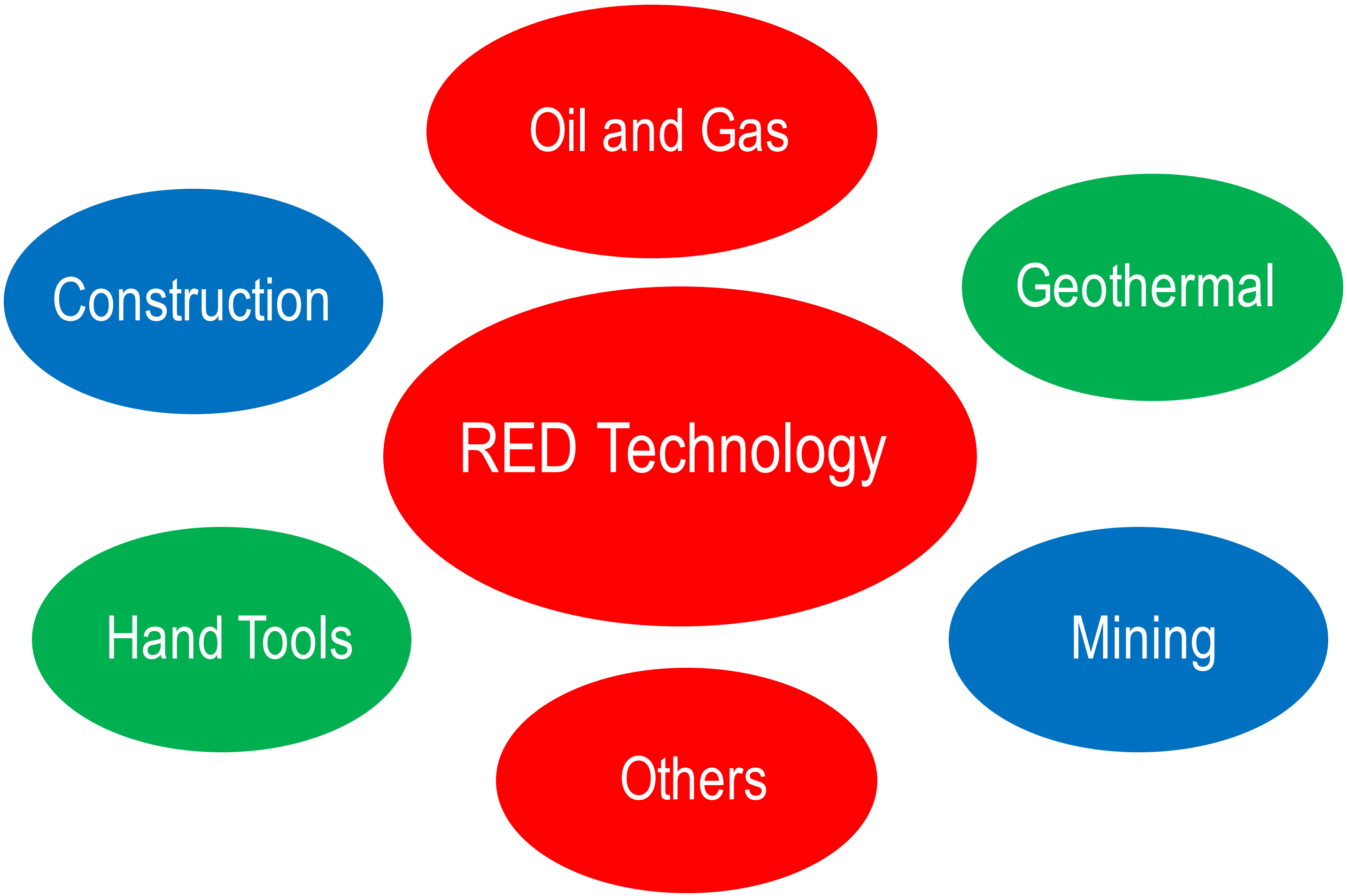
- For use with standard drill-bits (6” or larger)

- Variable frequency & amplitude of excitation
- Incorporates a vibro-isolator to prevent transmission of vibration to the drill-string
- Permits drilling in the conventional and RED configuration

New Experimental Rig

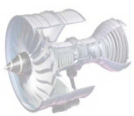


Bussines Potentials

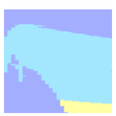


Presentation Outline

Nonlinear dynamics for engineering design: Introduction

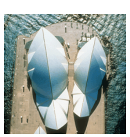


Rotor dynamics



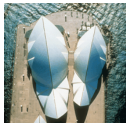
Drill-string vibration

Resonance Enhanced Drilling (RED)



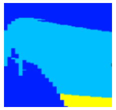
Closing remarks

Closing Remarks



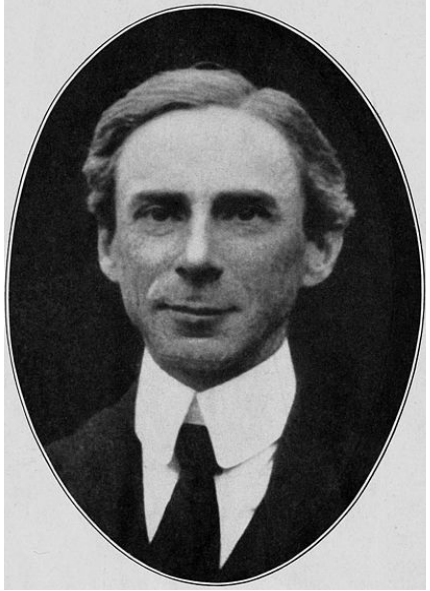
Nonlinear dynamics and engineering design have a natural synergy, which should be explored much further.

Nonlinear behaviour should be harvested not avoided.



The most advanced technologies will use nonlinear behaviour or nonlinear dynamic interactions.

ND*f*ED has huge bussines potentials.



Bertrand Russell

‘Against my will, in the course of my travels, the belief that everything worth knowing was known at Cambridge gradually wore off. ’.

‘Do not fear to be eccentric in opinion, for every opinion now accepted was once eccentric.’

‘The time you enjoy wasting is not wasted time.’

Thank you for your attention!

- Alistair Robertson,Vahid Vaziri, Marcin Kapitaniak, Marko Keber, James Ing, Ekaterina Pavlovskaia, Olusegun Ajibose, Yang Liu, Jerzy Wojewoda, Krishnan Nandakumar, Gyorgy Karolyi (University of Aberdeen); Alan Lowdon, Gary Senior, Frank Wright (ITI Energy), Qingjie Cao (Harbin Institute of Technology), Anton Krivtsov (Russian Academy of Sciences), Sergei Mikhailov (Brunel University), Paul Lurie (BP), Chee-Hoe Foong (Subsea 7), Soumitro Banerjee (Indian Institute of Technology)
- Engineering and Physical Science Research Council, Rolls-Royce plc, British Petroleum, Centre for Marine and Petroleum Technology, Scottish Enterprise, ITI Energy, The Royal Society

Centre for Applied Dynamics Research (CADR)