Grazing Induced Bifurcations:
Innocent or Sinister?

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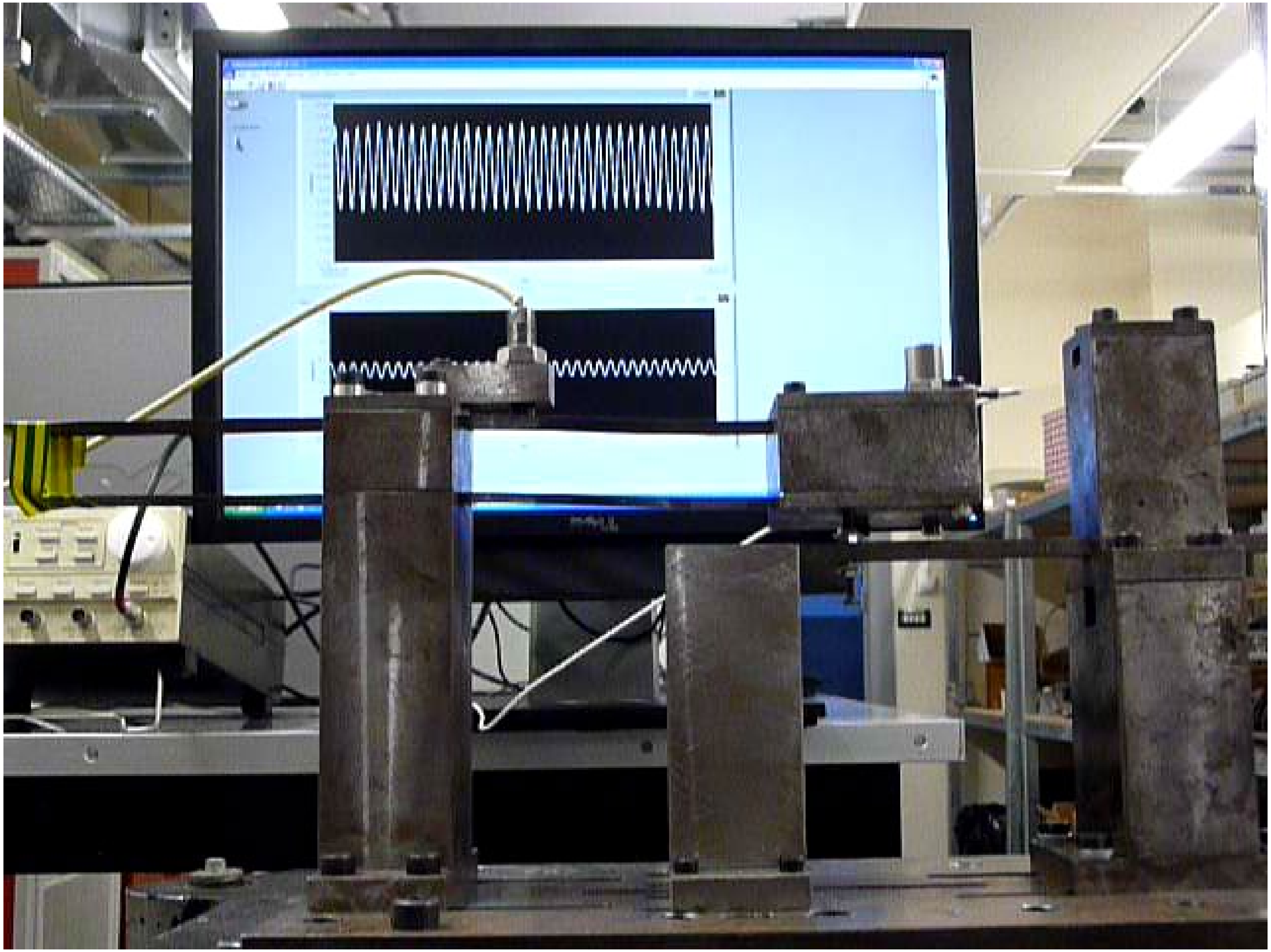
Grazing: Experimental Study



Experimental Results

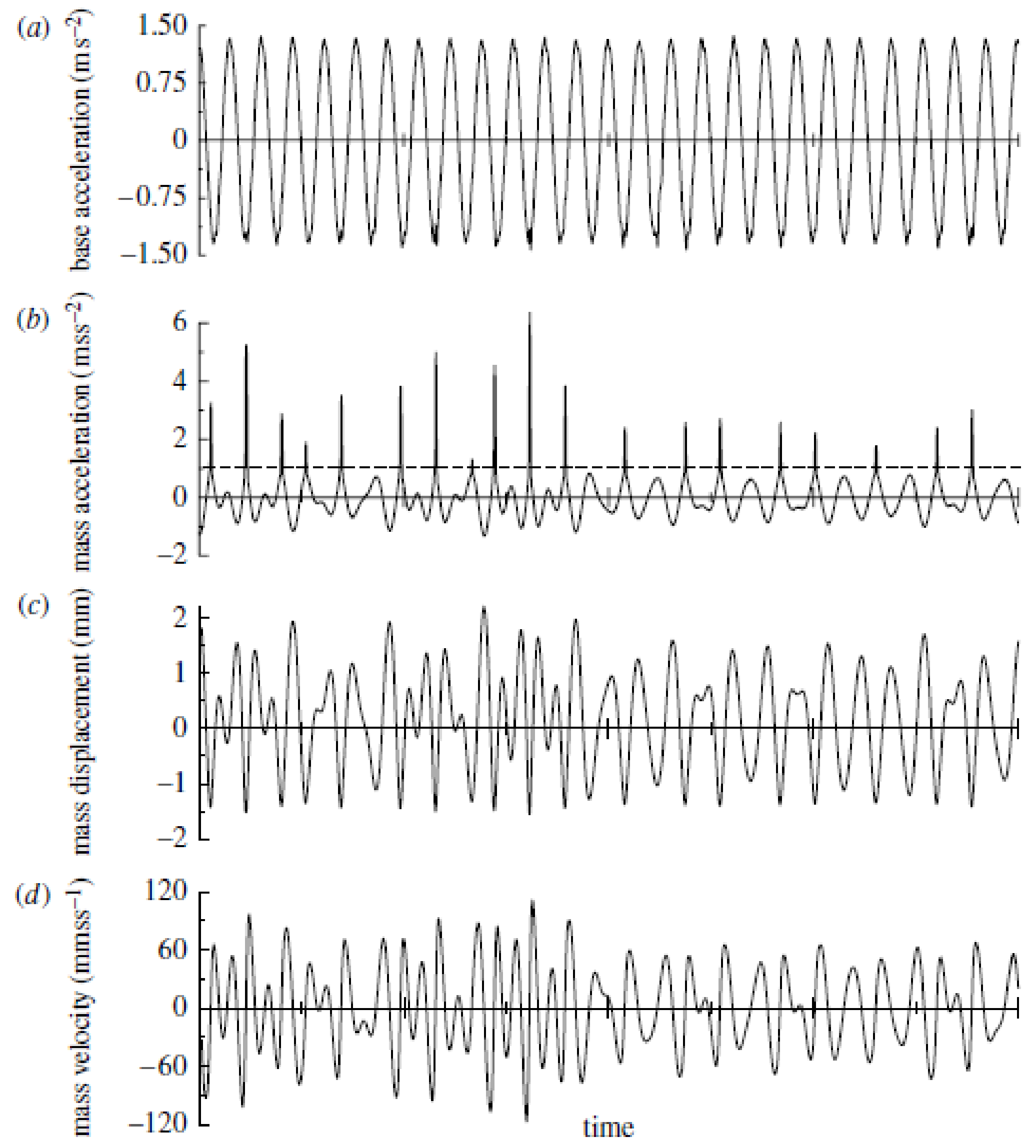
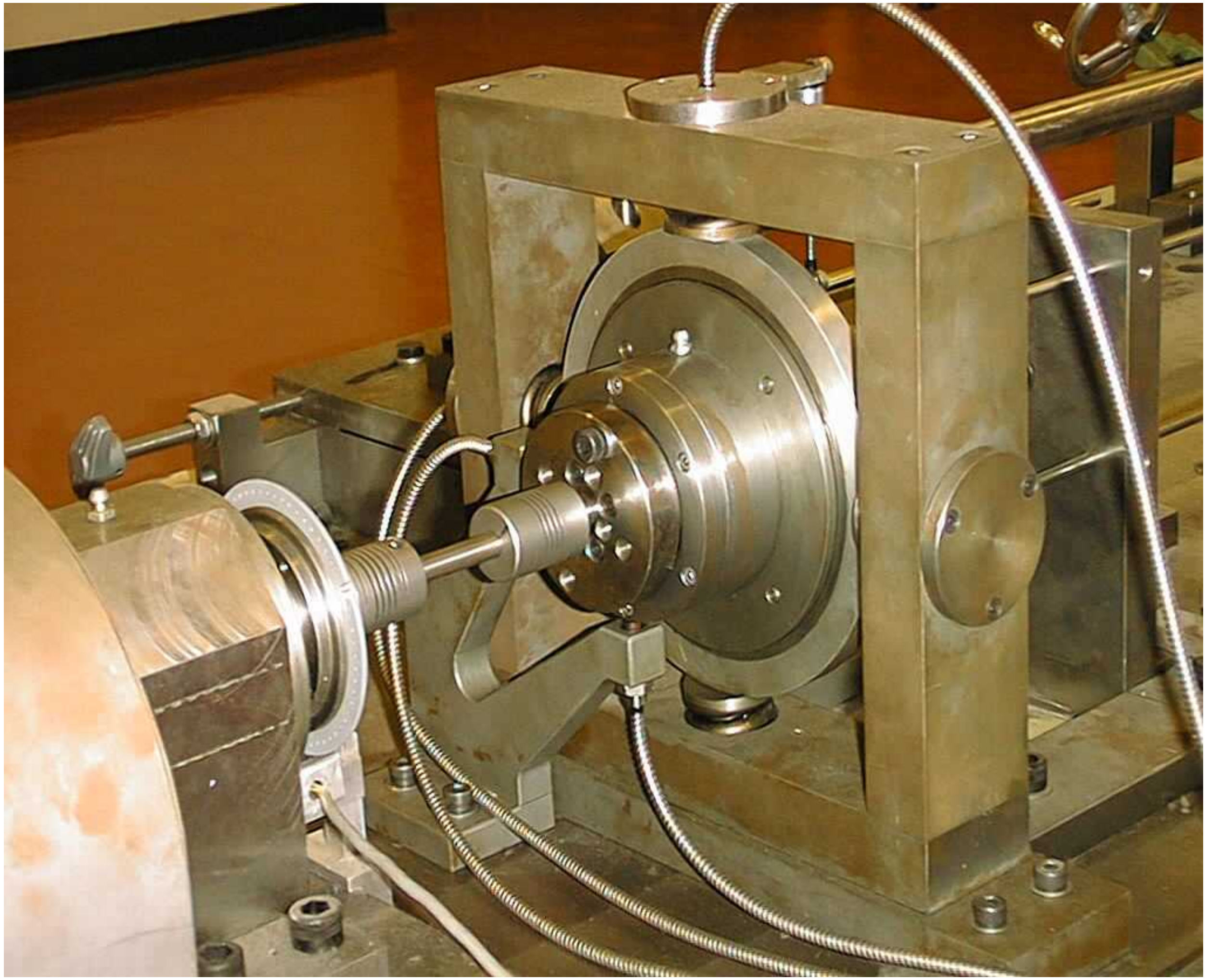
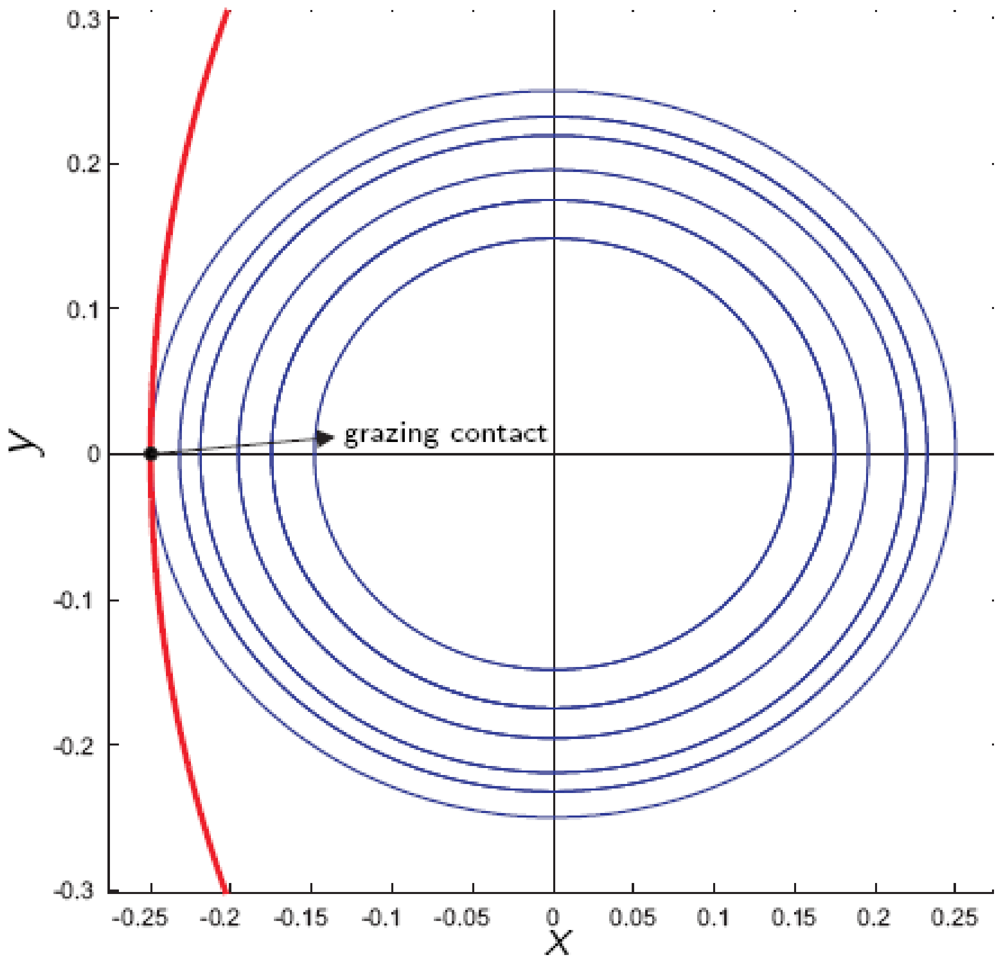


Figure 2. A sample of the recorded experimental time histories for f_n=9.38 Hz, f=6.5 Hz,

PTRSA **366** (2008) 679 –704

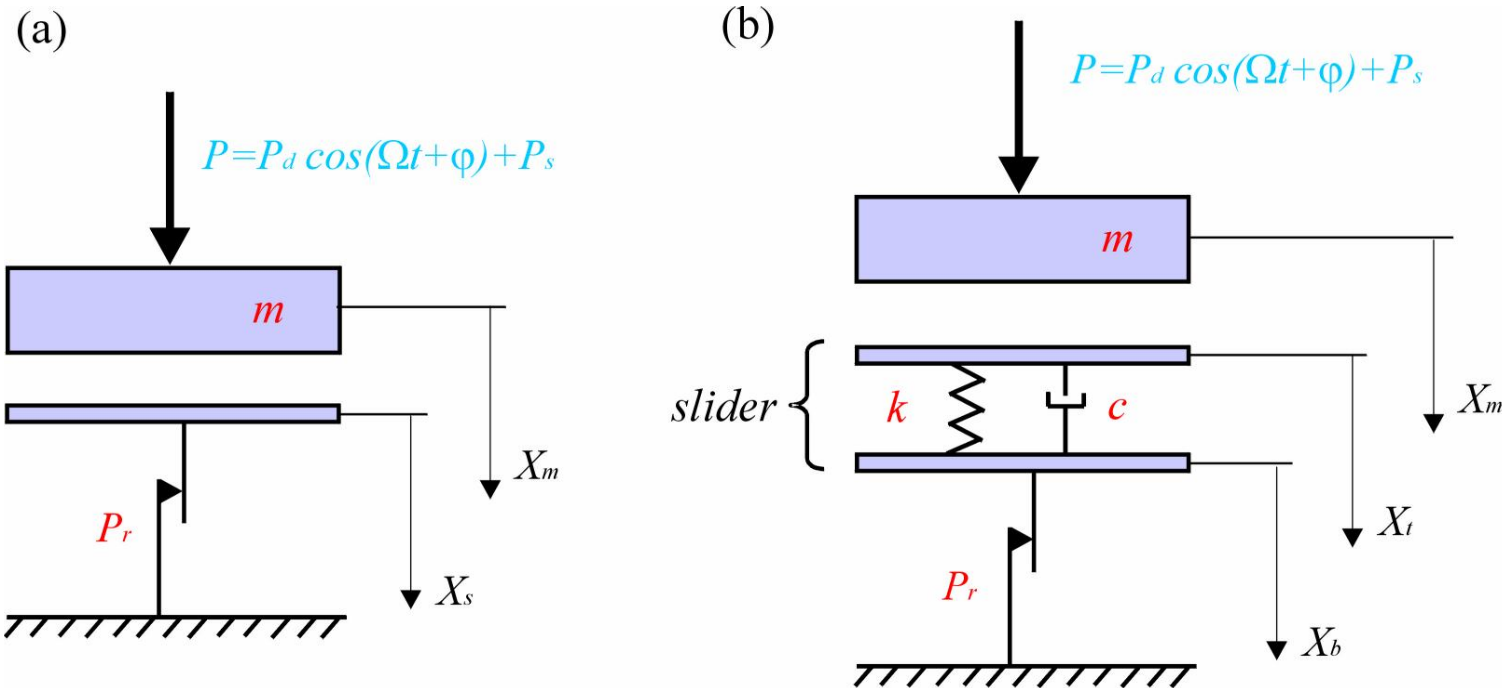
Grazing – Rotor Systems



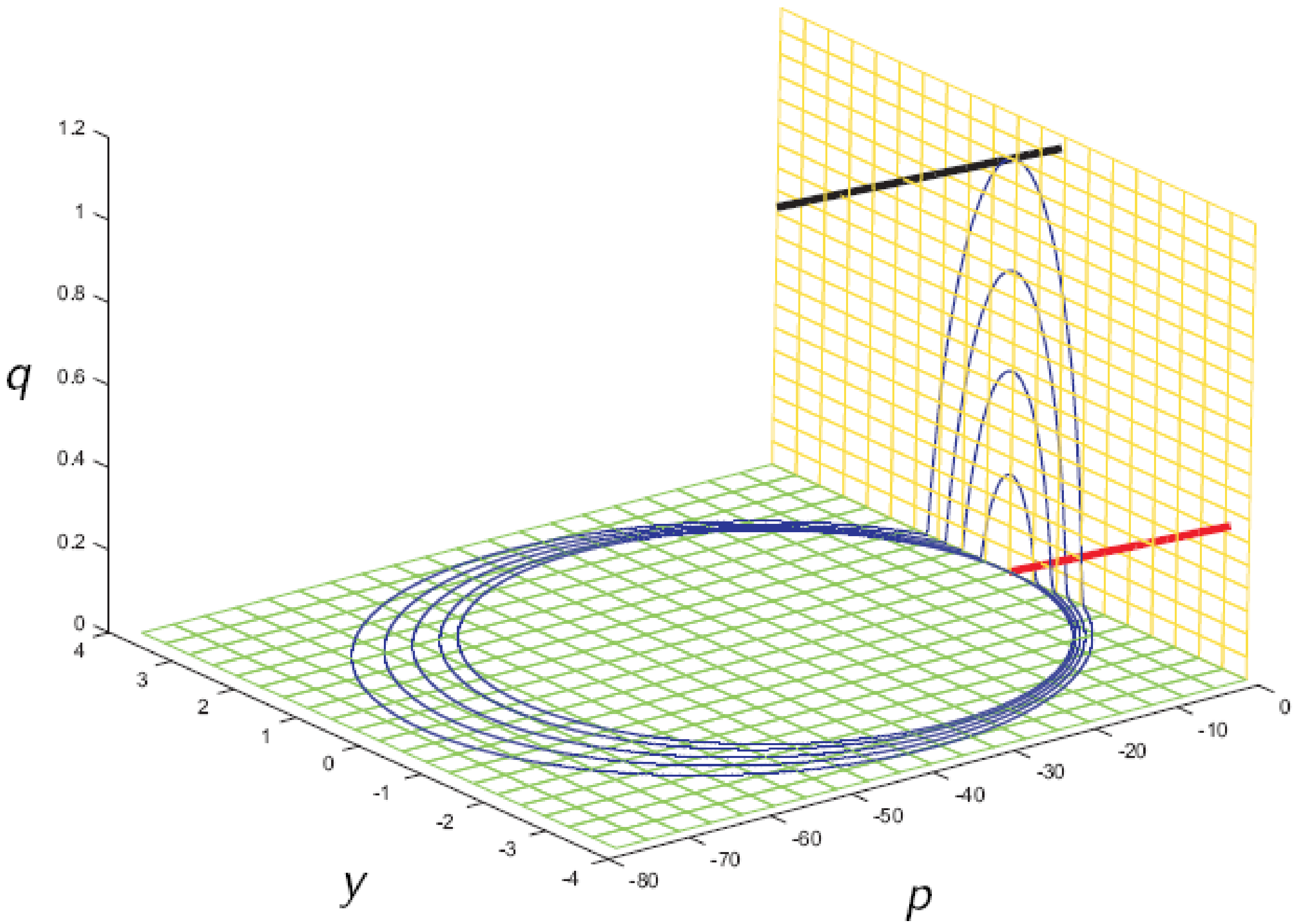


CNSNS **18** (2013) 2571–2580

Grazing – Drifting Impact Oscillator

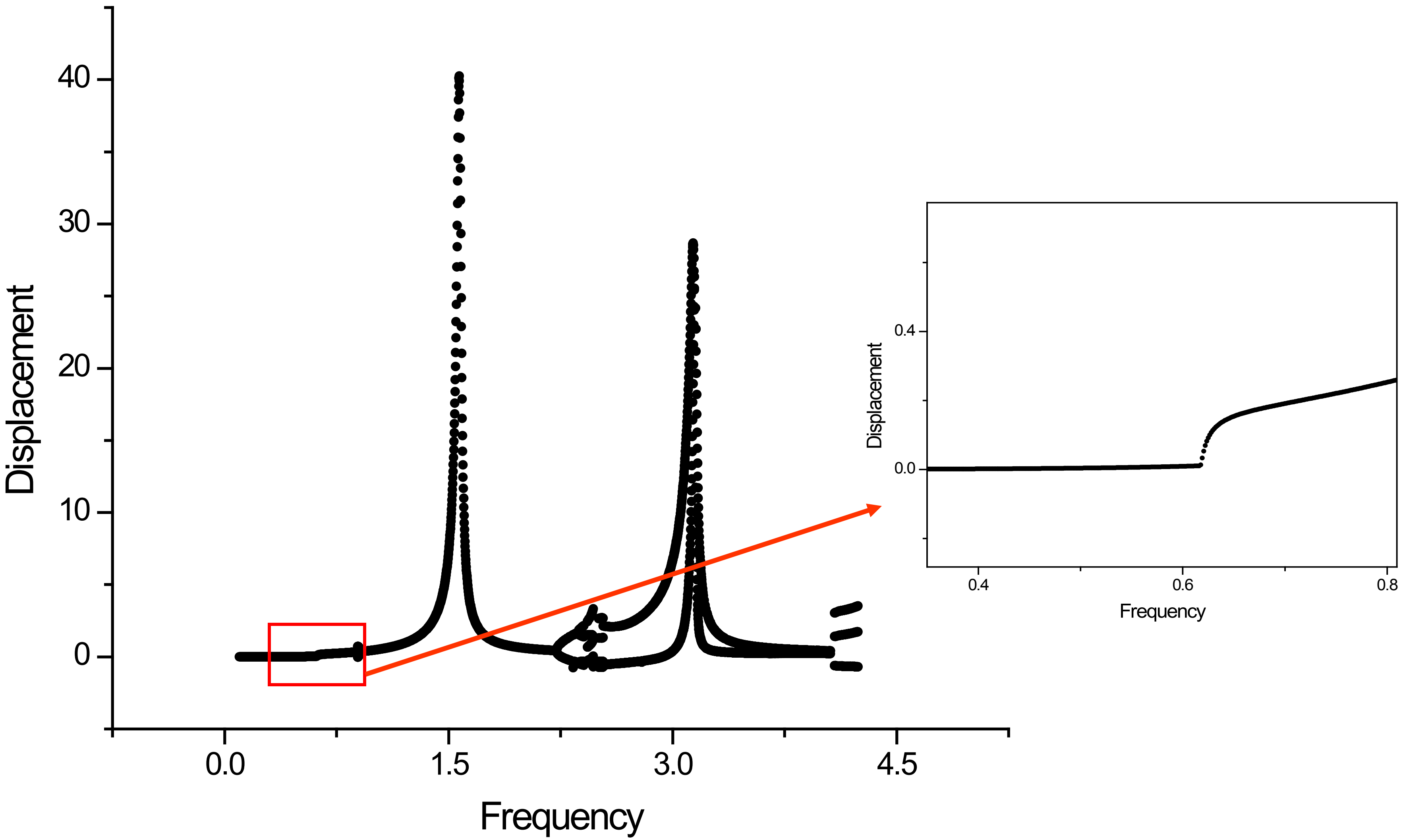


(a) *Meccanica* **34** (1999) 425-434
(b) *PRE* **64** (2001) 056224

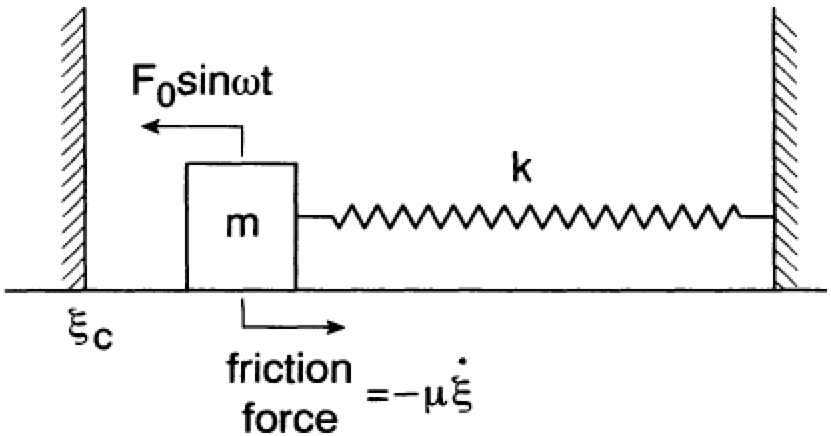


PRE **70** (2004) 036201

Grazing: Experimental Study

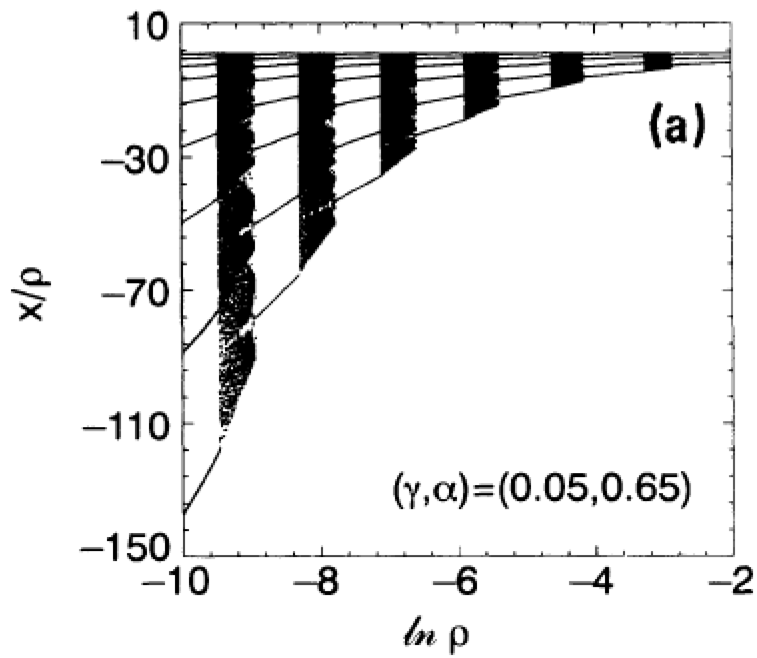


Grazing Phenomena



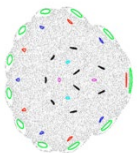
$$\begin{cases} x_{n+1} = \alpha x_n + y_n + \rho \\ y_{n+1} = -\gamma x_n \end{cases} \quad \text{for } x_n \le 0 \, ,$$

$$\begin{cases} x_{n+1} = -\sqrt{x_n} + y_n + \rho \\ y_{n+1} = -\gamma \tau^2 x_n \end{cases} \quad \text{for } x_n > 0 \, .$$

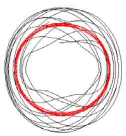


Chin W *et al. PRE* **50** (1994) 4427-4444
Nordmark AB *JSV* **145** (1991), *279–297*

Presentation Outline



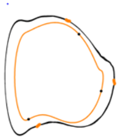
Motivation – Grazing in Impact Oscillators



Elastic Impact Oscillator

Drifting Impact Oscillator – RED

Rotor System



Closing Remarks

$$x' = \upsilon$$

$$\upsilon' = \Gamma \sin\left(\tilde{\omega}\tau\right) - 2\xi\upsilon - x - \beta(x-e)H(x-e)$$

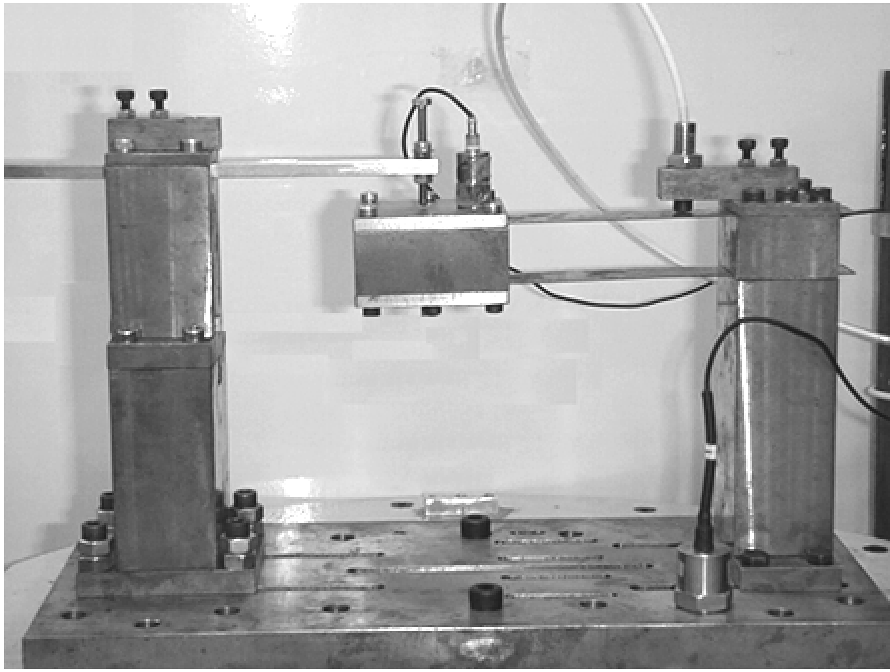
$$\tau' = 1$$

where

$H(\cdot)$ is Heaviside step function

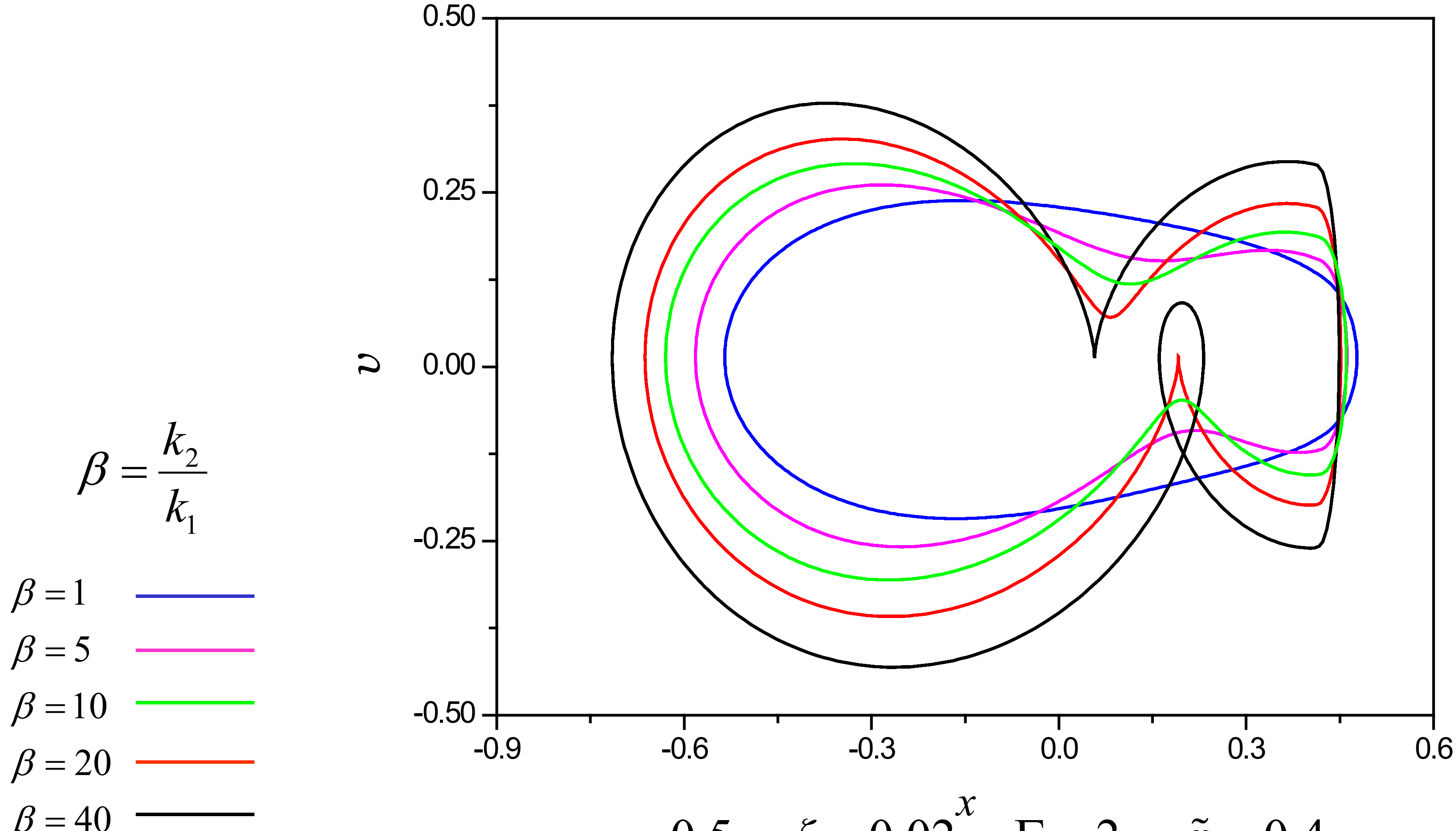
$$x = \frac{y}{x_0}, \quad \beta = \frac{k_2}{k_1}, \quad e = \frac{g}{x_0},$$

$$\Gamma = \frac{A\omega^2}{x_0 k}, \quad \tilde{\omega} = \frac{\omega}{\sqrt{k_1/m}}, \quad \xi = \frac{c\sqrt{k_1}}{2\sqrt{m}}$$



From Elastic to Rigid

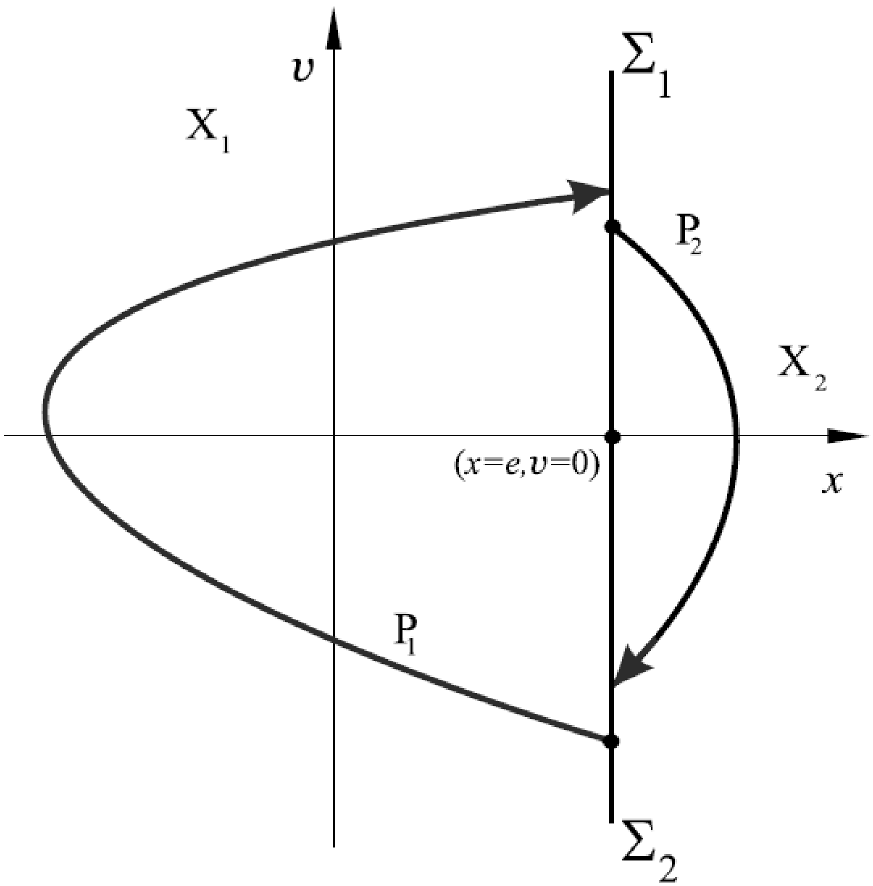
$$\beta = \frac{k_2}{k_1}$$



$$e = 0.5, \quad \xi = 0.02, \quad \Gamma = 2, \quad \tilde{\omega} = 0.4$$

$$\Sigma_1 = \left\{ \left(\tau_i, x_i, \upsilon_i \right) \middle| x_i = e, \upsilon_i > 0 \right\}$$

$$\Sigma_2 = \left\{ \left(\tau_i, x_i, \upsilon_i \right) \middle| x_i = e, \upsilon_i < 0 \right\}$$



Local maps

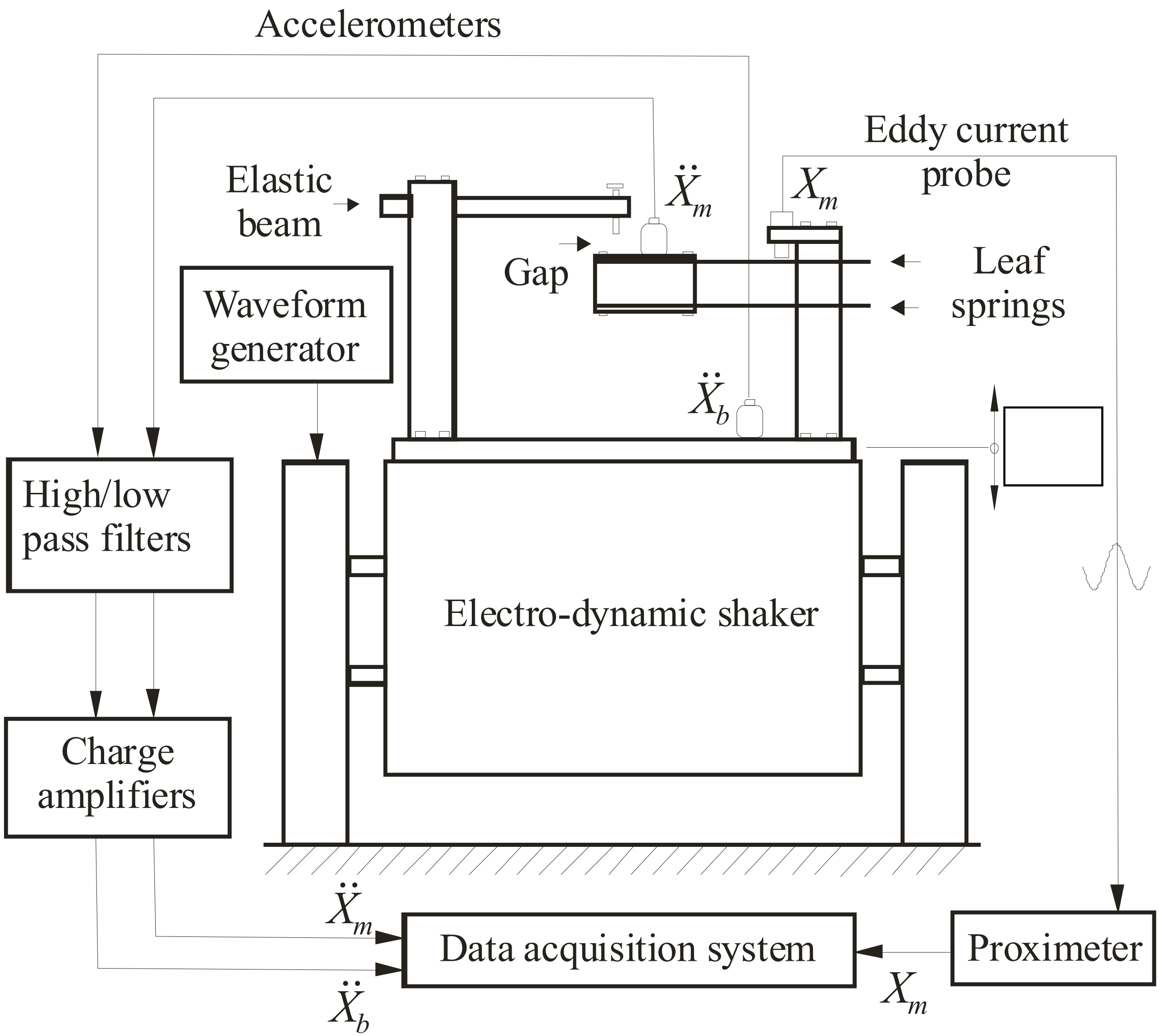
$$P_1 : \quad \Sigma_1 \to \Sigma_2$$

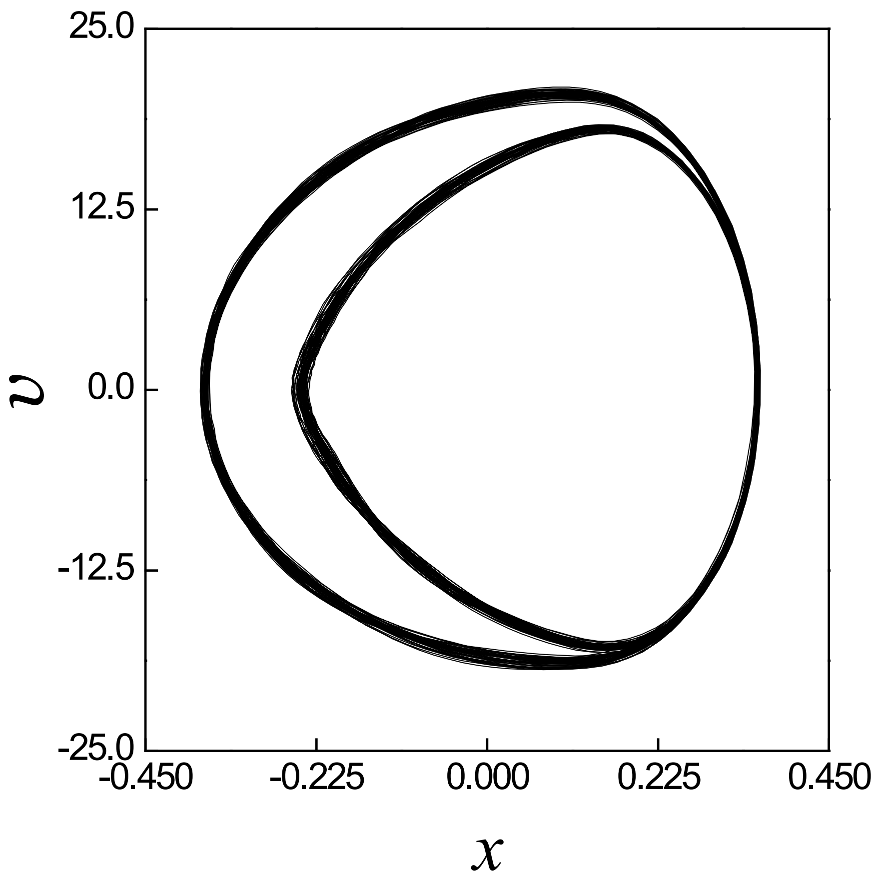
$$P_2 : \quad \Sigma_2 \to \Sigma_1$$

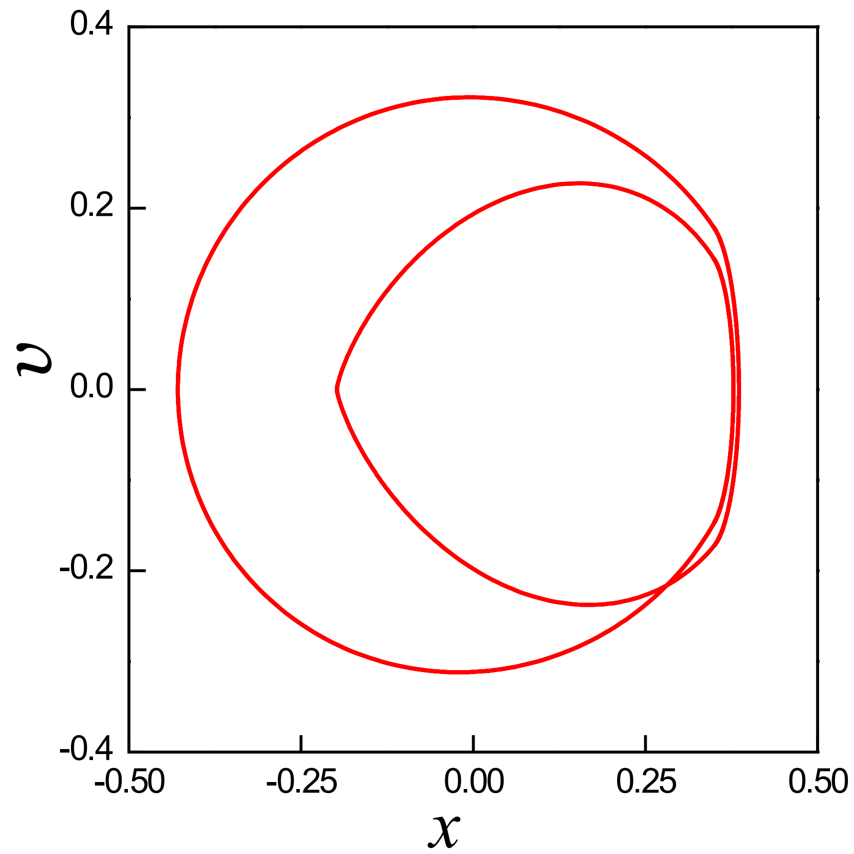
Global map

$$\hat{P} = P_2 \circ P_1 : \quad \Sigma_1 \to \Sigma_1$$

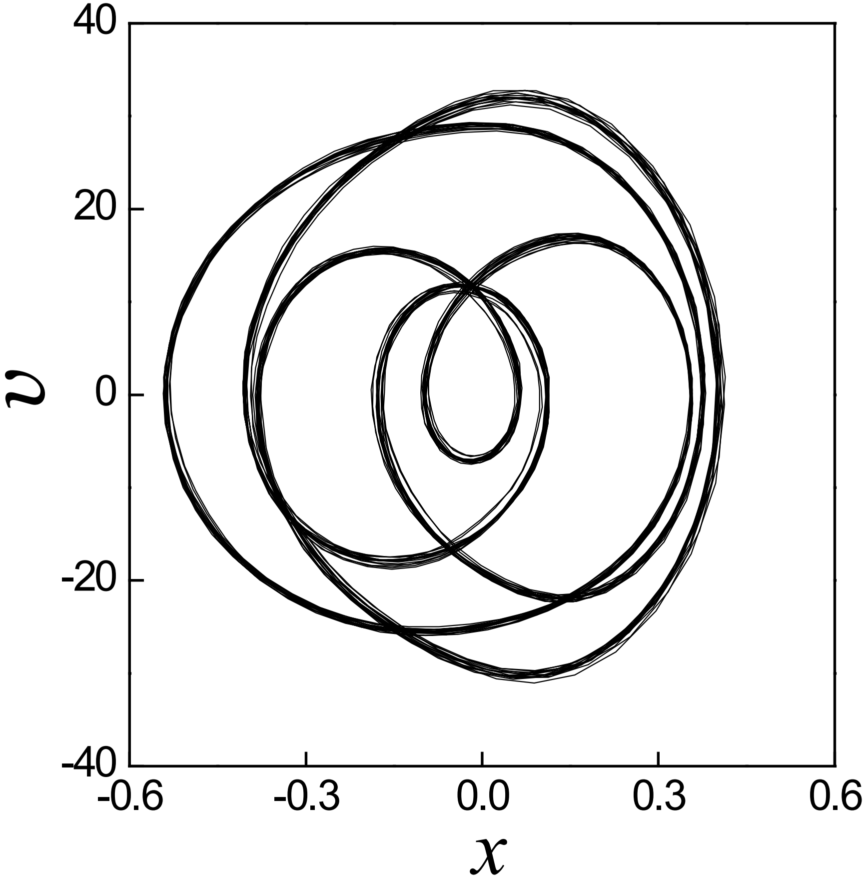
Elastic Impact Oscillator - Experiments

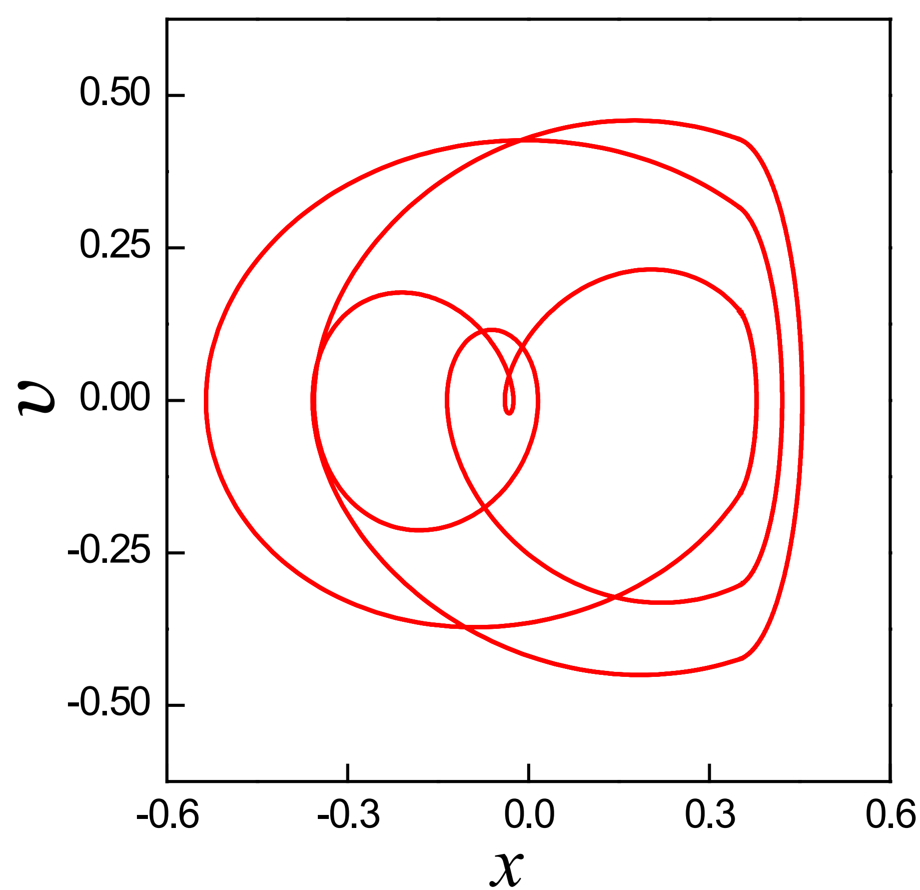



25.0
12.5
0.0
-12.5
-25.0
-0.450
-0.225
0.000
0.225
0.450
x
v

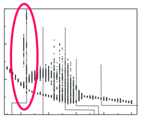

0.4
0.2
0.0
-0.2
-0.4
-0.50
-0.25
0.00
0.25
0.50
x
v

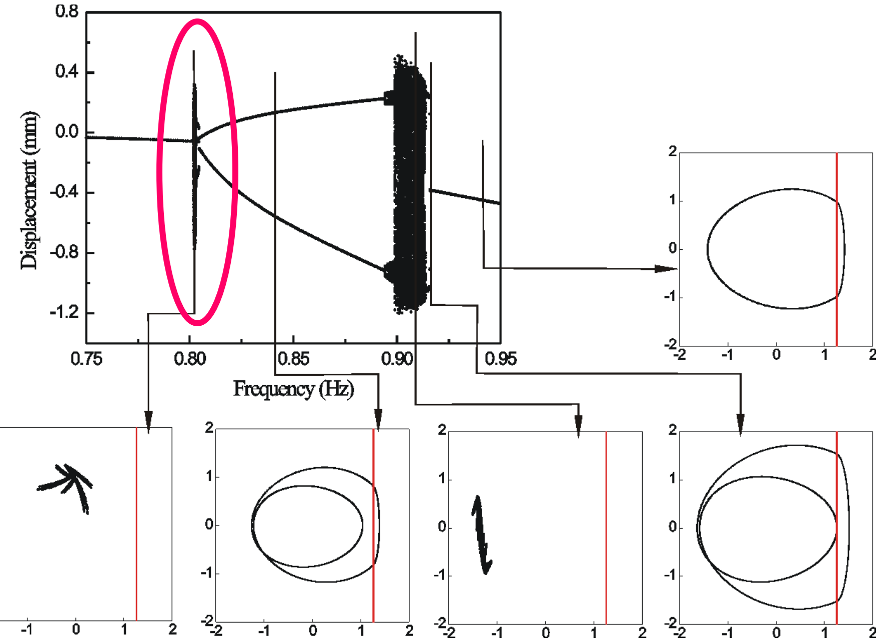
Experimental Verification



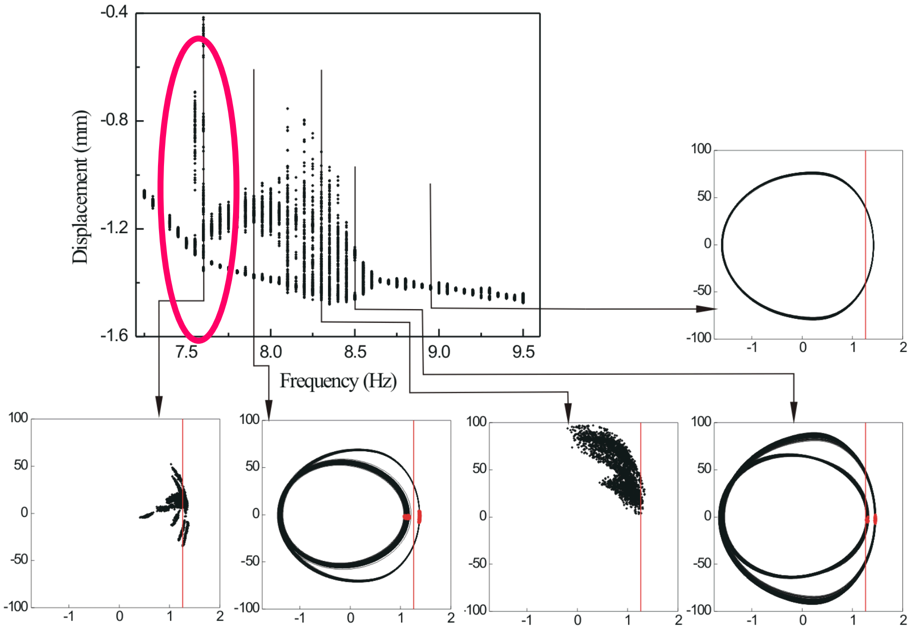


Experiments vs Numerics

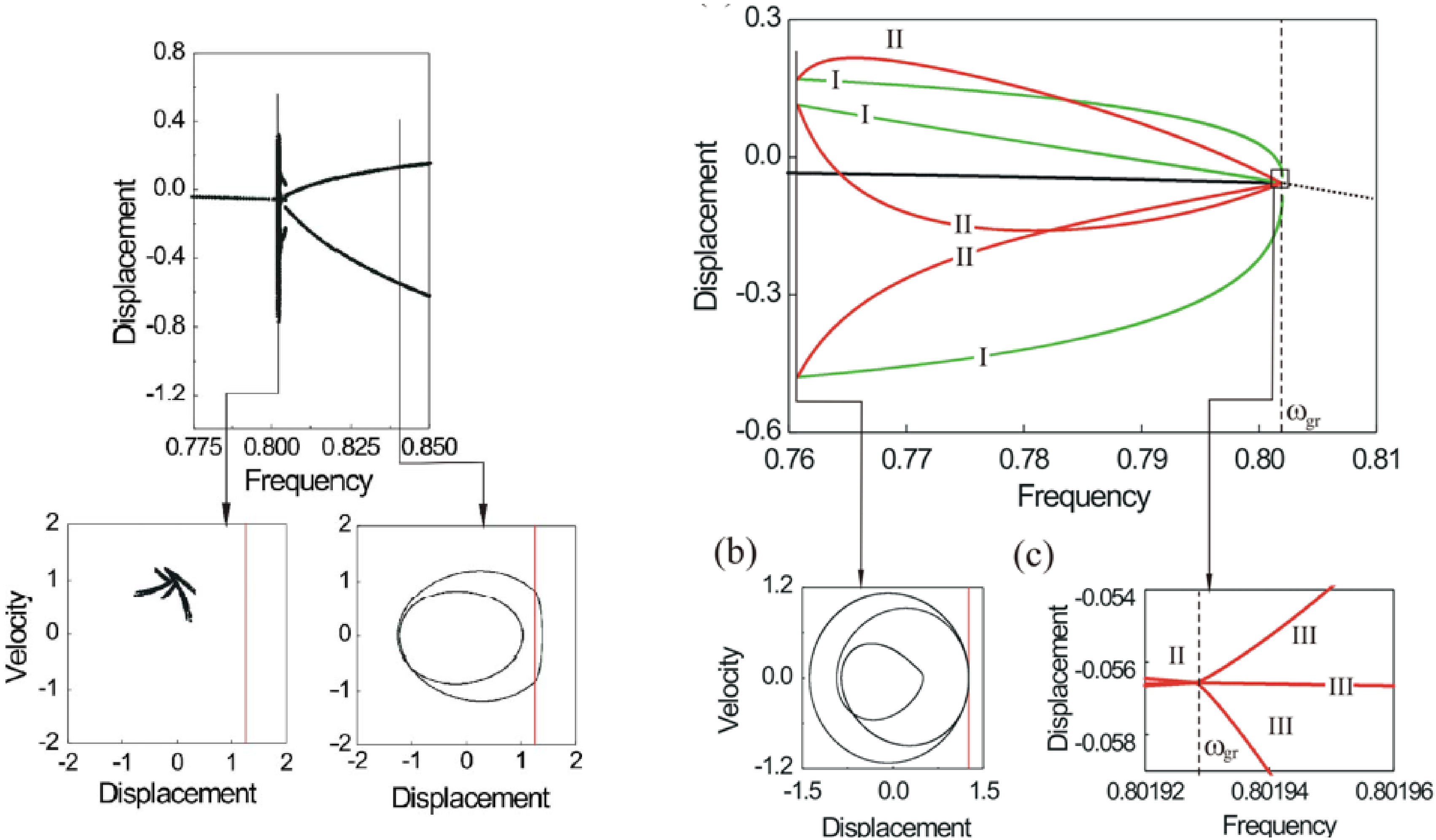




$$\beta = 29, \quad e = 1.26, \quad \xi = 0.01, \quad \Gamma = 0.7$$

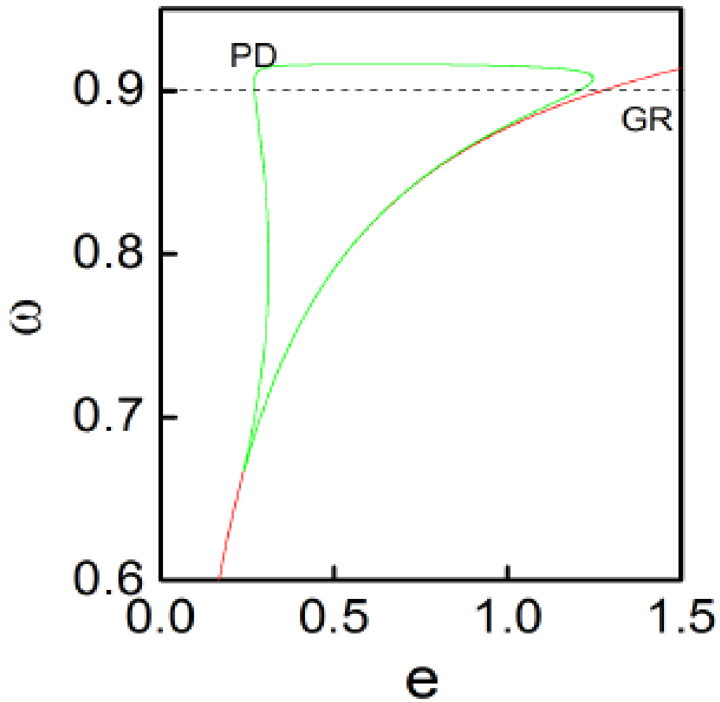


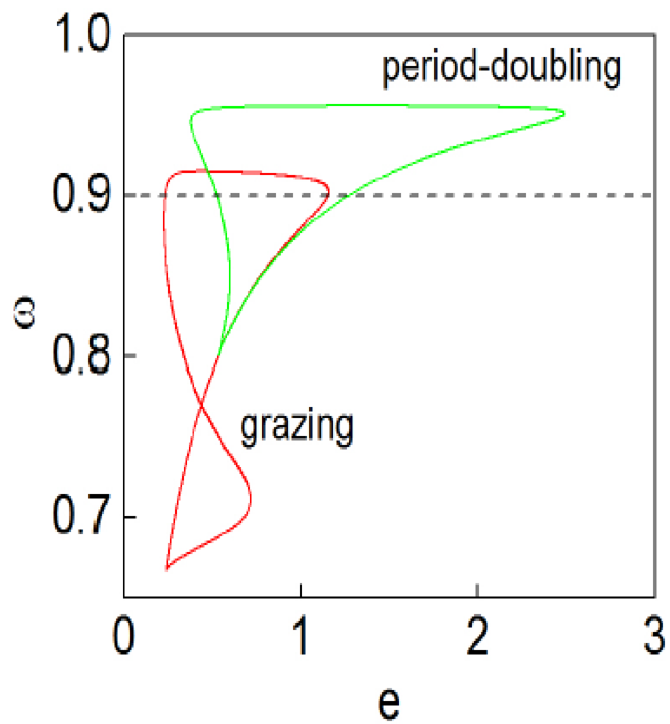
Narrow Band of Chaos & Invisible Grazing



PRE **79** (2009) 037201

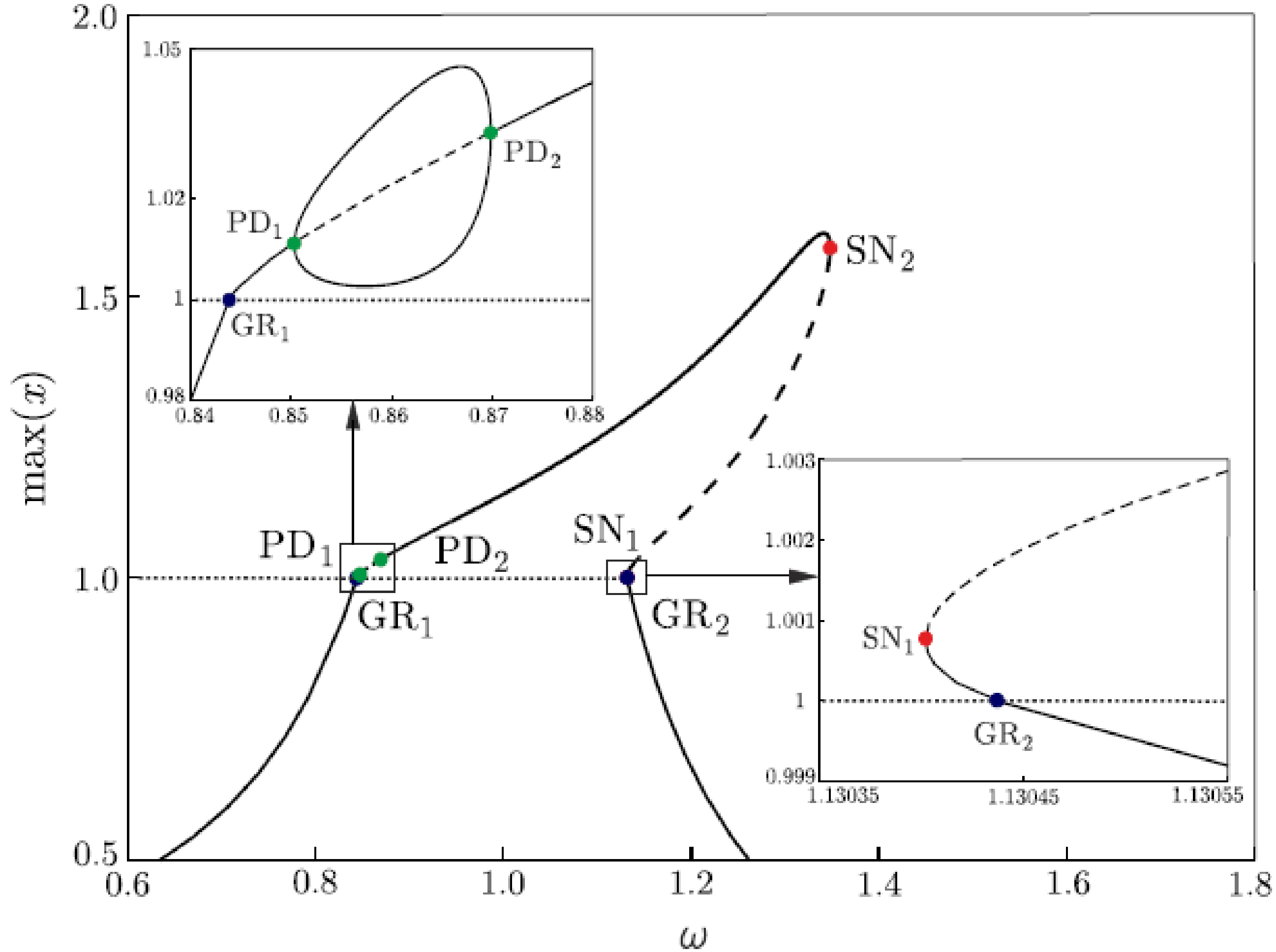
Grazing Induced Bifurcations





IMA JAM **81** (2016) 662-678

Grazing Induced Bifurcations

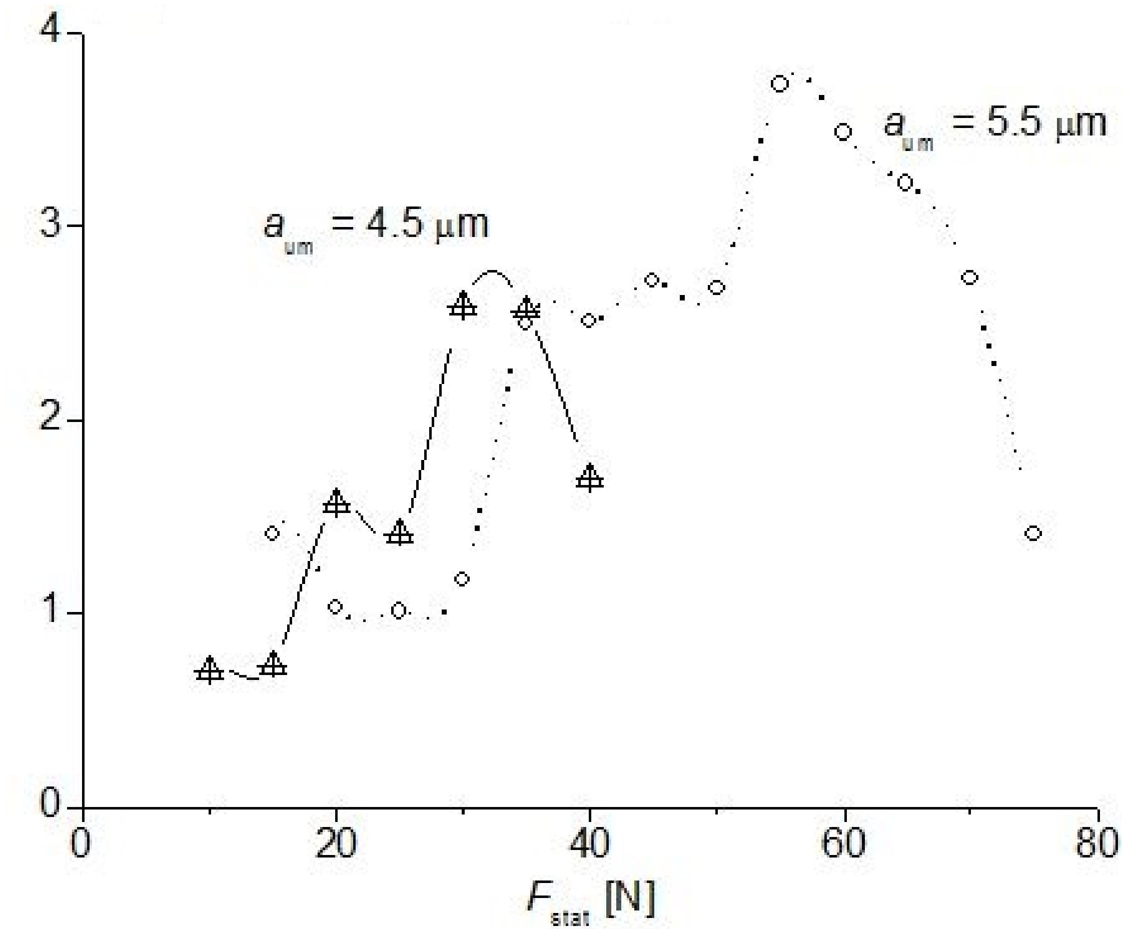


IJMS **127** (2017) 204-214

Presentation Outline

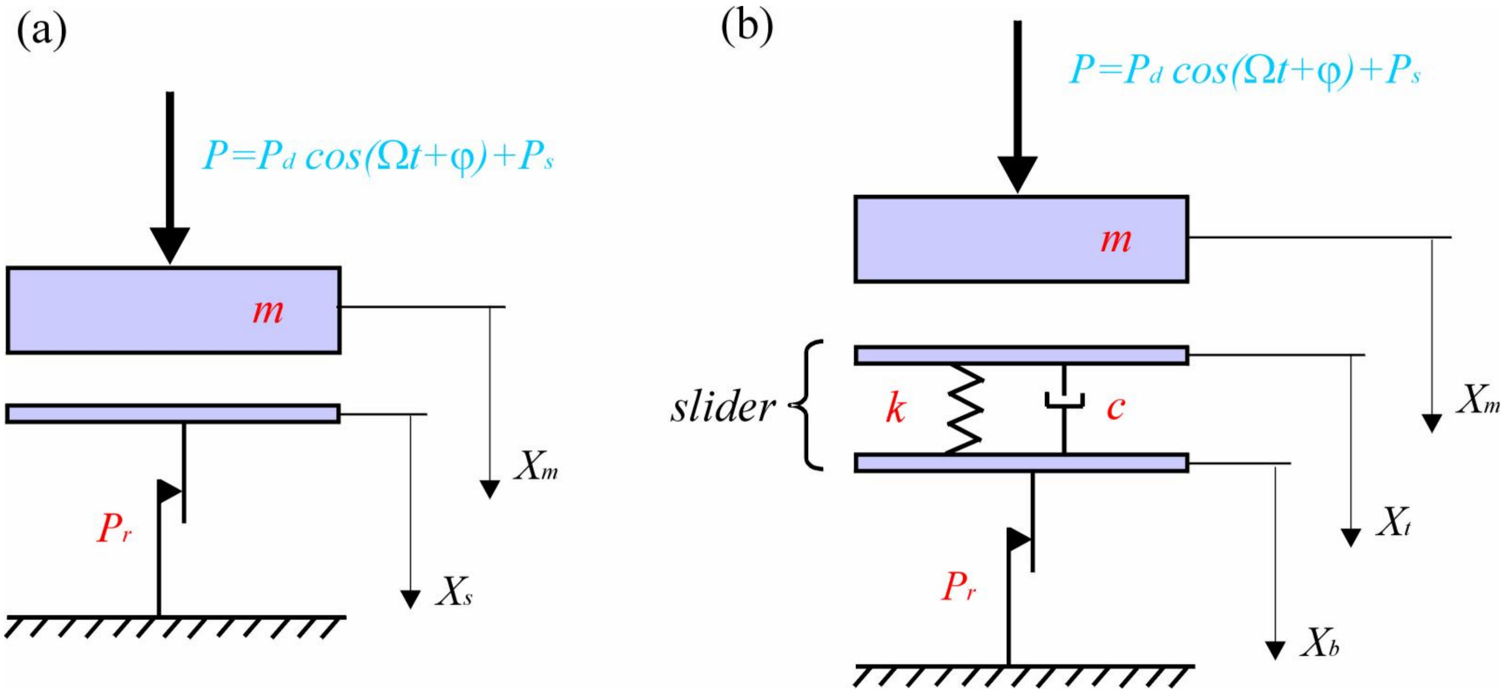
- Motivation – Grazing in Impact Oscillators
- Soft Impact Oscillator
- Drifting Impact Oscillator - RED
- Rotor System
- Closing Remarks

Drifting Impact Oscillator – RED



JSV **280** (2005) 739 - 757

Drifting Impact Oscillator – RED



(a) *Meccanica* **34** (1999) 425-434.

(b) *PRE* **64** (2001) 056224.

Modelling of Impact Dynamics

- No contact

$$x < z + g \qquad\qquad p < q + g$$

$$x' = y \qquad\qquad p' = y$$

$$y' = a\cos(\omega\tau + \varphi) + b \qquad\qquad y' = a\cos(\omega\tau + \varphi) + b$$

$$z' = -\frac{1}{2\xi}(z - v) \quad \Rightarrow \quad q' = -\frac{1}{2\xi}q$$

$$v' = 0 \qquad\qquad v' = 0$$

- Contact without progression

$$x \geq z + g \quad \text{and} \quad 0 < 2\xi y + (z - v) < 1 \qquad\qquad p \geq q + g \quad \text{and} \quad 0 < 2\xi y + q < 1$$

$$x' = y \qquad\qquad p' = y$$

$$y' = -2\xi y - (z - v) + a\cos(\omega\tau + \varphi) + b \quad \Rightarrow \quad y' = -2\xi y - q + a\cos(\omega\tau + \varphi) + b$$

$$z' = y \qquad\qquad q' = y$$

$$v' = 0 \qquad\qquad v' = 0$$

- Contact with progression

$$x \geq z + g \quad \text{and} \quad 2\xi z' + (z - v) \geq 1$$

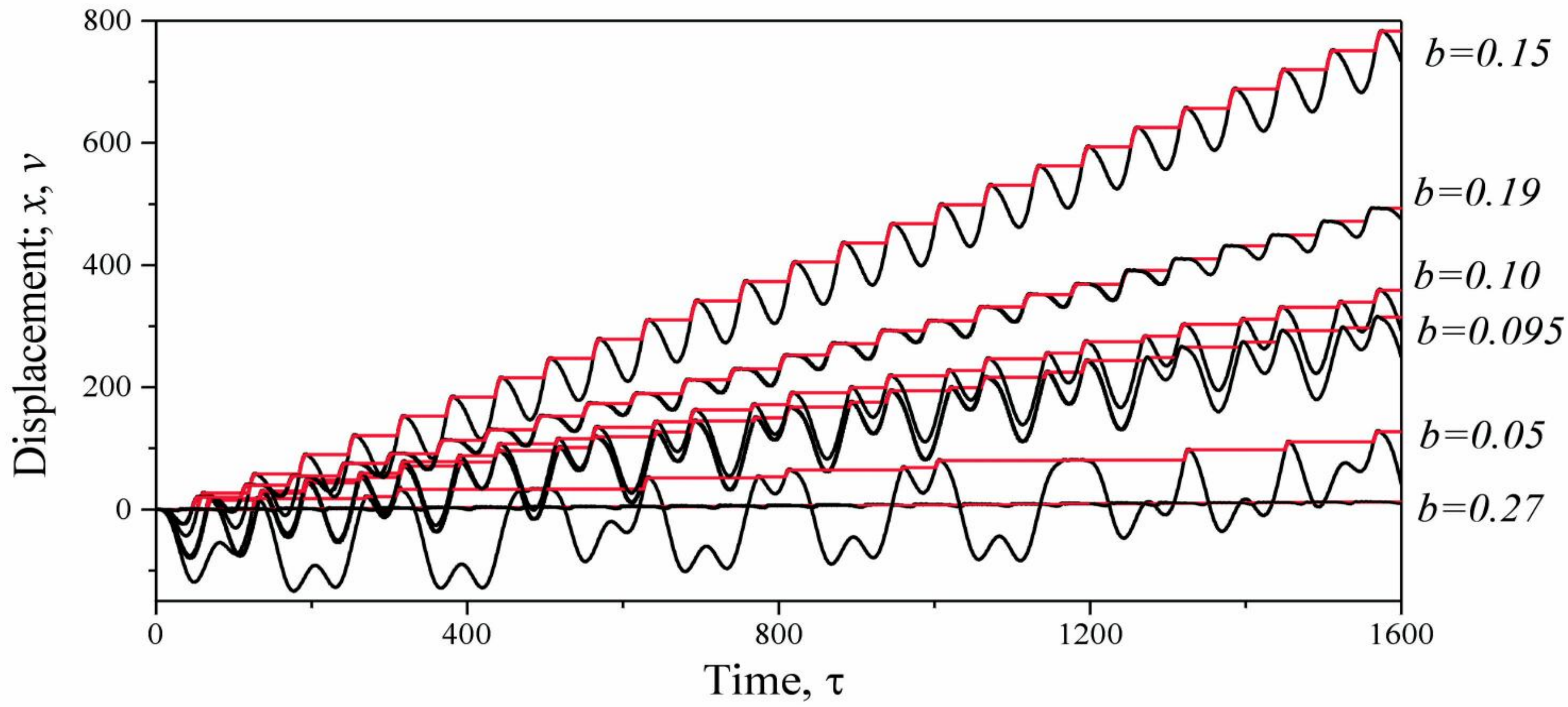
$$x' = y$$
$$y' = a\cos(\omega\tau + \varphi) + b - 1$$
$$z' = v$$
$$v' = y + \frac{1}{2\xi}(z - v - 1)$$

$\Rightarrow$

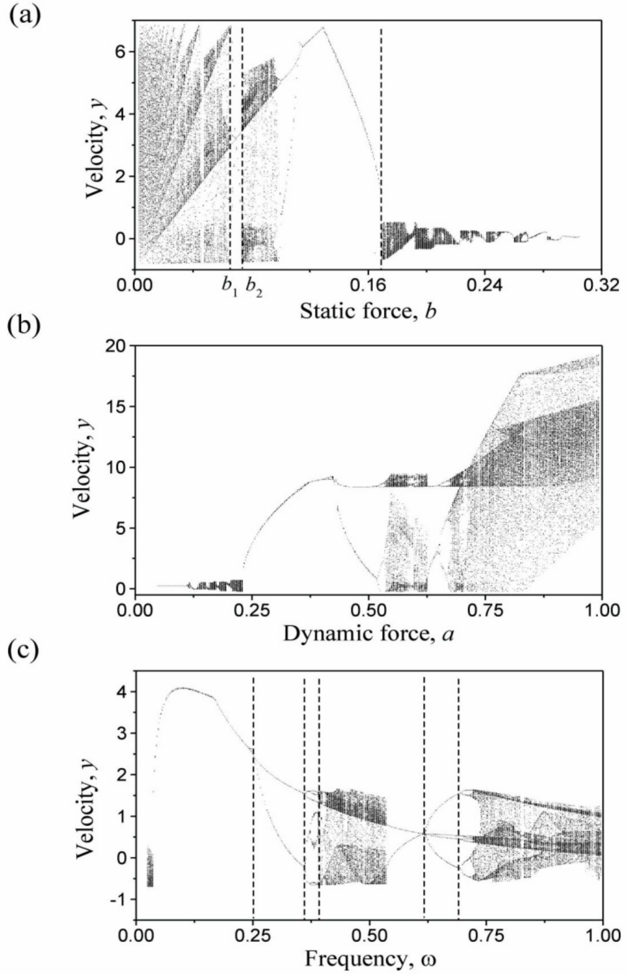
$$p \geq q + g \quad \text{and} \quad 2\xi y + q \geq 1$$

$$p' = -\frac{1}{2\xi}(q - 1)$$
$$y' = a\cos(\omega\tau + \varphi) + b - 1$$
$$q' = -\frac{1}{2\xi}(q - 1)$$
$$v' = y + \frac{1}{2\xi}(q - 1)$$

Parametric Studies



Bifurcation Analysis



$a = 0.3$

$\xi = 0.05$

$\omega = 0.1$

$b = 0.15$

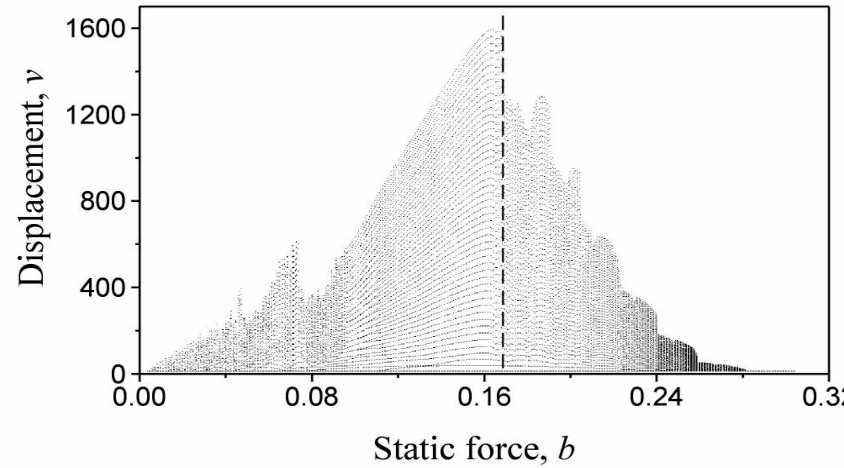
$\xi = 0.05$

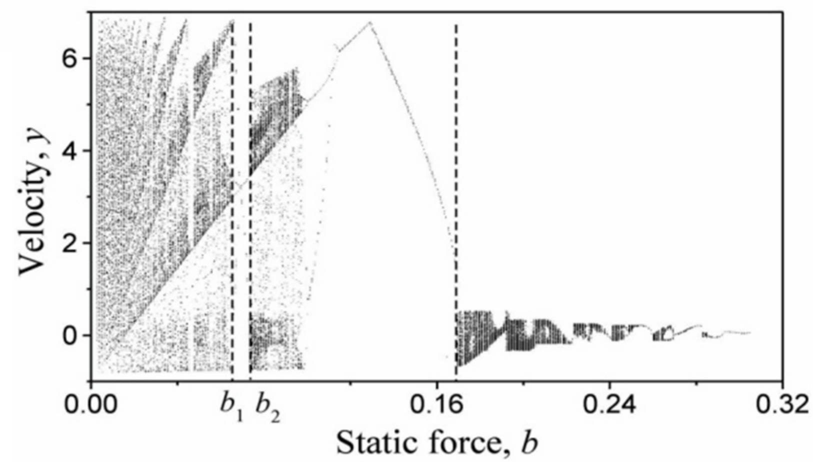
$\omega = 0.1$

$a = 0.3$

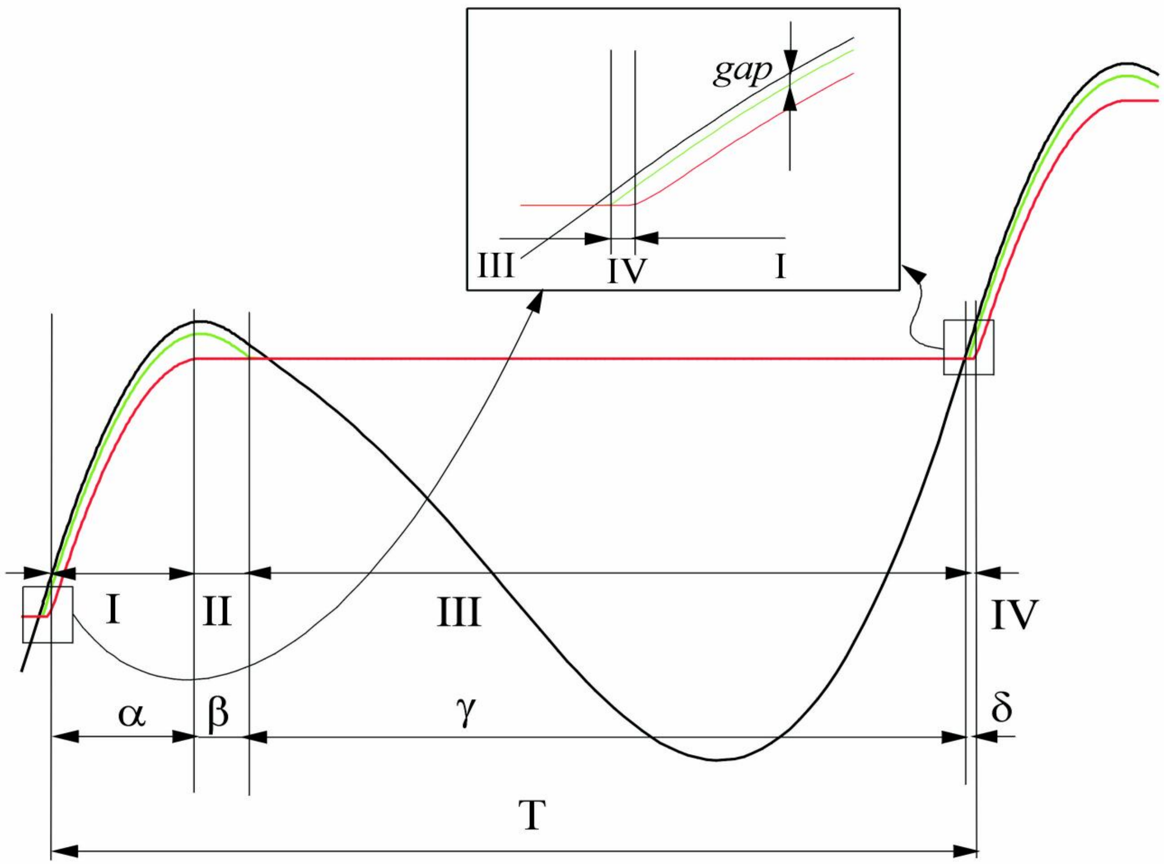
$b = 0.15$

$\xi = 0.05$

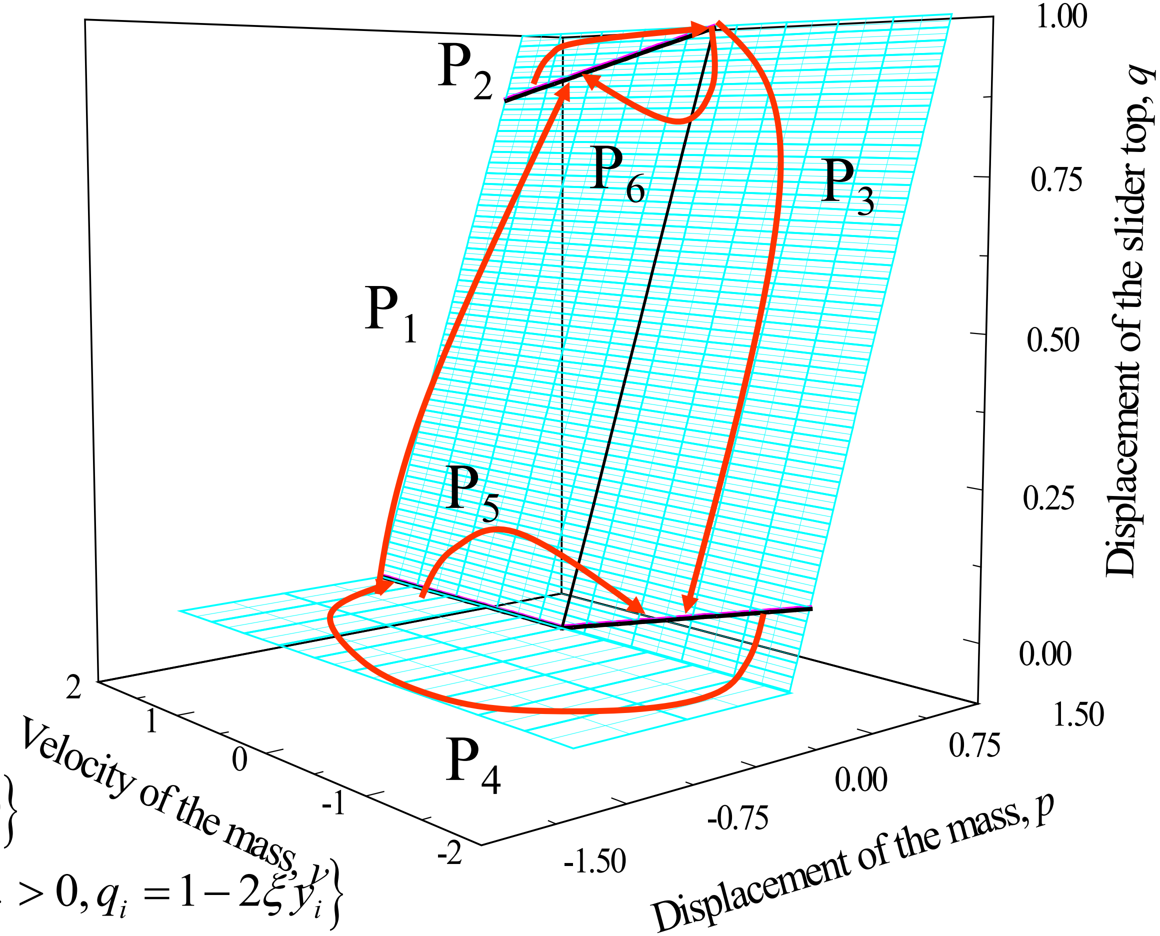




Reconstruction of Periodic Orbits



$$P_1: \quad \Sigma_1 \to \Sigma_2$$
$$P_2: \quad \Sigma_2 \to \Sigma_3$$
$$P_3: \quad \Sigma_3 \to \Sigma_4$$
$$P_4: \quad \Sigma_4 \to \Sigma_1$$
$$P_5: \quad \Sigma_1 \to \Sigma_4$$
$$P_6: \quad \Sigma_3 \to \Sigma_2$$



$$\Sigma_1 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = e, y_i > 0, q_i = 0 \right\}$$
$$\Sigma_2 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 + e - 2\xi y_i, y_i > 0, q_i = 1 - 2\xi y_i \right\}$$
$$\Sigma_3 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 + e - 2\xi y_i, y_i < 0, q_i = 1 - 2\xi y_i \right\}$$
$$\Sigma_4 = \left\{ (\tau_i, p_i, y_i, q_i) \middle| \, p_i = 1 - 2\xi y_i, y_i < 0, q_i = -2\xi y_i \right\}$$

Reduction to 1D

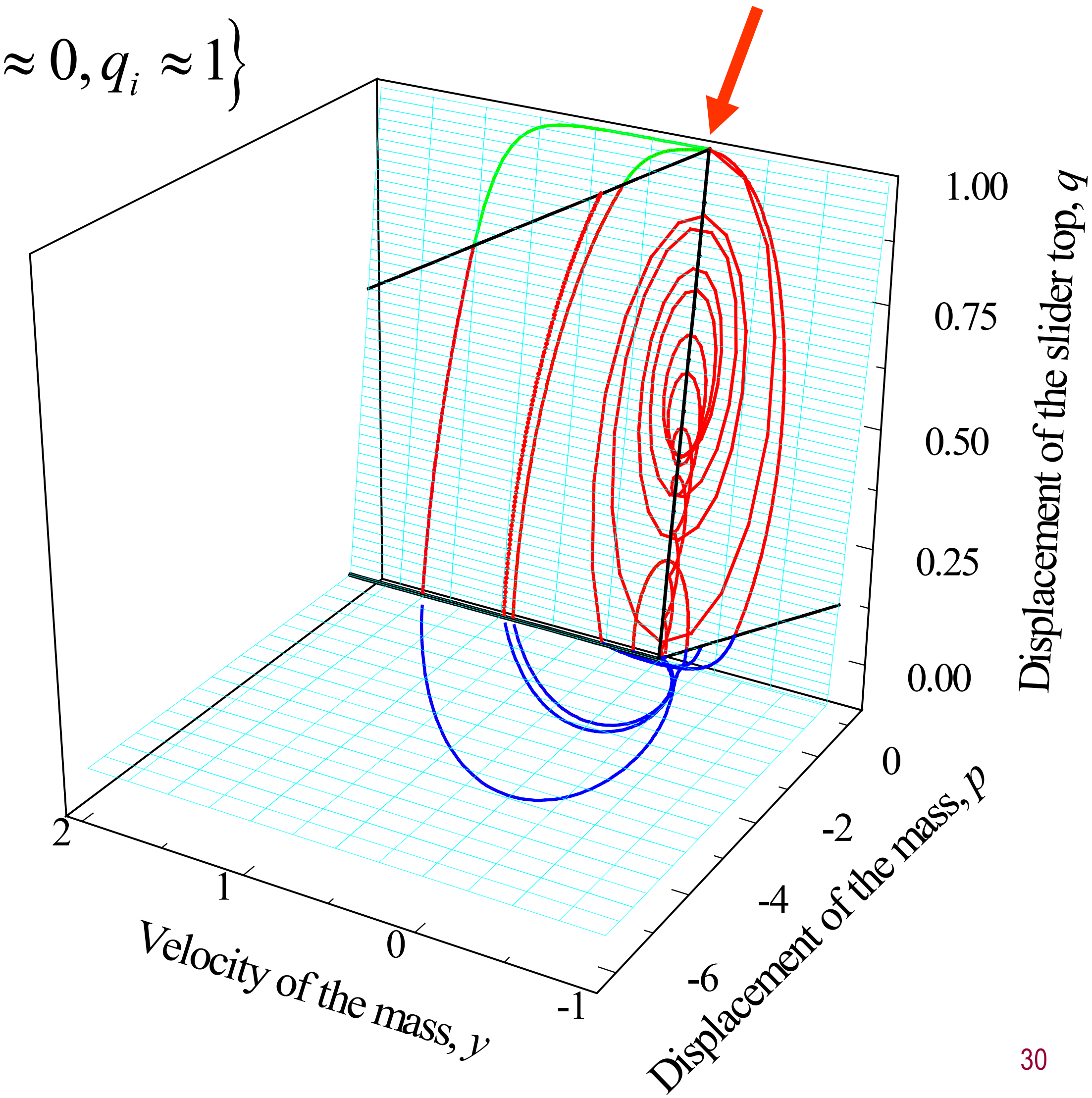
$$\tilde{\Sigma}_3 = \left\{ \left(\tau_i, p_i, y_i, q_i \right) \middle| \, p_i \approx 1 + e, y_i \approx 0, q_i \approx 1 \right\}$$

Approximate 1D map P

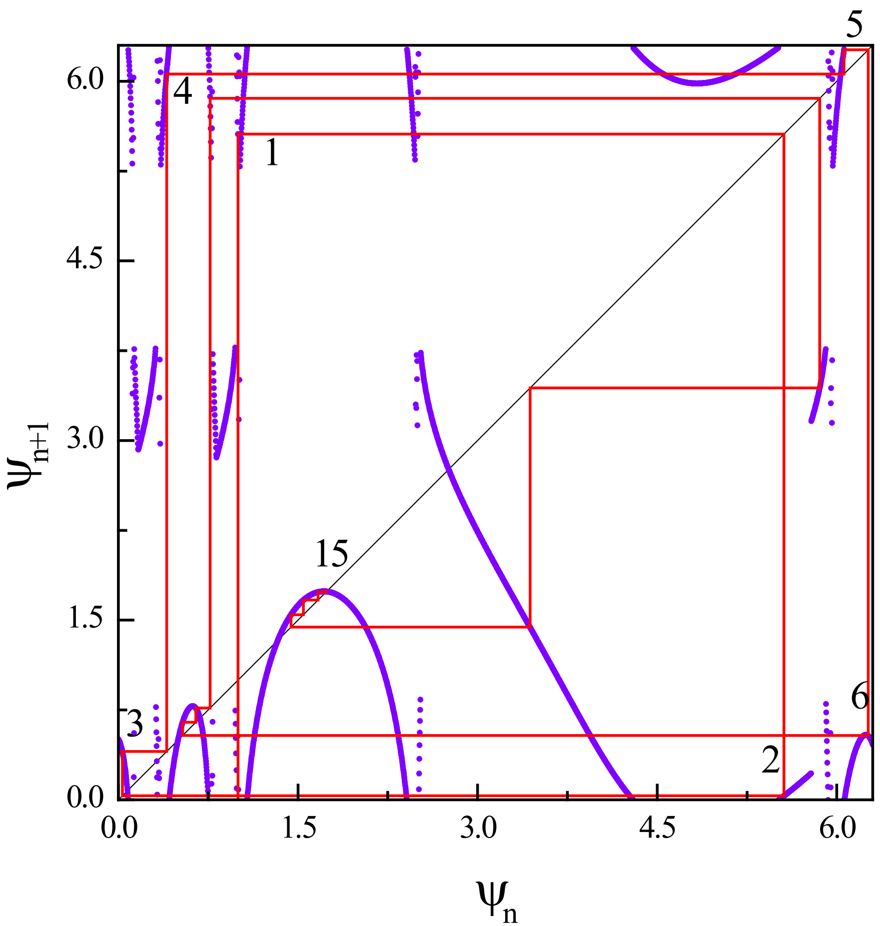
$$\mathrm{P} : \quad \tilde{\Sigma}_3 \to \tilde{\Sigma}_3$$

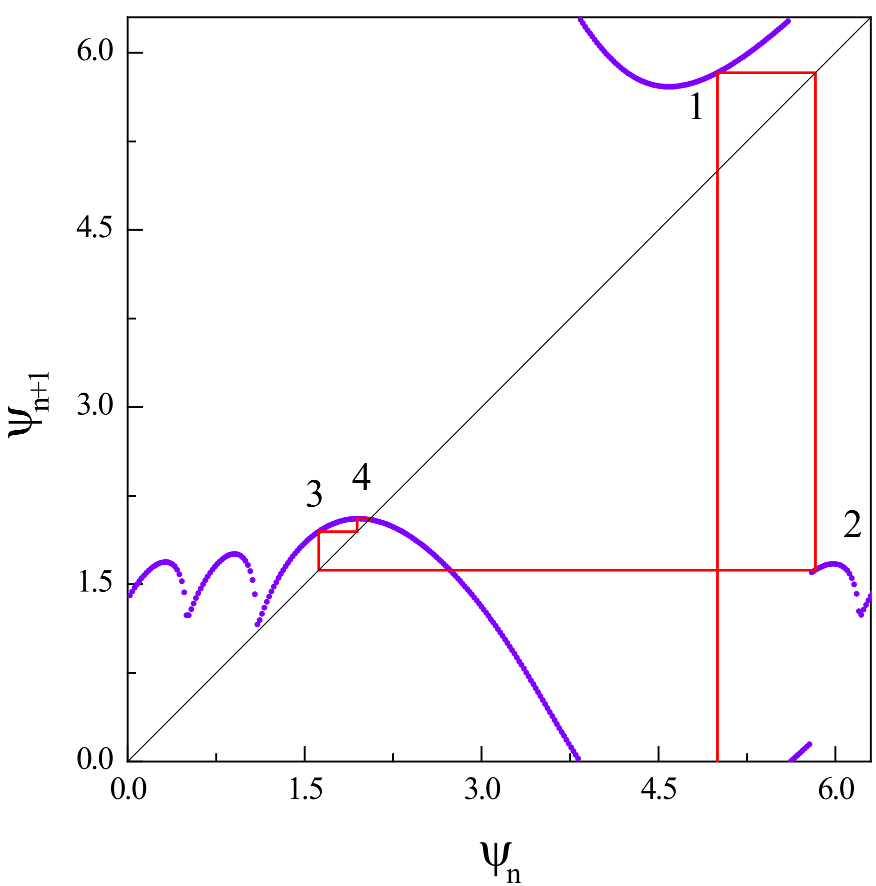
$$\tau_n \to \psi_n$$

$$\psi_{n+1} = \mathrm{P}\left(\psi_n\right)$$



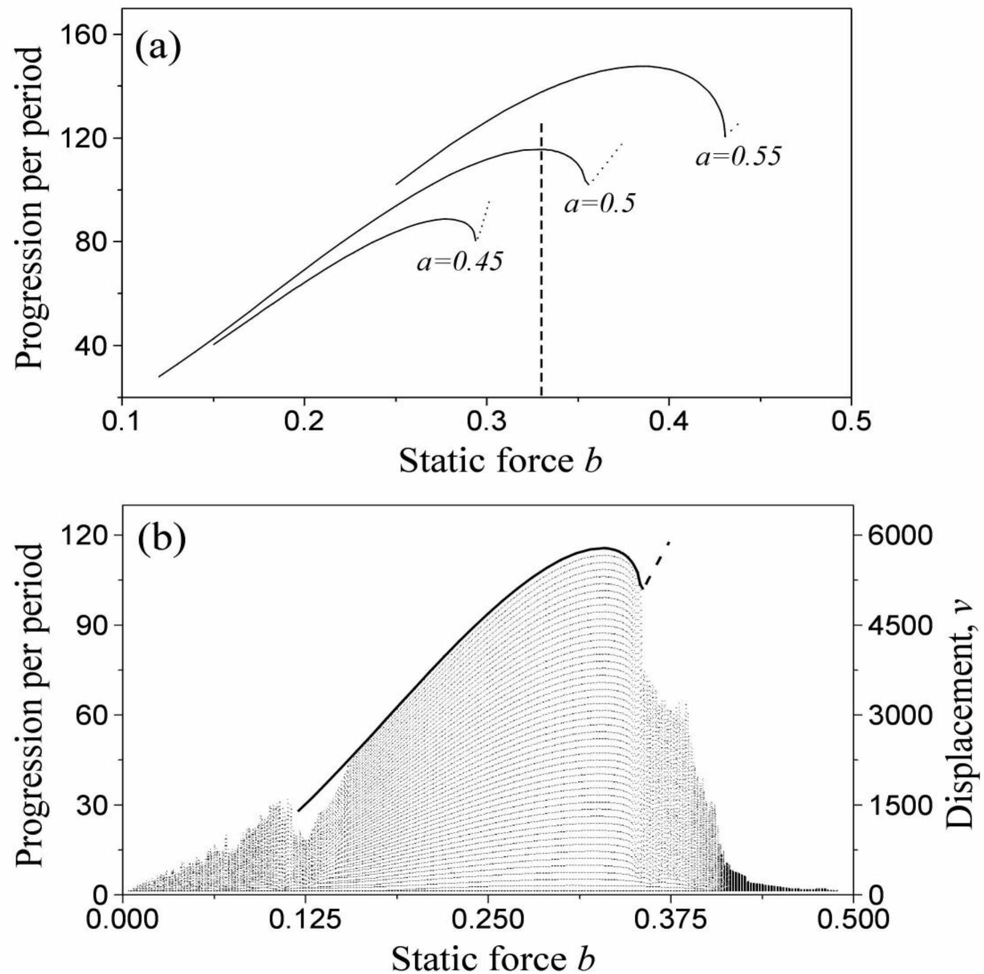
b=0.07 Period one motion *b=0.15*



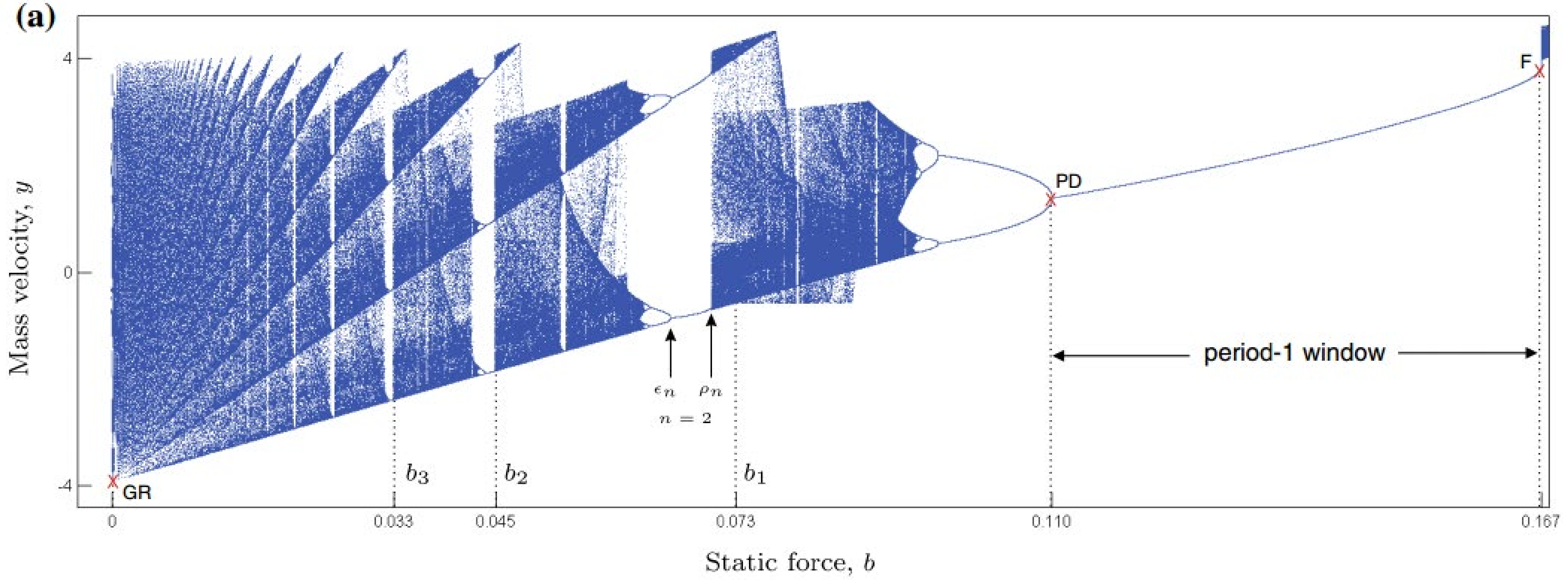


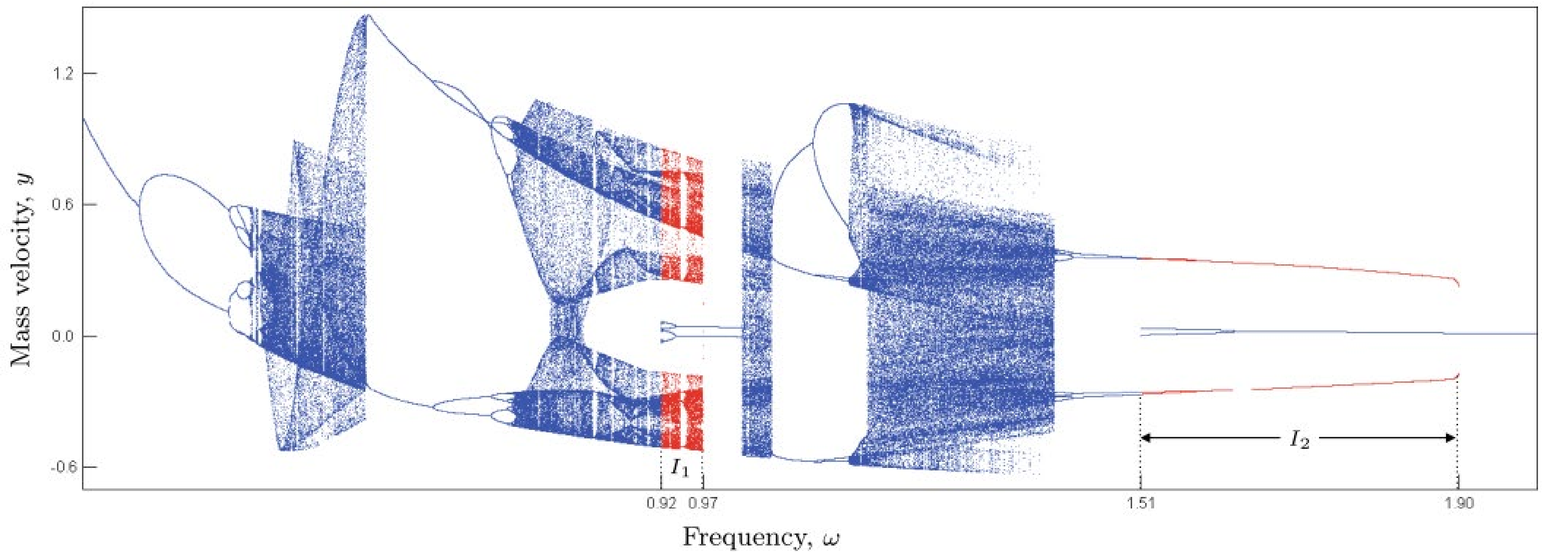
JSV **305** (2007) 58 - 77

Predictions via 1D Map

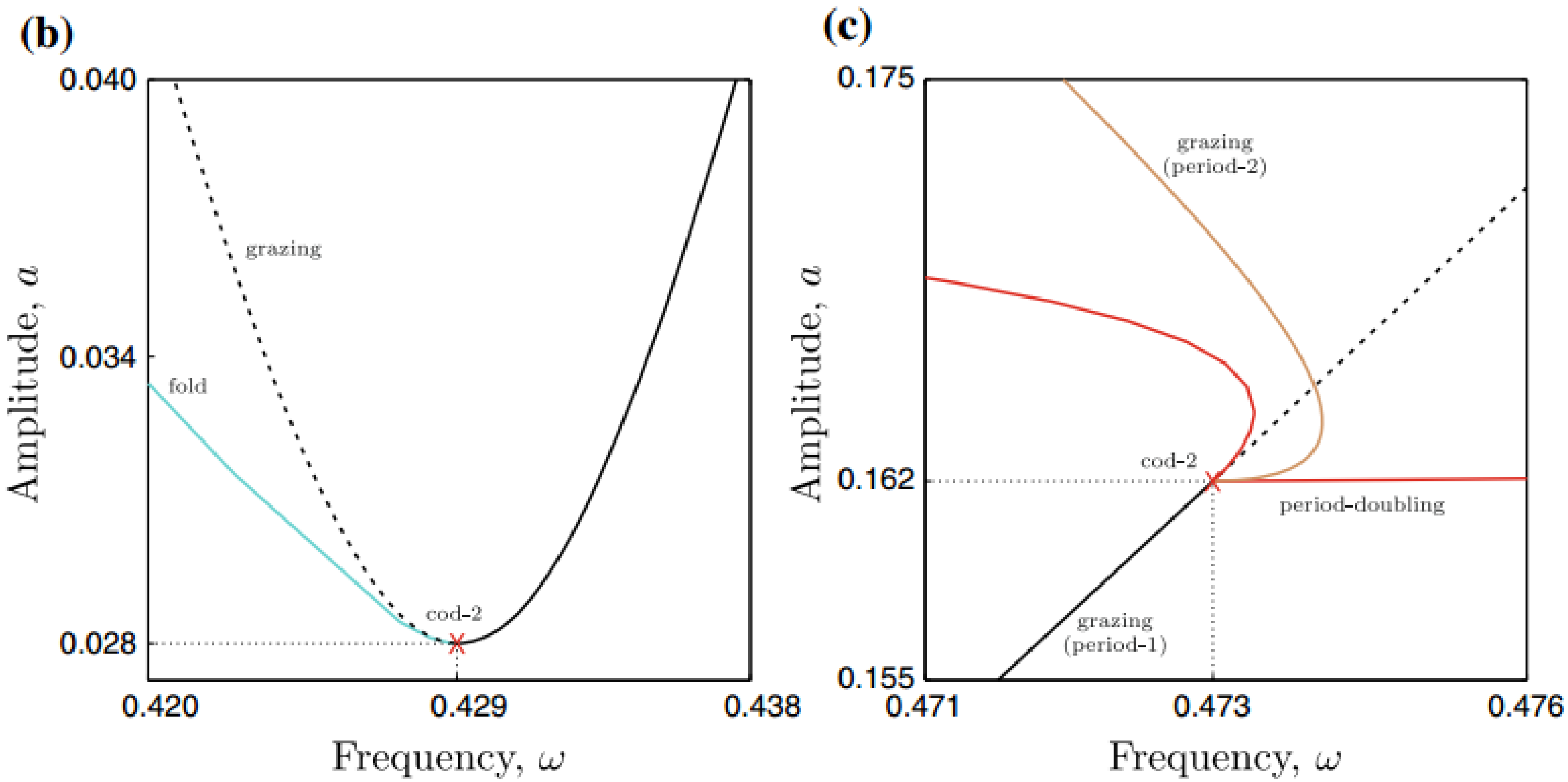


Drifting Impact Oscillator – Smooth Bifurcations



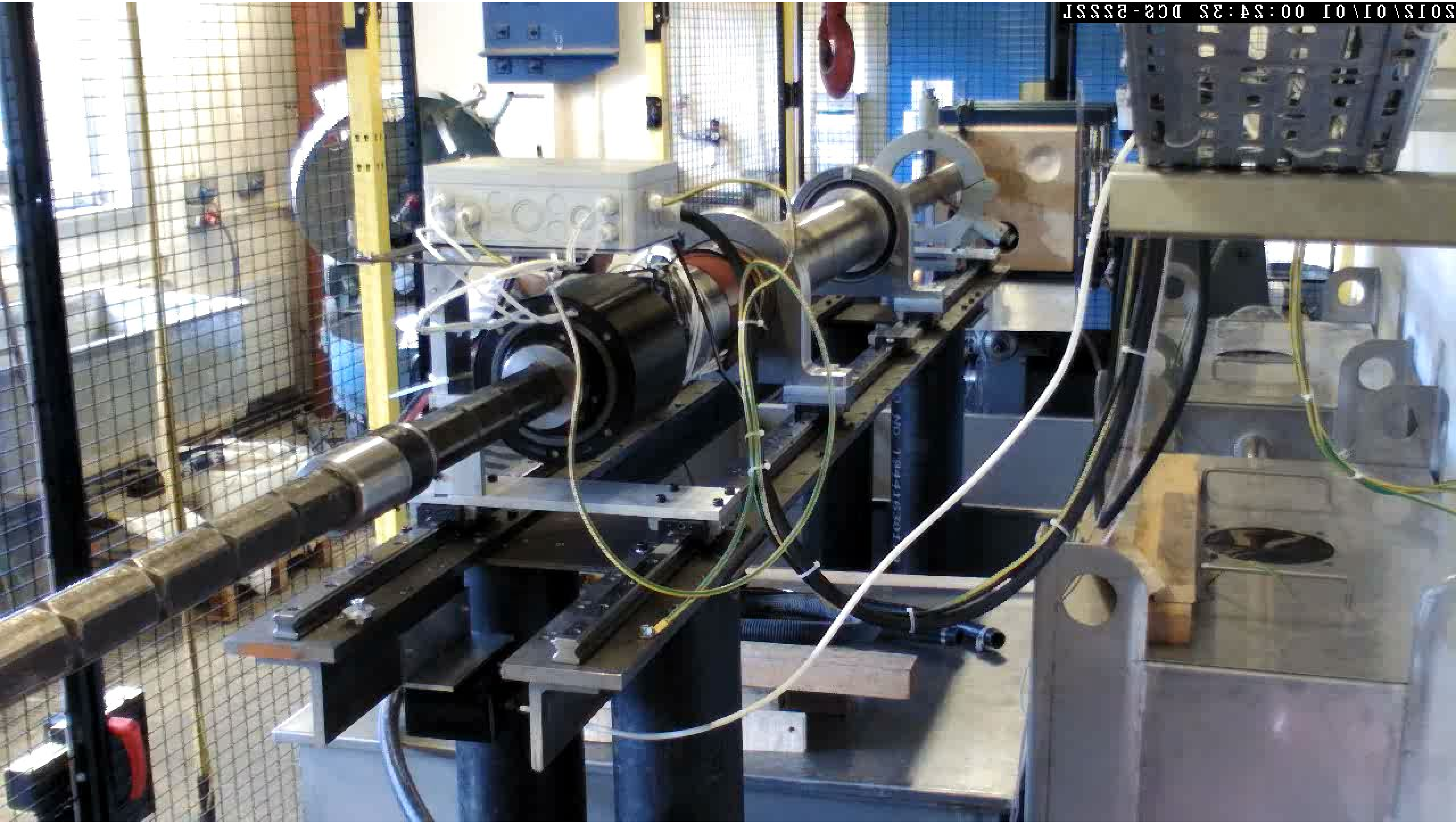


Grazing Induced Bifurcations

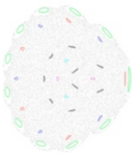


ND **77** (2014) 213 - 227

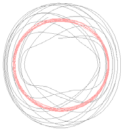
New Drilling Technology – Impacts Dynamics



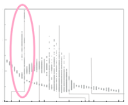
Presentation Outline



Motivation – Grazing in Impact Oscillators



Soft Impact Oscillator

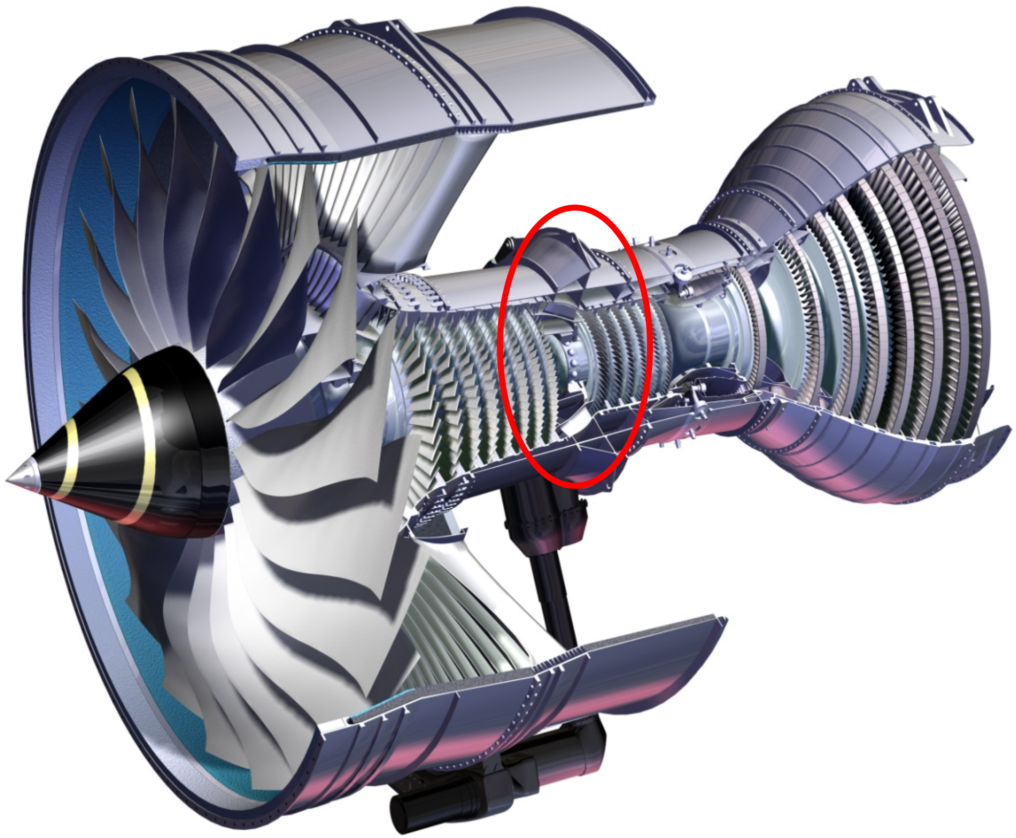


Drifting Impact Oscillator

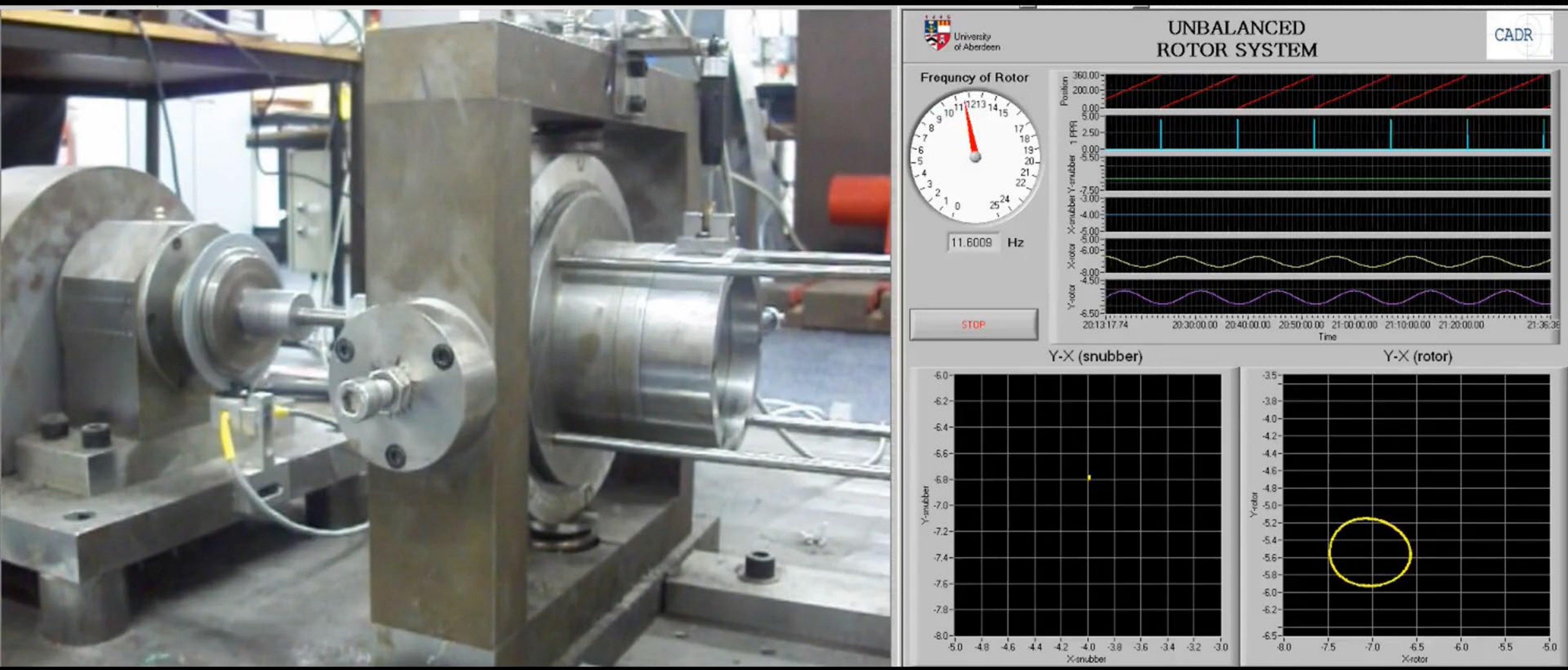
Rotor System

Closing Remarks

RR Jet Engine



Rolls-Royce Jet Engine



For $D < \gamma$ (no contact), we integrate the system

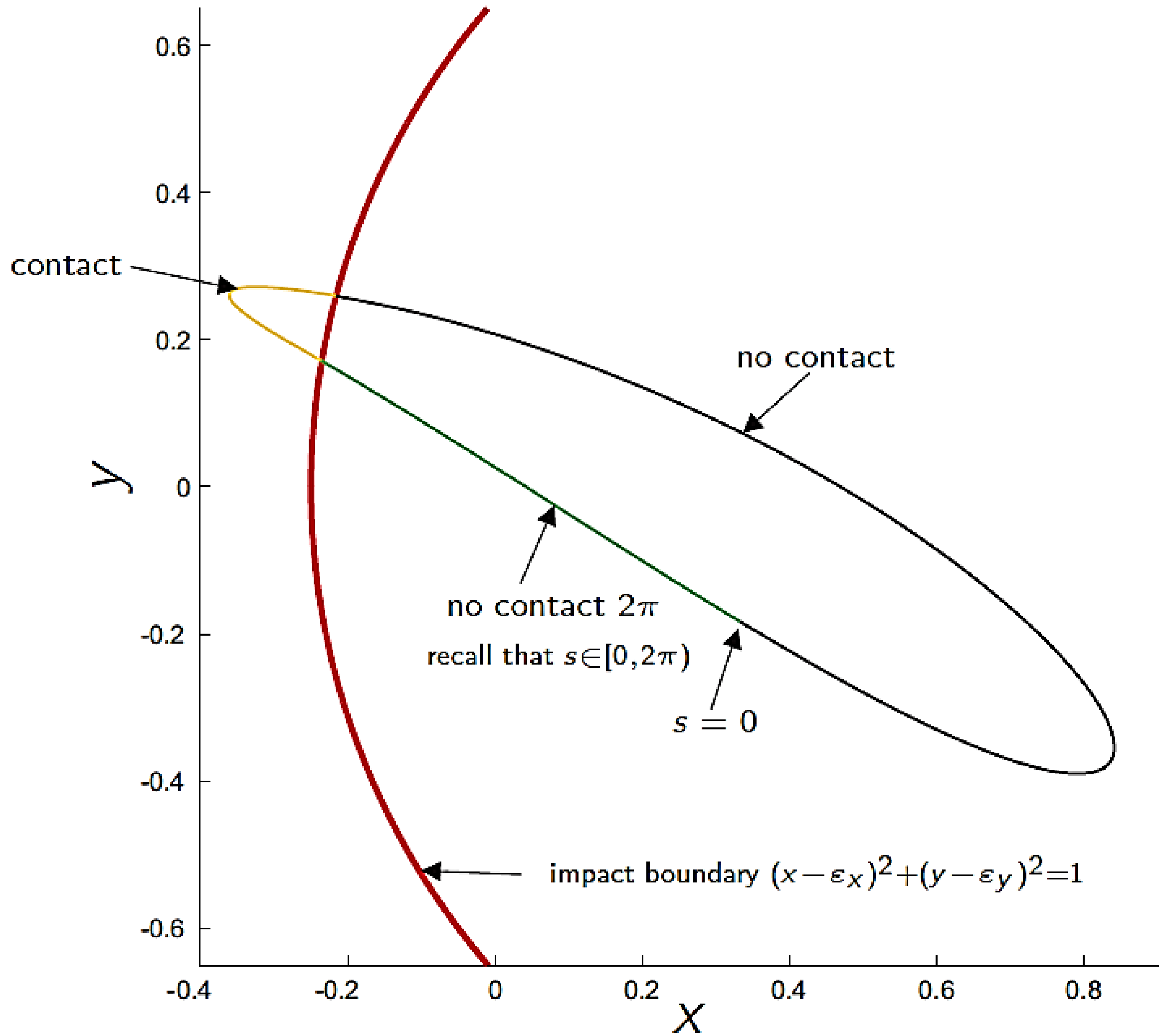
$$M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) = m\rho\omega^2\cos(\omega t + \theta_0),$$
$$M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) = m\rho\omega^2\sin(\omega t + \theta_0),$$
$$k_{sx}x_s + c_{sx}\dot{x}_s = 0, \quad k_{sy}y_s + c_{sy}\dot{y}_s = 0.$$

At the moment when $D = \gamma$ ($t = t_{\mathrm{imp}}$), an impact with the snubber has occurred and the motion of the system is obtained by integrating the equations

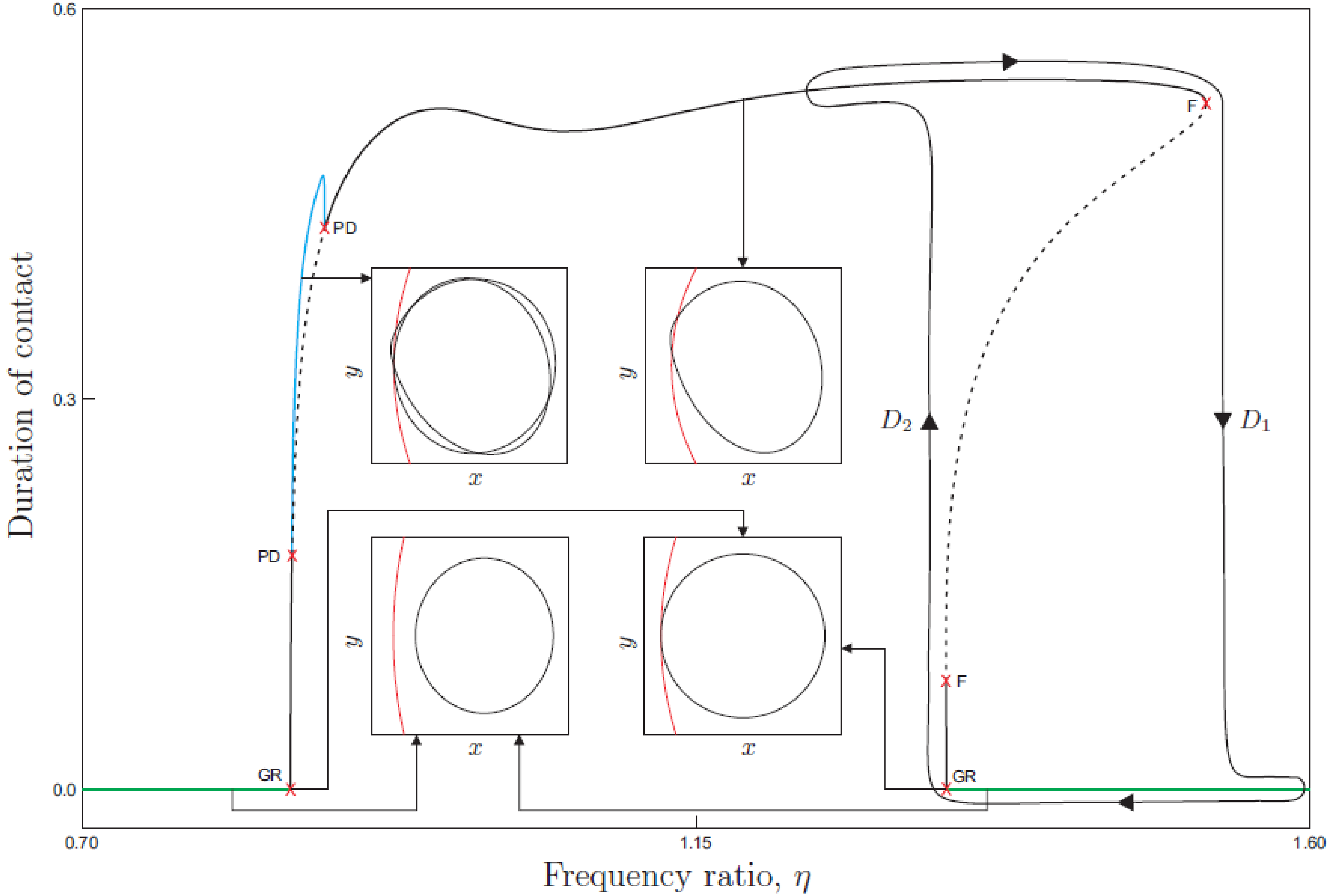
$$M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) + N_{sx} = m\rho\omega^2\cos(\omega t + \theta_0),$$
$$M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) + N_{sy} = m\rho\omega^2\sin(\omega t + \theta_0),$$
$$\dot{x}_s = \chi_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K), \quad \dot{y}_s = \Upsilon_t(x_r, y_r, \dot{x}_r, \dot{y}_r, x_s, y_s, K),$$

with the initial position of the snubber ring $x_s(t^+_{\mathrm{imp}}) = \chi(x_r(t_{\mathrm{imp}}), y_r(t_{\mathrm{imp}}), K, \gamma)$, $y_s(t^+_{\mathrm{imp}}) = \Upsilon(x_r(t_{\mathrm{imp}}), y_r(t_{\mathrm{imp}}), K, \gamma)$.

The system returns to the no contact regime when $\langle F_s, \widehat{N}\rangle \leq 0$.


contact
no contact
no contact 2π
recall that $s\in[0,2\pi)$
$s = 0$
impact boundary $(x-\varepsilon_X)^2+(y-\varepsilon_Y)^2=1$
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
y
-0.4
-0.2
0
0.2
0.4
0.6
0.8
X

Grazing Induced Bifurcations



Closing Remarks

- Grazing phenomena and the difference between grazing and **grazing induced bifurcations** were introduced.
- Very rich dynamical behaviour (e.g. invisible grazing) is **not** caused by **standard grazing bifurcations** but an interplay between smooth and non-smooth dynamics, where smooth bifurcations are triggered by grazing.
- Grazing [induced] bifurcations are definitely not innocent – they can be equally dangerous as useful.

Thank you for your attention!

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